

# Query languages for property graphs

## From RPQs to Cypher

NoSQL and New SQL course  
M2 LID, Université Gustave-Eiffel

2023 - 2024

version 2

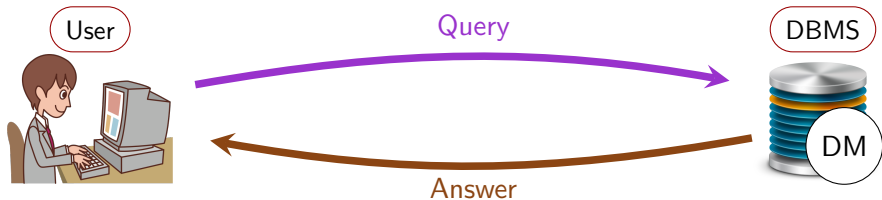
# Introduction

## Navigation

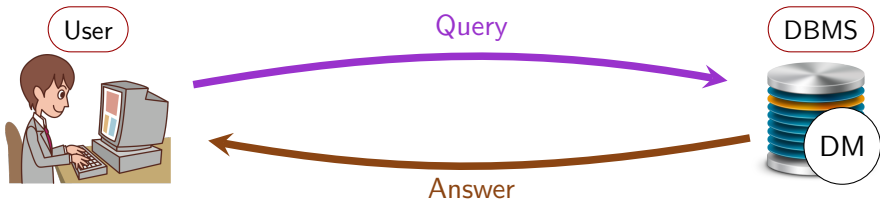
From any frame, the [page number](#) is a link to the navigable outline

## Term translations

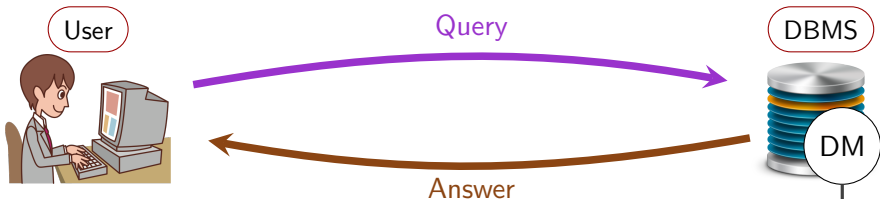
There is a [French/English lexicon](#) at the end.



- **DBMS** (DataBase Management System)



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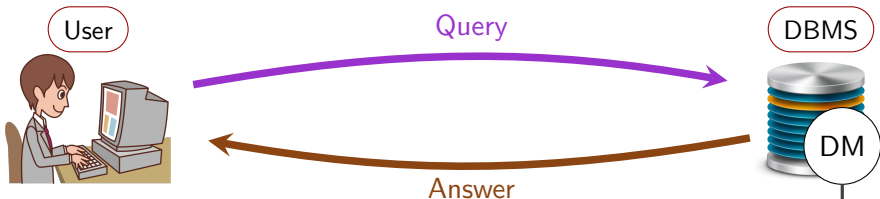
- **DM (Data Model)**

- “How is the data structured?” “What data is representable?”
- Ex: Relations (SQL), Trees (XML, JSON), Graphs (PGs, RDF),

- **DBMS** (DataBase Management System)

- **Query language**

- *“What can the user write?”*



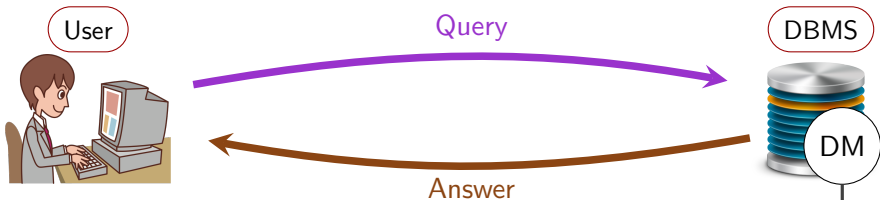
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- **DBMS** (DataBase Management System)

- **Query language**

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- **Semantics**

- *“What does the query mean?” “What is the correct answer?”*
- Ex: Set semantics (duplicate elimination)

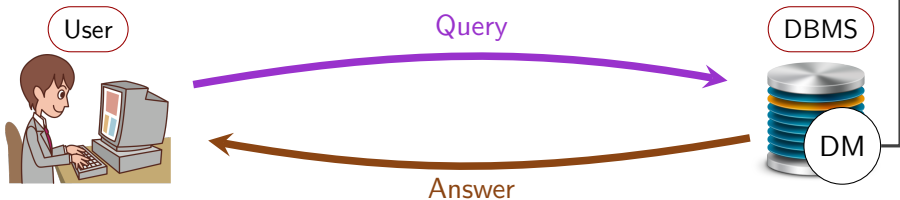
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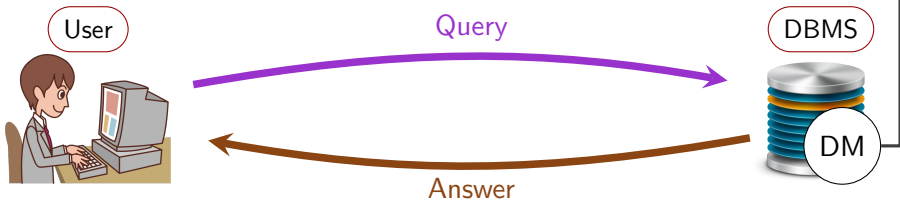
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- The **data model** is **Property Graph (PG)**



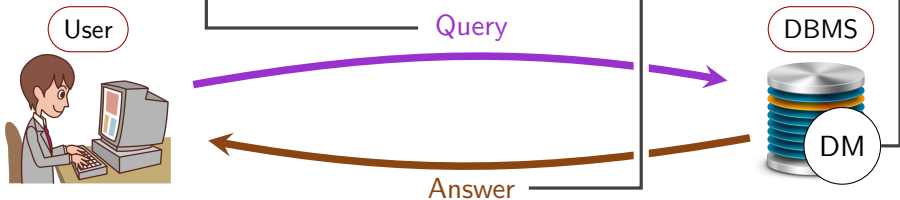
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- The **data model** is **Property Graph (PG)**
- The **DBMS** we will use (**Neo4j**) implements this DM



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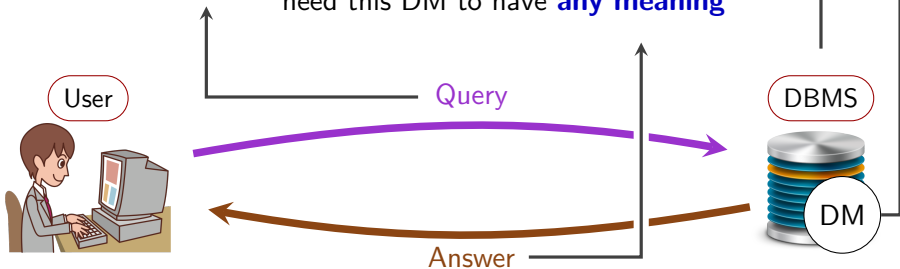
- The **data model** is **Property Graph (PG)**
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- The **query languages** we consider (**Cypher**, GQL, etc.) need this DM to have **any meaning**



## This segment is about **query languages for property graphs**

### In part II:

- The **data model** is **Property Graph (PG)**
- The **DBMS** we will use (**Neo4j**) implements this DM
- The **query languages** we consider (**Cypher**, GQL, etc.) need this DM to have **any meaning**



# Popularity of Graph DBMS's (1)

Vast majority of DBMS's are relational, not graph

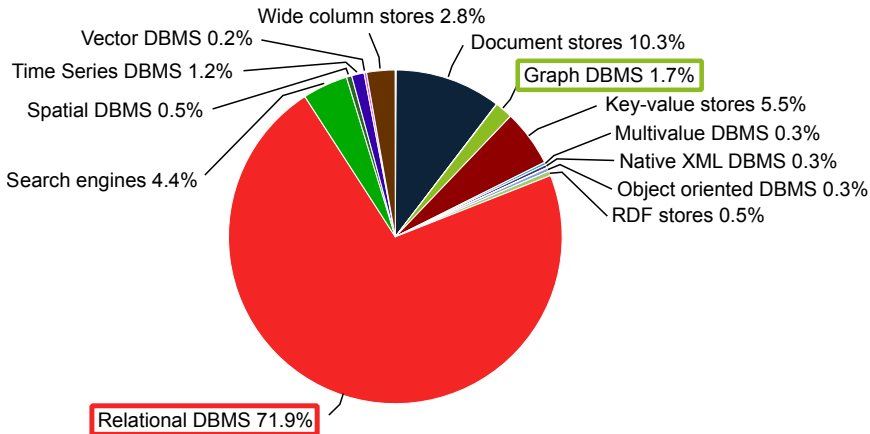


Figure and data from [db-engines.com](https://db-engines.com), August 2023

# Popularity of Graph DBMS's (2)

Graph DBMS's has grown in popularity for ten years  
Relational DBMS's continued their slow decline

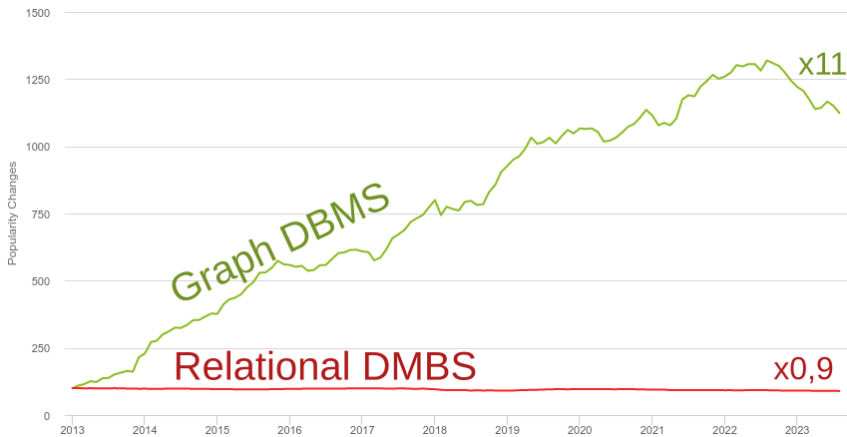
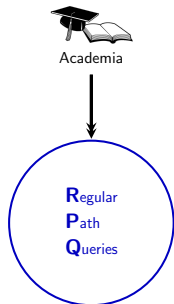
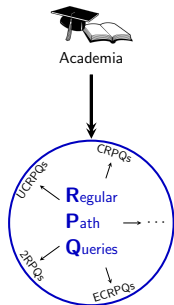


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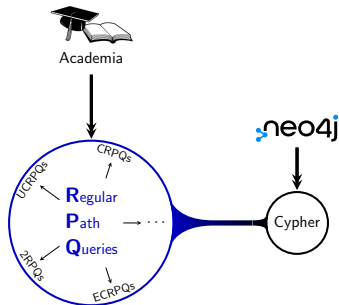


Late 1980's – RPQs are invented

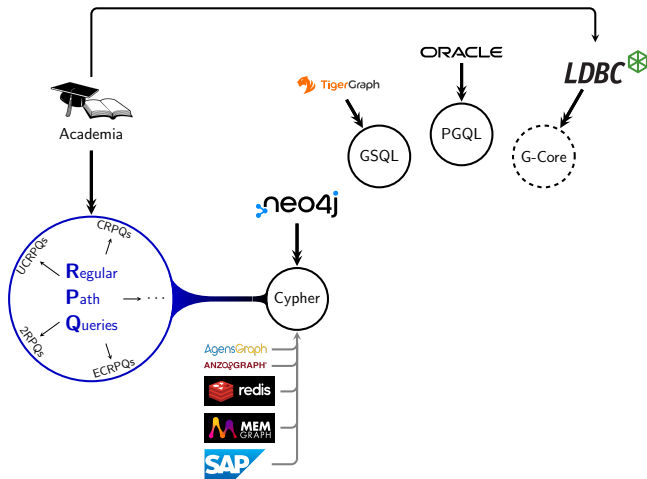


Since 1990's – RPQs are studied and extended in academia

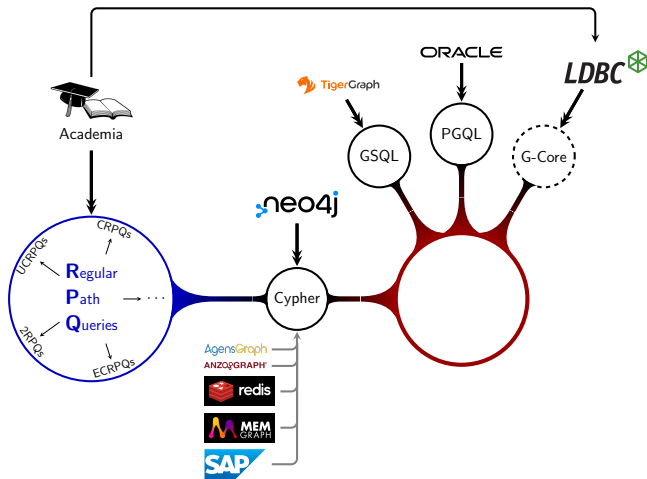




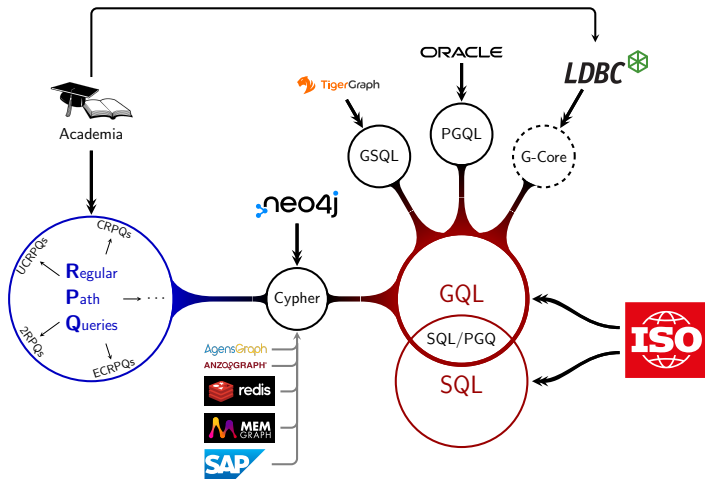
2011 – The query language Cypher is released with the DBMS Neo4j



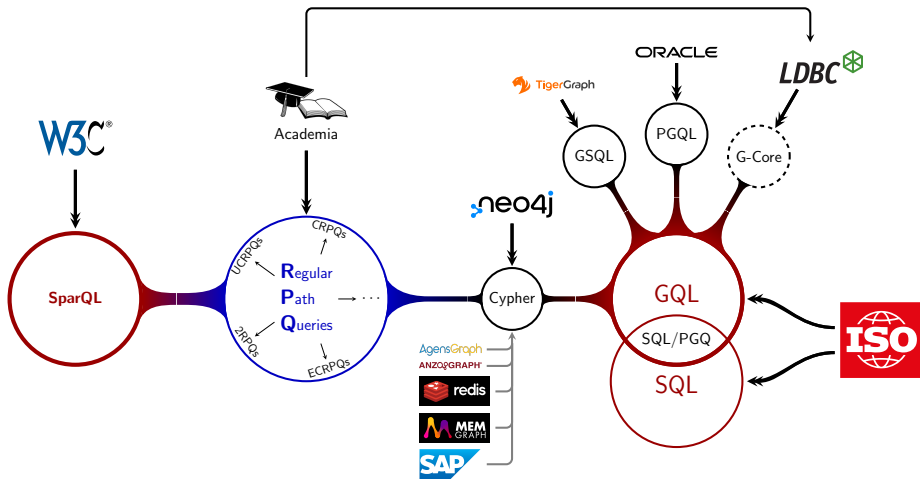
Mid 2010's – Cypher is successful and new graph DBMS's appear. Some use Cypher, some come with their own query language.



Late 2010's – Idea to merge existing languages for interoperability



2023 – SQL/PGQ support for querying PG's in SQL  
2024? – GQL, standard query language for PG's



Side note: In SPARQL, the standard language for the RDF DM, features *Property paths* which are also based on RPQ's.

## Part I: Theoretical Foundations

- Data model: Graphs
- Query language: RPQs

## Part II: A practical application

- Data model: Property graphs
- Query language: Cypher

# **Part I: Theoretical foundations**

A **set** is a finite or infinite collection of elements such that:

- order does not matter
- duplicates do not matter

Example sets:

- $\{1\} = \{1, 1\}$
- $\{4, 8, 15, 16, 23, 42\}$   
 $= \{8, 4, 23, 15, 42, 16\}$
- $\{(4, 15), (16, 42)\}$
- $\mathbb{N} = \{0, 1, 2, \dots\}$
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The **union** of two sets  $A$  and  $B$  is the set of elements which are in  $A$ , in  $B$ , or in both  $A$  and  $B$ .

$$\{1, 3\} \cup \{2, 1\} = \{1, 3, 2\}$$

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The **Cartesian product** of two sets  $A$  and  $B$  is the set of all pairs  $(x, y)$  for  $x \in A$  and  $y \in B$

$$\{1, 2, 3\} \times \{a, b\} = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$$

Part I: Theoretical foundations

# 1. **Data model: labeled graphs**

## Example

A graph consists of ...

- Vertices
- Edges
- Edge labels

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①

②

③

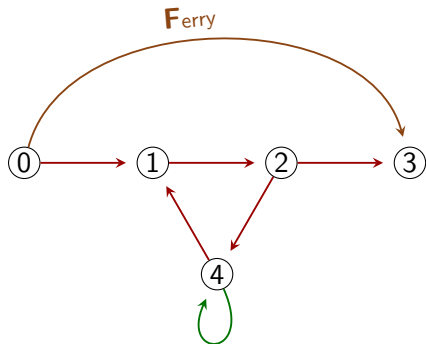
④

⑤

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A graph consists of ...

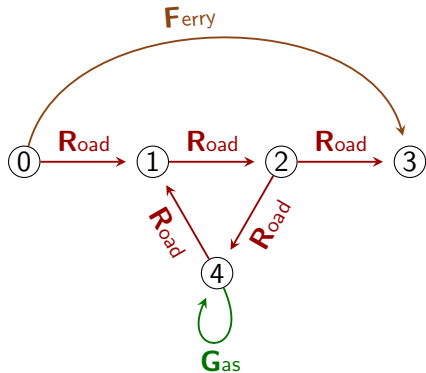
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# Our data model : (Labeled) graphs (2)

## Formalisation

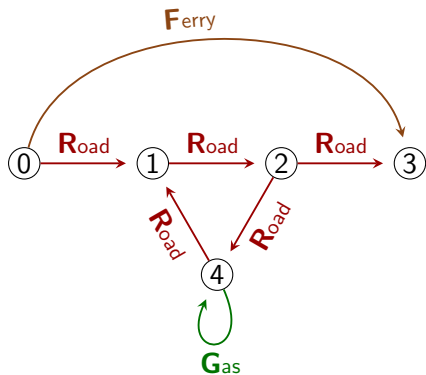
### Definition

A labeled graph is a triplet  $(V, L, E)$

- $V$  is a finite set of **vertices**
- $L$  is a finite set of **labels**
- $E \subseteq V \times L \times V$  is a finite set of **edges**

### Formal representation of $G$

- $V = \{0, 1, 2, 3, 4\}$
- $L = \{\mathbf{R}, \mathbf{F}, \mathbf{G}\}$
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Example graph  $G$

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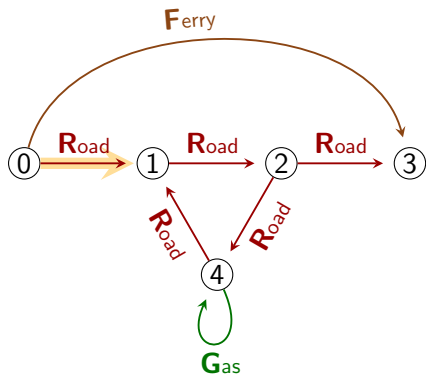
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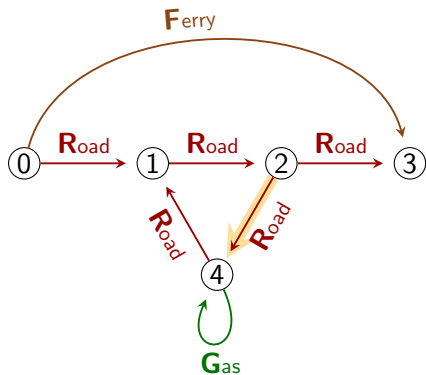
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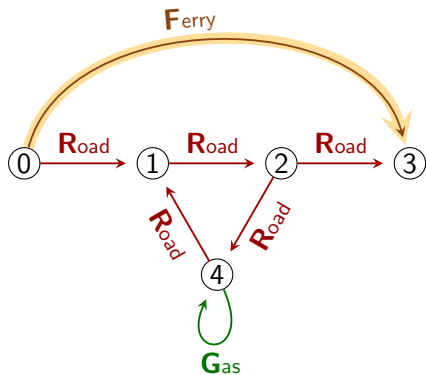
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Example graph  $G$

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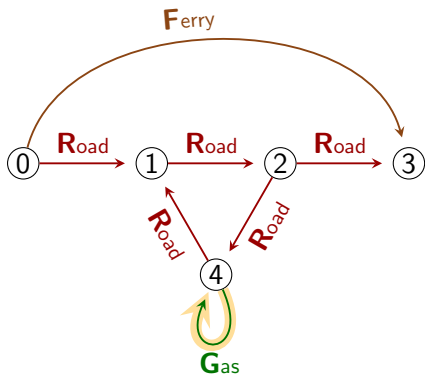
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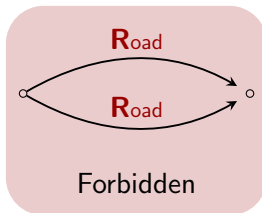
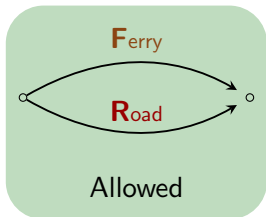
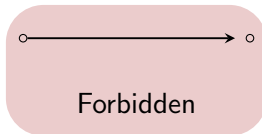
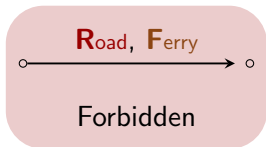


Example graph  $G$

# Limits to the graph data model (1)

Our graphs are single-labeled and single-edge

- Each edge has exactly one label.
- There cannot be two identical edges.



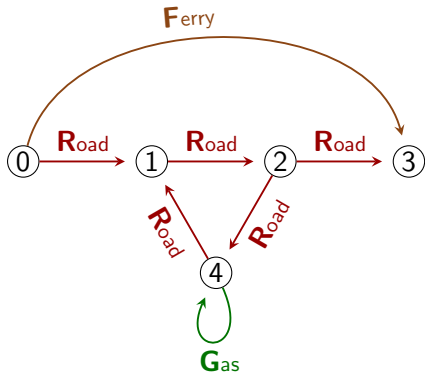
The graph DM is about topology, not data

- We encode the existence of entities and of relations between entities  
Ex: cities, roads
- We don't encode specific data of an entity or relation  
Ex: names, distances

## Examples

Our model cannot encode that

- the road from 0 to 1 is 2km long
- the gas price is 2€ in vertex 4



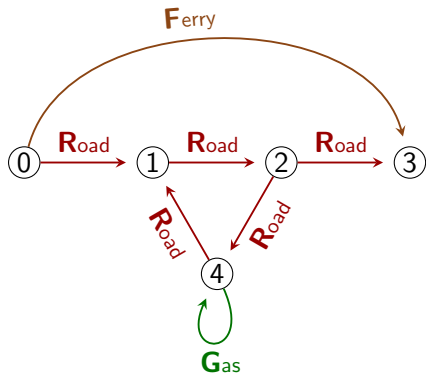
Part I: Theoretical foundations

## **2. Graph DM vs Relational DM**



# Translation: Graph to Tables (1)

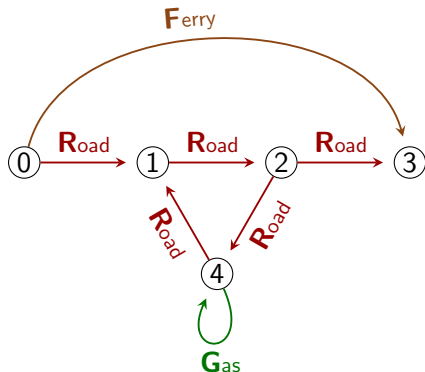
Can a graph be stored in tables?



# Translation: Graph to Tables (1)

Example – One **Vertex** table with one row per vertex in the graph

<b>Vertex</b>	<b>Road</b>	
<u>id</u>	<u>#src</u>	<u>#tgt</u>
0	0	1
1	1	2
2	2	3
3	2	4
4	4	1



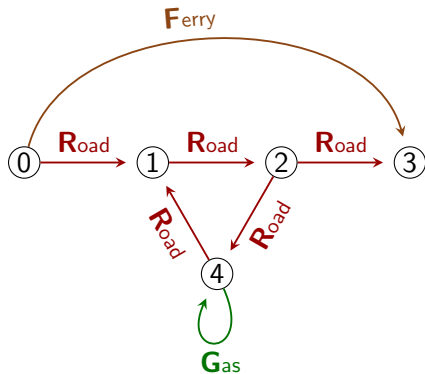
<b>Ferry</b>	
<u>#src</u>	<u>#tgt</u>
0	3

<b>Gas</b>	
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4	4

# Translation: Graph to Tables (1)

Example – One table for each different label in the graph

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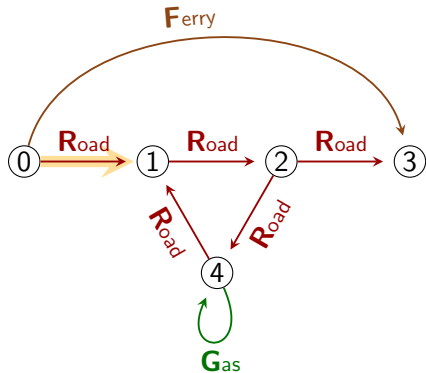
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# Translation: Graph to Tables (1)

Example – For each edge  $(i, \ell, j)$  in the graph add row  $(i, j)$  in table  $\ell$

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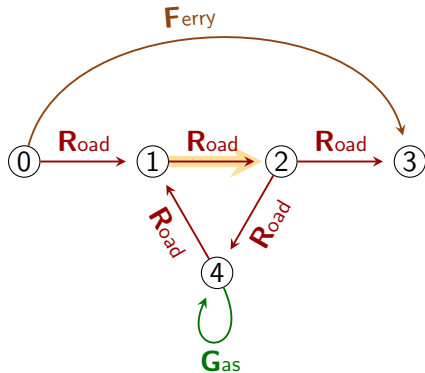
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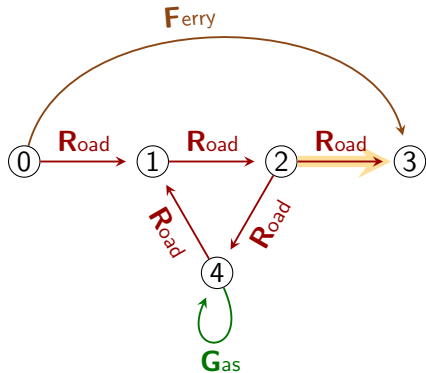
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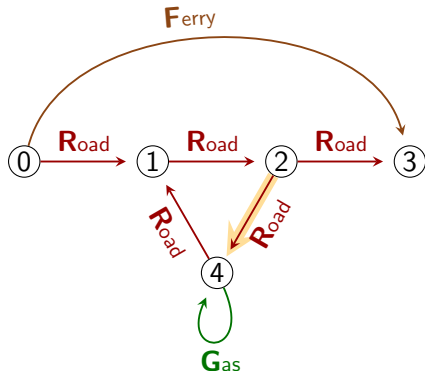
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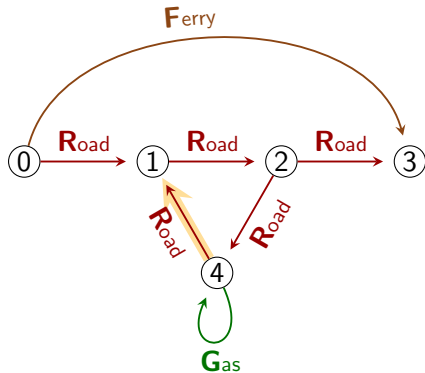
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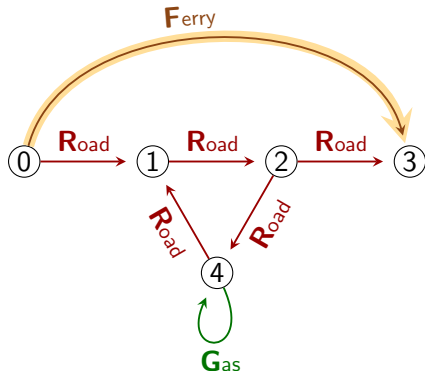
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3	2	4
4	4	1



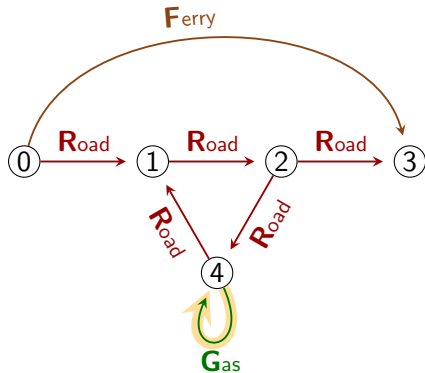
<b>Ferry</b>	
<u>#src</u>	<u>#tgt</u>
0	3

<b>Gas</b>	
<u>#src</u>	<u>#tgt</u>
4	4

# Translation: Graph to Tables (1)

Example – For each edge  $(i, \ell, j)$  in the graph add row  $(i, j)$  in table  $\ell$

<u>Vertex</u>	<u>Road</u>	
<u>id</u>	<u>#src</u>	<u>#tgt</u>
0	0	1
1	1	2
2	2	3
3	2	4
4	4	1



<u>Ferry</u>	
<u>#src</u>	<u>#tgt</u>
0	3

<u>Gas</u>	
<u>#src</u>	<u>#tgt</u>
4	4

## Principles of the translation

We start from a graph  $(V, L, E)$

Since  $V$  is finite we may enumerate it:  $V = \{v_1, \dots, v_n\}$

### One table for vertices

<b>Vertex</b>
<u>id</u>
0
1
$\vdots$
$n$

### One table per label $\ell$ in $L$

$\ell$	
<u>#src</u>	<u>#tgt</u>
$\vdots$	$\vdots$
$i$	$j$
$\vdots$	$\vdots$

Table  $\ell$  contains  $(i, j)$

$$\iff (v_i, \ell, v_j) \in E$$

The relational database we want to encode in a graph

## Client

<u>login</u>	address
"Alice"	"Wonderland"
"Bob"	"124 Conch St."
"Eve"	"WALL-E's Truck"

## Product

<u>name</u>	price
"Pocket Watch"	42
"Rabbit"	0
"Pants"	8
"Broom&Bucket"	4

\_\_\_ : part of primary key

# Translation: Tables to Graph (1)

The relational database we want to encode in a graph

## Client

<u>login</u>	address
"Alice"	"Wonderland"
"Bob"	"124 Conch St."
"Eve"	"WALL-E's Truck"

## Order

<u>id</u>	#buyer	date
0	"Alice"	01-11-1865
1	"Bob"	07-07-2022
2	"Bob"	07-11-2023

## Product

<u>name</u>	price
"Pocket Watch"	42
"Rabbit"	0
"Pants"	8
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    : part of primary key

#     : foreign keys

# Translation: Tables to Graph (1)

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## Product

<u>name</u>	price
"Pocket Watch"	42
"Rabbit"	0
"Pants"	8
"Broom&Bucket"	4

## Contains

<u>#order</u>	#product
0	"Rabbit"
0	"Pocket Watch"
1	"Pants"
2	"Pants"

    : part of primary key


# : foreign keys

### Condition for the translation to be possible

Relational DB consists of tables  $T_1, \dots, T_k$ .

Each table  $T_i$

- has a primary key, consisting of several columns
- has columns that are foreign keys


 Foreign keys can be part of the primary key.

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Relational DB consists of tables  $T_1, \dots, T_k$ .

Each table  $T_i$

- has a primary key, consisting of several columns
- has columns that are foreign keys

 Foreign keys can be part of the primary key.

## Conditions for the database to be encodable in a graph

Each table  $T_i$  satisfies one of the following.

- 0 Zero foreign key is part of the primary key of  $T_i$ .
- 1 One foreign key is part of the primary key of  $T_i$ .
- 2 Two foreign keys are part of the primary key of  $T_i$ , and no other column is part of the primary key.



# Translation: Tables to Graph (4)

The relational database we want to encode in a graph

## Client

<u>login</u>	address
"Alice"	"Wonderland"
"Bob"	"124 Conch St."
"Eve"	"WALL-E's Truck"

## Order

<u>id</u>	#buyer	date
0	"Alice"	01-11-1865
1	"Bob"	07-07-2022
2	"Bob"	07-11-2023

## Product

<u>name</u>	price
"Pocket Watch"	42
"Rabbit"	0
"Pants"	8
"Broom&Bucket"	4

## Contains

# <u>order</u>	# <u>product</u>
0	"Rabbit"
0	"Pocket Watch"
1	"Pants"
2	"Pants"

\_\_\_ : part of primary key

# : foreign keys

- **Client, Product and Order** satisfy **0**
- **Contains** satisfies **2**

# Translation: Tables to Graph (5)

One vertex per row in table satisfying 0 or 1



Client  
row 1

Order  
row 1

Product  
row 1



Client  
row 2

Order  
row 2

Product  
row 2



Client  
row 3

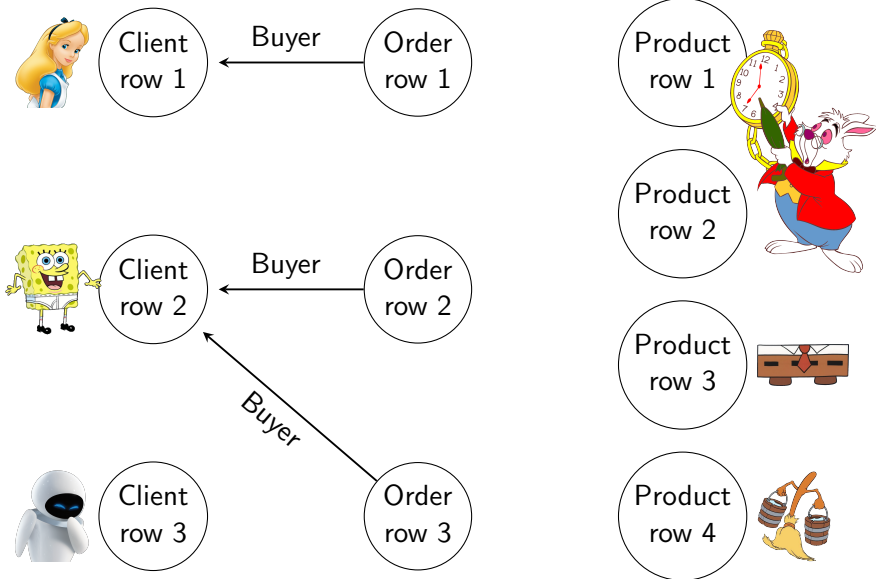
Order  
row 3

Product  
row 4



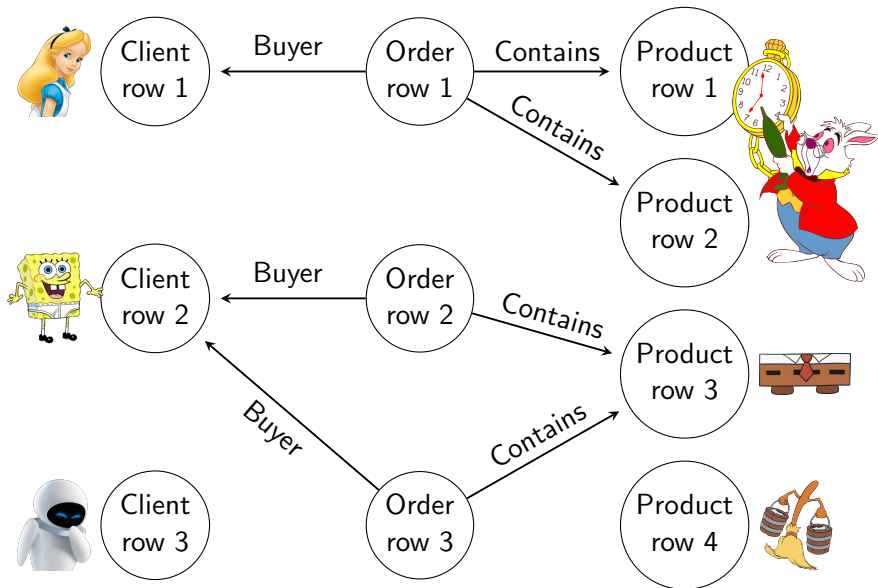
# Translation: Tables to Graph (5)

One edge per row and per foreign-key column in each table satisfying 0 or 1



# Translation: Tables to Graph (5)

One edge per row of tables satisfying **2**



## Takeaway

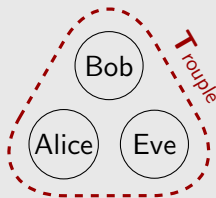
### Wedding is edgy...

<b>W</b> edding	
<u>#person1</u>	<u>#person2</u>
Alice	Bob
⋮	⋮



### ..but trouple is trouble

<b>T</b> rouple		
<u>#person1</u>	<u>#person2</u>	<u>#person3</u>
Alice	Bob	Eve
⋮	⋮	⋮



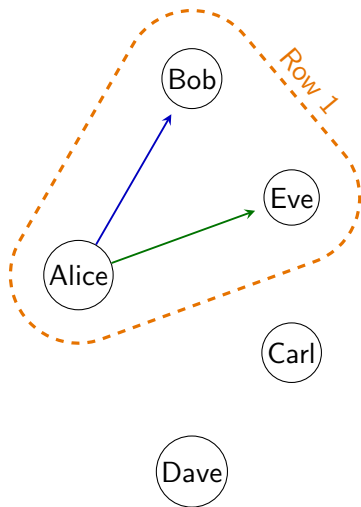
# How to encode non-binary relations in a graph (1)

The **wrong** ways: adding more edges

**Trouple**

<u>#pers1</u>	<u>#pers2</u>	<u>#pers3</u>
Alice	Bob	Eve
Alice	Carl	Dave

Let us try to add two edges per row of table **Trouple**.



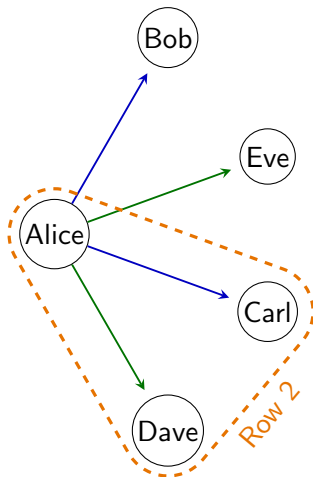
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The **wrong** ways: adding more edges

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# How to encode non-binary relations in a graph (1)

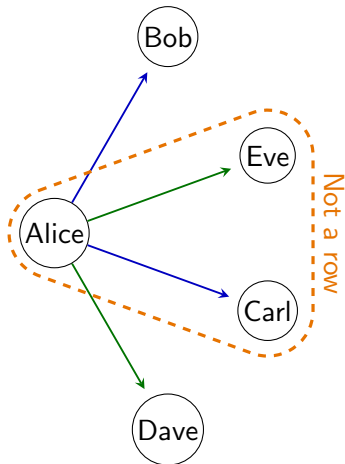
The **wrong** ways: adding more edges

**Trouple**

<u>#pers1</u>	<u>#pers2</u>	<u>#pers3</u>
Alice	Bob	Eve
Alice	Carl	Dave

Let us try to add two edges per row of table **Trouple**.

⚠ (Alice, Carl, Eve) is not a row of table **Trouple**





The **right** way : Reification

## Reification

- Literally, make into an object
- For us, transform into a vertex

The **right** way : Reification

## Reification

- Literally, make into an object
- For us, transform into a vertex

Troupe		
<u>#pers1</u>	<u>#pers2</u>	<u>#pers3</u>
Alice	Bob	Eve
Alice	Carl	Dave

Bob

Eve

Alice

Carl

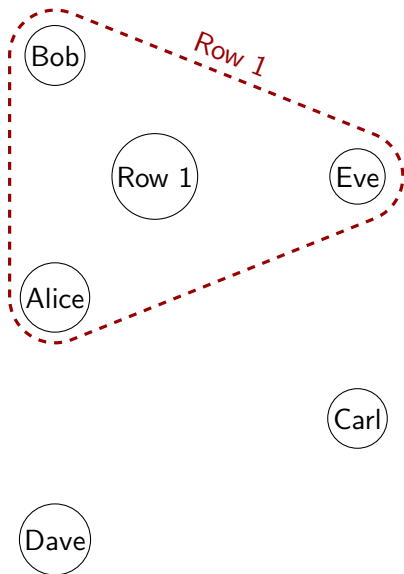
Dave

The **right** way : Reification

## Reification

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- For us, transform into a vertex

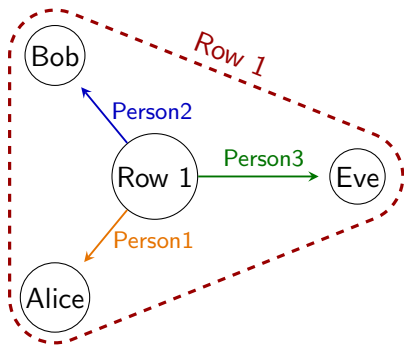
Troupe		
<u>#pers1</u>	<u>#pers2</u>	<u>#pers3</u>
Alice	Bob	Eve
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The **right** way : Reification

## Reification

- Literally, make into an object
- For us, transform into a vertex



## Trouple

#pers1

#pers2

#pers3

Alice

Bob

Eve

Alice

Carl

Dave

The **right** way : Reification

## Reification

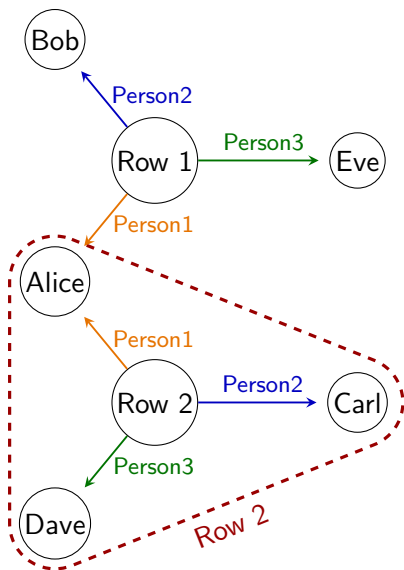
- Literally, make into an object
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### Trouple

#pers1 #pers2 #pers3

Alice Bob Eve

Alice Carl Dave

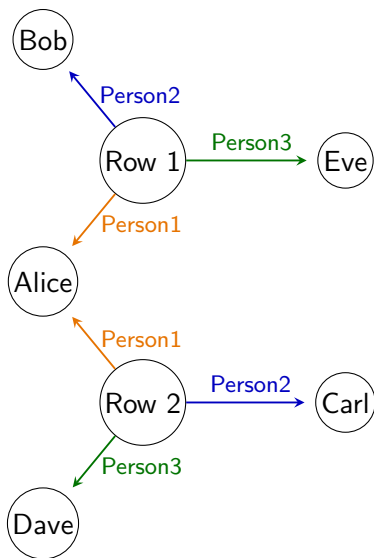


The **right** way : Reification

## Reification

- Literally, make into an object
- For us, transform into a vertex

Troupe		
#pers1	#pers2	#pers3
Alice	Bob	Eve
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Reification is **cheating**

Reification works...

- Reversible (one may reconstruct the **Trouble** table)
- Easy to generalize to any arity

...but, it is contrary to the spirit of graphs:

- The graph requires extra knowledge and maintenance:
  - Special vertices/edges/labels
  - Implicitly linked labels/edges (Person1/Person2/Person3)
  - Integrity constraints
- Query languages designed for graphs will not expect them

Part I: Theoretical foundations

### **3. Regular Path Queries**



A **letter** is a symbol coming from a finite set, the **alphabet**.

In our case, the alphabet is the label-set of the graph.

Examples:

- $\{\mathbf{R}, \mathbf{F}, \mathbf{G}\}$  is an alphabet
- $\mathbf{R}$  and  $\mathbf{G}$  are letters

A **word** is a finite sequence of letters

Examples words:

- $\mathbf{RGRR}$
- $\mathbf{R}$
- $\varepsilon$ , the empty word

A **language** is a finite or infinite set of words

Example languages:

- $\{\mathbf{R}, \mathbf{RG}\}$
- $\{\mathbf{R}, \mathbf{RR}, \mathbf{RRR}, \dots\}$
- The words with one  $\mathbf{G}$
- The words with a prime number of  $\mathbf{G}$

## Atoms

- Each letter is a regexp
- $\epsilon$  is a regexp

Ex:  $\epsilon$ , **R**, and **F** are regexps

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Ex:  $\varepsilon$ , **R**, and **F** are regexps

## Concatenation $\cdot$

**If**  $Q_1$  and  $Q_2$  are regexps

**Then**  $Q_1 \cdot Q_2$  is a regexp

Ex: **R**  $\cdot$  **R** and **G**  $\cdot$  **F** are regexps

**(R**  $\cdot$  **R)**  $\cdot$  **(G**  $\cdot$  **F)** is a regexp

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Ex: **R**  $\cdot$  **R** and **G**  $\cdot$  **F** are regexps  
**(R  $\cdot$  R)  $\cdot$  (G  $\cdot$  F)** is a regexp

## Disjunction $+$

**If**  $Q_1$  and  $Q_2$  are regexps  
**Then**  $Q_1 + Q_2$  is a regexp

Ex: **R**  $+$  **R** and **G**  $+$  **F** are regexps  
**(R  $\cdot$  R)  $+$  (G  $\cdot$  F)** is a regexp

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**(R  $\cdot$  R)  $+$  (G  $\cdot$  F)** is a regexp

## Kleene star $*$

**If**  $Q$  is a regexp  
**Then**  $Q^*$  is a regexp

Ex: **R** $^*$  and **G** $^*$  are regexps  
**((R $^*$   $\cdot$  G)  $+$  F) $^*$**  is a regexp

Each regexp  $Q$  **describes** a language  $L(Q)$

Examples:

- $L(\mathbf{R}) = \{\mathbf{R}\}$

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“words of even length”

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*“words of even length”*
- $L(\mathbf{R}^* \cdot \mathbf{G} \cdot \mathbf{R}^*) = \{\mathbf{G}, \mathbf{RG}, \mathbf{GR}, \mathbf{RGR}, \mathbf{RRG}, \dots\}$   
*“words over  $\{\mathbf{G}, \mathbf{R}\}$  with exactly one  $\mathbf{G}$ ”*

Each regexp  $Q$  **describes** a language  $L(Q)$

Examples:

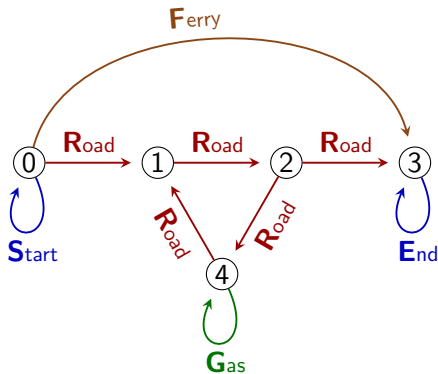
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Any language described by a regexp is called **regular**



## A Regular Path Query (RPQ)

- queries a graph  $\mathcal{D} = (V, L, E)$
- is a **regexp** over  $L$
- **matches** a set of **walks** in  $\mathcal{D}$



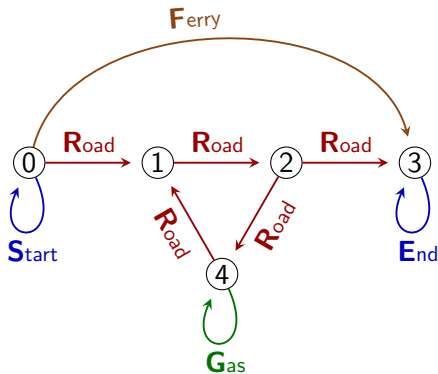


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A **walk** in  $\mathcal{D}$  is a consistent sequence of edges in  $\mathcal{D}$ .

The **label of a walk** is the **word** formed by the label of its edges.



Example walk

$0 \xrightarrow{R} 1 \xrightarrow{R} 2 \xrightarrow{R} 4$

Label

**RRR**

$0 \xrightarrow{S} 0 \xrightarrow{F} 3$

**SF**

$0 \xrightarrow{R} 1 \xrightarrow{R} 2 \xrightarrow{R} 4 \xrightarrow{G}$

$4 \xrightarrow{R} 1 \xrightarrow{R} 2 \xrightarrow{R} 3$

**RRRGRRR**

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**RRR**

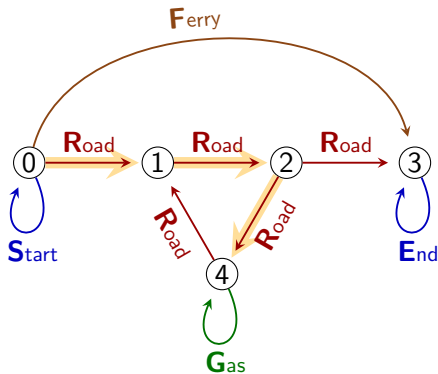
$0 \xrightarrow{S} 0 \xrightarrow{F} 3$

**SF**

$0 \xrightarrow{R} 1 \xrightarrow{R} 2 \xrightarrow{R} 4 \xrightarrow{G}$

$4 \xrightarrow{R} 1 \xrightarrow{R} 2 \xrightarrow{R} 3$

**RRRGRRR**

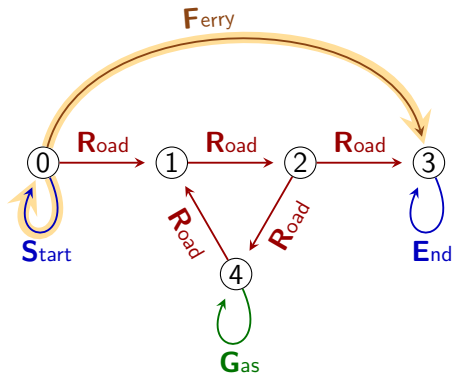


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Label

**RRR**

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**SF**

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$4 \xrightarrow{R} 1 \xrightarrow{R} 2 \xrightarrow{R} 3$  **RRRGRRR**

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Example walk

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Label

**RRR**

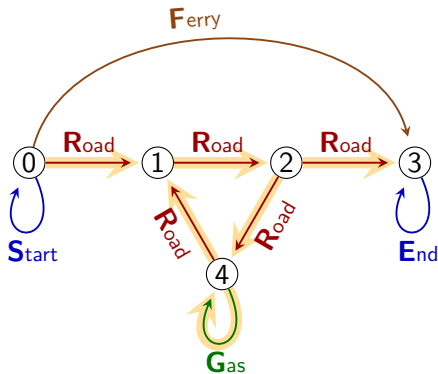
$0 \xrightarrow{S} 0 \xrightarrow{F} 3$

**SF**

$0 \xrightarrow{R} 1 \xrightarrow{R} 2 \xrightarrow{R} 4 \xrightarrow{G}$

$4 \xrightarrow{R} 1 \xrightarrow{R} 2 \xrightarrow{R} 3$

**RRRGRRR**



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- matches** a set of **walks** in  $\mathcal{D}$

A **walk** in  $\mathcal{D}$  is a consistent sequence of edges in  $\mathcal{D}$ .

The **label of a walk** is the **word** formed by the label of its edges.

Example walk

$0 \xrightarrow{R} 1 \xrightarrow{R} 2 \xrightarrow{R} 4$

Label

**RRR**

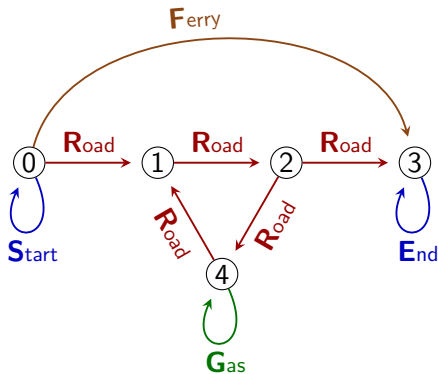
$0 \xrightarrow{S} 0 \xrightarrow{F} 3$

**SF**

$0 \xrightarrow{R} 1 \xrightarrow{R} 2 \xrightarrow{R} 4 \xrightarrow{G}$

$4 \xrightarrow{R} 1 \xrightarrow{R} 2 \xrightarrow{R} 3$

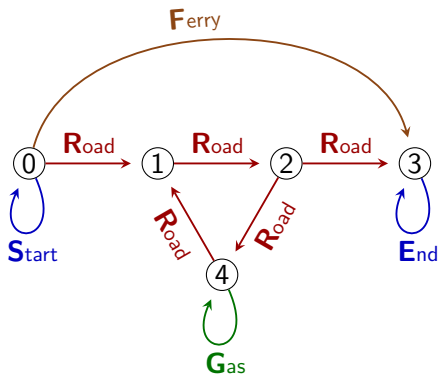
**RRRGRRR**



A **walk**  $w$  is a **match** to an **RPQ**  $Q$  if the **label** of  $w$  is in  $L(Q)$ .

Matching query  $Q_1 = \mathbf{R}$

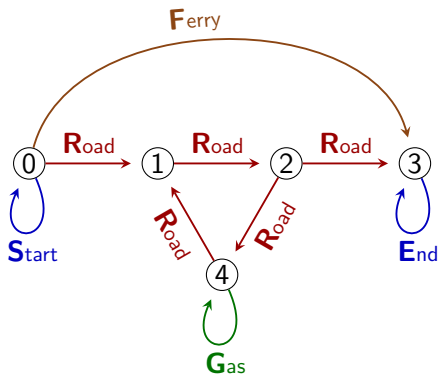
$L(Q_1) = \{\mathbf{R}\}$



Matching query  $Q_1 = \mathbf{R}$

$$L(Q_1) = \{\mathbf{R}\}$$

The matches to  $Q_1$  are the walks labeled by some word in  $L(Q_1)$ , that is labeled by  $\mathbf{R}$ .

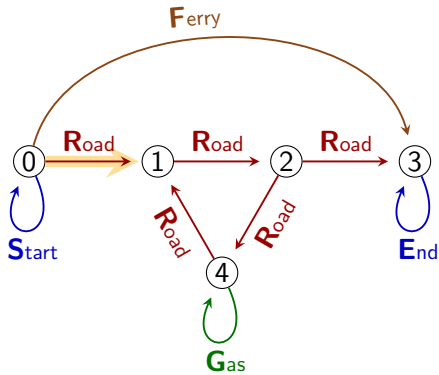


Matching query  $Q_1 = \mathbf{R}$

$L(Q_1) = \{\mathbf{R}\}$

The matches to  $Q_1$  are the walks labeled by some word in  $L(Q_1)$ , that is labeled by  $\mathbf{R}$ .

Match for $Q_1$	Label
$0 \rightarrow 1$	$\mathbf{R}$





Matching query  $Q_1 = \mathbf{R}$

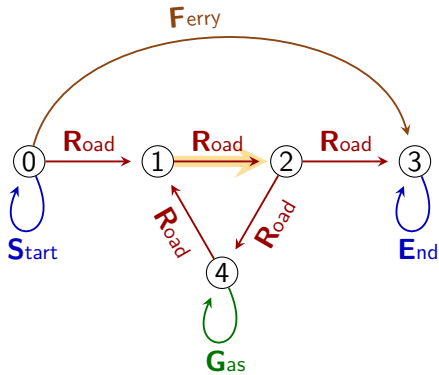
$L(Q_1) = \{\mathbf{R}\}$

The matches to  $Q_1$  are the walks labeled by some word in  $L(Q_1)$ , that is labeled by  $\mathbf{R}$ .

Match for $Q_1$	Label
-----------------	-------

$0 \rightarrow 1$	$\mathbf{R}$
-------------------	--------------

$1 \rightarrow 2$	$\mathbf{R}$
-------------------	--------------

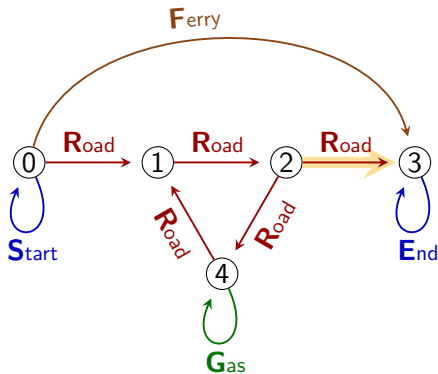


Matching query  $Q_1 = \mathbf{R}$

$L(Q_1) = \{\mathbf{R}\}$

The matches to  $Q_1$  are the walks labeled by some word in  $L(Q_1)$ , that is labeled by  $\mathbf{R}$ .

Match for $Q_1$	Label
$0 \rightarrow 1$	$\mathbf{R}$
$1 \rightarrow 2$	$\mathbf{R}$
$2 \rightarrow 3$	$\mathbf{R}$

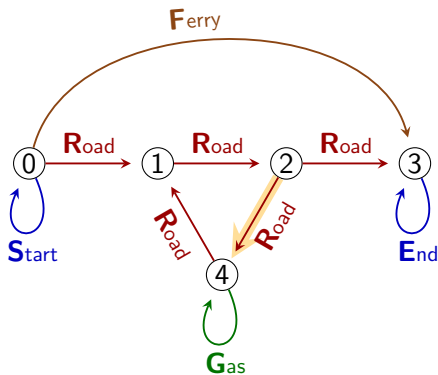


Matching query  $Q_1 = \mathbf{R}$

$L(Q_1) = \{\mathbf{R}\}$

The matches to  $Q_1$  are the walks labeled by some word in  $L(Q_1)$ , that is labeled by  $\mathbf{R}$ .

Match for $Q_1$	Label
$0 \rightarrow 1$	$\mathbf{R}$
$1 \rightarrow 2$	$\mathbf{R}$
$2 \rightarrow 3$	$\mathbf{R}$
$2 \rightarrow 4$	$\mathbf{R}$

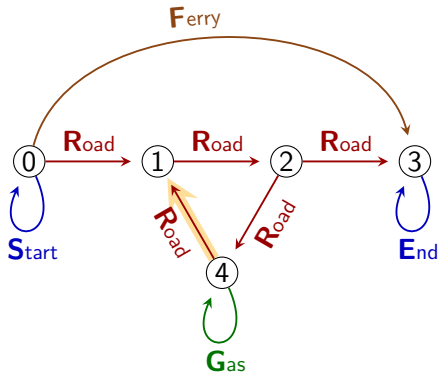


Matching query  $Q_1 = \mathbf{R}$

$L(Q_1) = \{\mathbf{R}\}$

The matches to  $Q_1$  are the walks labeled by some word in  $L(Q_1)$ , that is labeled by  $\mathbf{R}$ .

Match for $Q_1$	Label
$0 \rightarrow 1$	$\mathbf{R}$
$1 \rightarrow 2$	$\mathbf{R}$
$2 \rightarrow 3$	$\mathbf{R}$
$2 \rightarrow 4$	$\mathbf{R}$
$4 \rightarrow 1$	$\mathbf{R}$

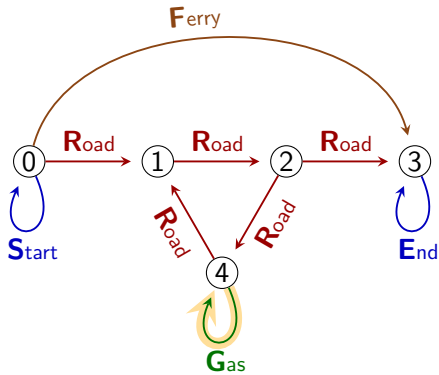


## Matching query $Q_1 = \mathbf{R}$

$$L(Q_1) = \{\mathbf{R}\}$$

The matches to  $Q_1$  are the walks labeled by some word in  $L(Q_1)$ , that is labeled by  $\mathbf{R}$ .

Match for $Q_1$	Label
$0 \rightarrow 1$	$\mathbf{R}$
$1 \rightarrow 2$	$\mathbf{R}$
$2 \rightarrow 3$	$\mathbf{R}$
$2 \rightarrow 4$	$\mathbf{R}$
$4 \rightarrow 1$	$\mathbf{R}$



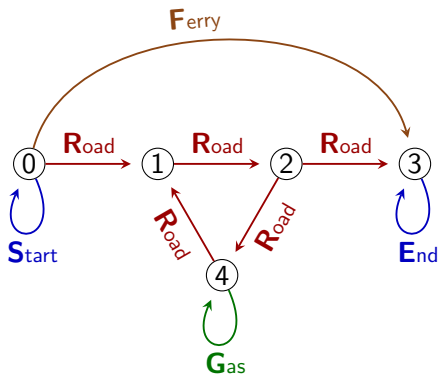
## Matching $Q_2 = \mathbf{G}$

$$L(Q_2) = \{\mathbf{G}\}$$

Match for $Q_2$	Label
$4 \rightarrow 4$	$\mathbf{G}$

$$Q_3 = \mathbf{R} + \mathbf{F}$$

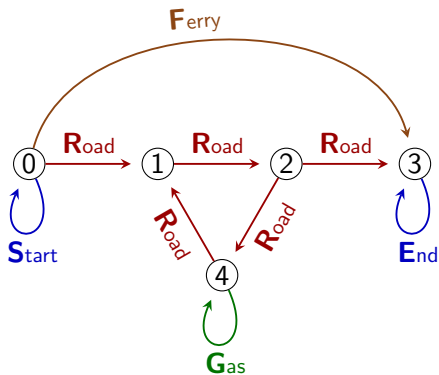
$$L(Q_3) = \{\mathbf{R}, \mathbf{F}\}$$



$$Q_3 = \mathbf{R} + \mathbf{F}$$

$$L(Q_3) = \{\mathbf{R}, \mathbf{F}\}$$

The matches to  $Q_3$  are the walks labeled by some word in  $L(Q_3)$ , that is labeled by  $\mathbf{R}$  or by  $\mathbf{F}$ .

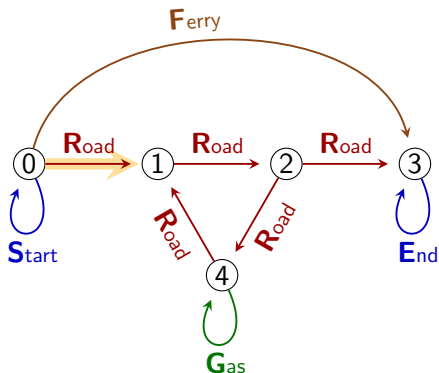


$$Q_3 = \mathbf{R} + \mathbf{F}$$

$$L(Q_3) = \{\mathbf{R}, \mathbf{F}\}$$

The matches to  $Q_3$  are the walks labeled by some word in  $L(Q_3)$ , that is labeled by  $\mathbf{R}$  or by  $\mathbf{F}$ .

Match for $Q_3$	Label
0 → 1	$\mathbf{R}$
1 → 2	$\mathbf{R}$
2 → 3	$\mathbf{R}$
2 → 4	$\mathbf{R}$
4 → 1	$\mathbf{R}$
0 → 3	$\mathbf{F}$



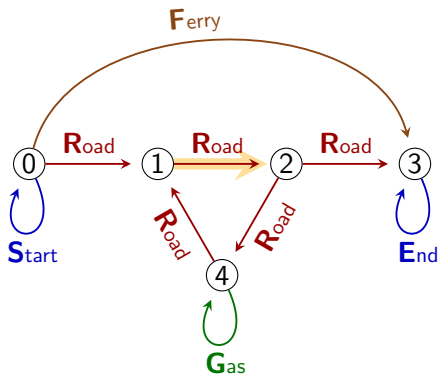


$$Q_3 = \mathbf{R} + \mathbf{F}$$

$$L(Q_3) = \{\mathbf{R}, \mathbf{F}\}$$

The matches to  $Q_3$  are the walks labeled by some word in  $L(Q_3)$ , that is labeled by  $\mathbf{R}$  or by  $\mathbf{F}$ .

Match for $Q_3$	Label
$0 \rightarrow 1$	$\mathbf{R}$
$1 \rightarrow 2$	$\mathbf{R}$
$2 \rightarrow 3$	$\mathbf{R}$
$2 \rightarrow 4$	$\mathbf{R}$
$4 \rightarrow 1$	$\mathbf{R}$
$0 \rightarrow 3$	$\mathbf{F}$

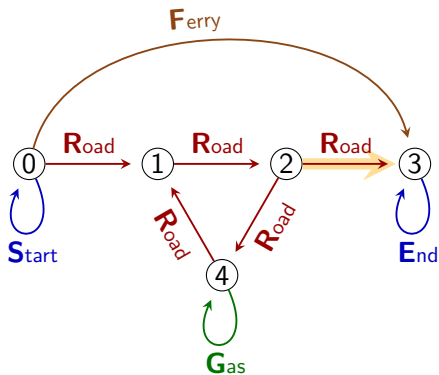


$$Q_3 = \mathbf{R} + \mathbf{F}$$

$$L(Q_3) = \{\mathbf{R}, \mathbf{F}\}$$

The matches to  $Q_3$  are the walks labeled by some word in  $L(Q_3)$ , that is labeled by  $\mathbf{R}$  or by  $\mathbf{F}$ .

Match for $Q_3$	Label
$0 \rightarrow 1$	$\mathbf{R}$
$1 \rightarrow 2$	$\mathbf{R}$
$2 \rightarrow 3$	$\mathbf{R}$
$2 \rightarrow 4$	$\mathbf{R}$
$4 \rightarrow 1$	$\mathbf{R}$
$0 \rightarrow 3$	$\mathbf{F}$

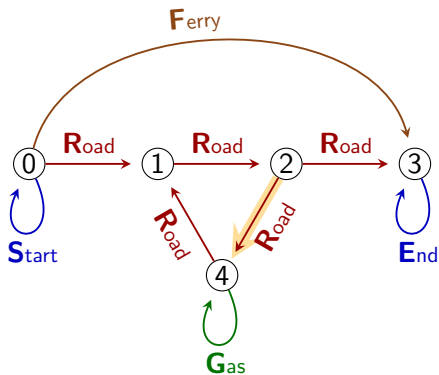


$$Q_3 = \mathbf{R} + \mathbf{F}$$

$$L(Q_3) = \{\mathbf{R}, \mathbf{F}\}$$

The matches to  $Q_3$  are the walks labeled by some word in  $L(Q_3)$ , that is labeled by  $\mathbf{R}$  or by  $\mathbf{F}$ .

Match for $Q_3$	Label
$0 \rightarrow 1$	$\mathbf{R}$
$1 \rightarrow 2$	$\mathbf{R}$
$2 \rightarrow 3$	$\mathbf{R}$
$2 \rightarrow 4$	$\mathbf{R}$
$4 \rightarrow 1$	$\mathbf{R}$
$0 \rightarrow 3$	$\mathbf{F}$

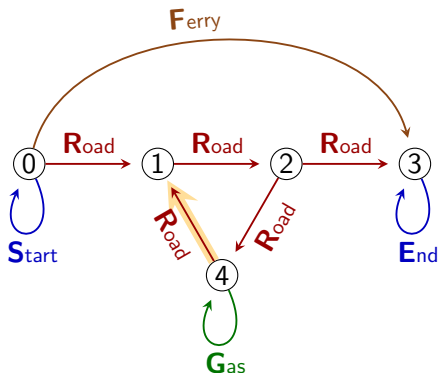


$$Q_3 = \mathbf{R} + \mathbf{F}$$

$$L(Q_3) = \{\mathbf{R}, \mathbf{F}\}$$

The matches to  $Q_3$  are the walks labeled by some word in  $L(Q_3)$ , that is labeled by  $\mathbf{R}$  or by  $\mathbf{F}$ .

Match for $Q_3$	Label
$0 \rightarrow 1$	$\mathbf{R}$
$1 \rightarrow 2$	$\mathbf{R}$
$2 \rightarrow 3$	$\mathbf{R}$
$2 \rightarrow 4$	$\mathbf{R}$
$4 \rightarrow 1$	$\mathbf{R}$
$0 \rightarrow 3$	$\mathbf{F}$

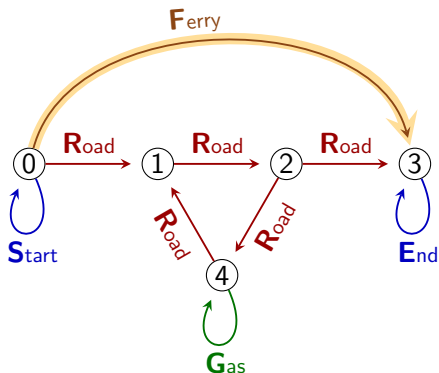


$$Q_3 = \mathbf{R} + \mathbf{F}$$

$$L(Q_3) = \{\mathbf{R}, \mathbf{F}\}$$

The matches to  $Q_3$  are the walks labeled by some word in  $L(Q_3)$ , that is labeled by  $\mathbf{R}$  or by  $\mathbf{F}$ .

Match for $Q_3$	Label
$0 \rightarrow 1$	$\mathbf{R}$
$1 \rightarrow 2$	$\mathbf{R}$
$2 \rightarrow 3$	$\mathbf{R}$
$2 \rightarrow 4$	$\mathbf{R}$
$4 \rightarrow 1$	$\mathbf{R}$
$0 \rightarrow 3$	$\mathbf{F}$

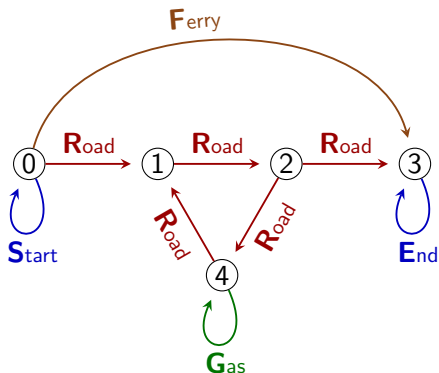


$$Q_3 = \mathbf{R} + \mathbf{F}$$

$$L(Q_3) = \{\mathbf{R}, \mathbf{F}\}$$

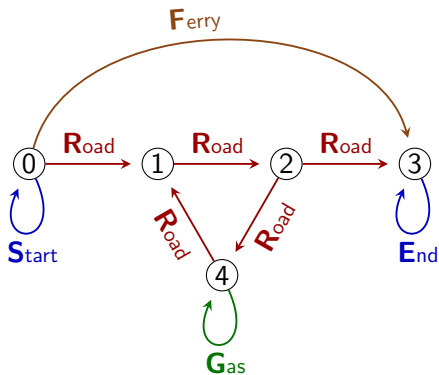
The matches to  $Q_3$  are the walks labeled by some word in  $L(Q_3)$ , that is labeled by  $\mathbf{R}$  or by  $\mathbf{F}$ .

Match for $Q_3$	Label
$0 \rightarrow 1$	$\mathbf{R}$
$1 \rightarrow 2$	$\mathbf{R}$
$2 \rightarrow 3$	$\mathbf{R}$
$2 \rightarrow 4$	$\mathbf{R}$
$4 \rightarrow 1$	$\mathbf{R}$
$0 \rightarrow 3$	$\mathbf{F}$



$$Q_4 = \mathbf{R} \cdot \mathbf{R}$$

$$L(Q_4) = \{\mathbf{RR}\}$$



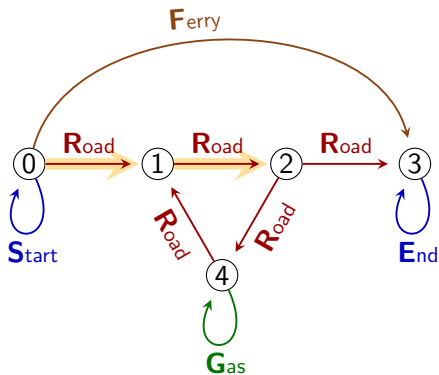
$$Q_4 = \mathbf{R} \cdot \mathbf{R}$$

$$L(Q_4) = \{\mathbf{RR}\}$$

Match for  $Q_4$       Label

0 → 1 → 2

RR

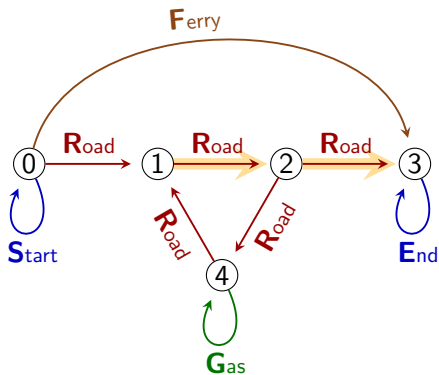




$$Q_4 = \mathbf{R \cdot R}$$

$$L(Q_4) = \{\mathbf{RR}\}$$

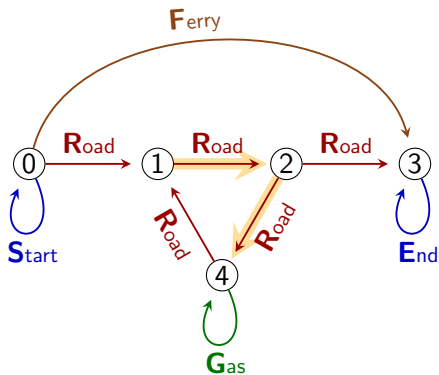
Match for $Q_4$	Label
$0 \rightarrow 1 \rightarrow 2$	<b>RR</b>
$1 \rightarrow 2 \rightarrow 3$	<b>RR</b>



$$Q_4 = \mathbf{R} \cdot \mathbf{R}$$

$$L(Q_4) = \{\mathbf{RR}\}$$

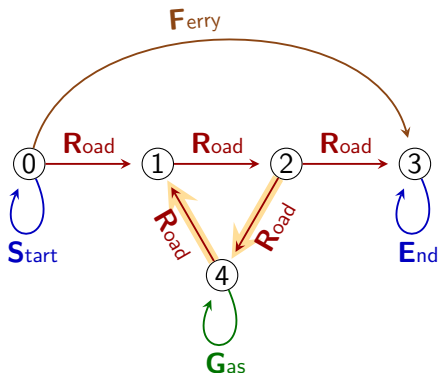
Match for $Q_4$	Label
$0 \rightarrow 1 \rightarrow 2$	<b>RR</b>
$1 \rightarrow 2 \rightarrow 3$	<b>RR</b>
$1 \rightarrow 2 \rightarrow 4$	<b>RR</b>



$$Q_4 = \mathbf{R} \cdot \mathbf{R}$$

$$L(Q_4) = \{\mathbf{RR}\}$$

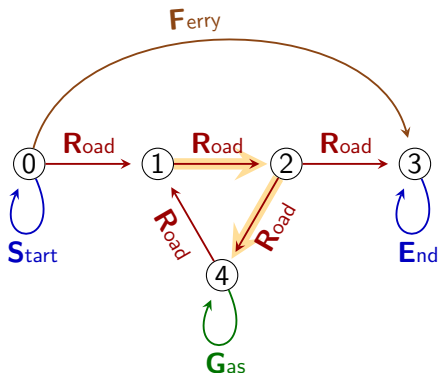
Match for $Q_4$	Label
$0 \rightarrow 1 \rightarrow 2$	<b>RR</b>
$1 \rightarrow 2 \rightarrow 3$	<b>RR</b>
$1 \rightarrow 2 \rightarrow 4$	<b>RR</b>
$2 \rightarrow 4 \rightarrow 1$	<b>RR</b>



$$Q_4 = \mathbf{R} \cdot \mathbf{R}$$

$$L(Q_4) = \{\mathbf{RR}\}$$

Match for $Q_4$	Label
$0 \rightarrow 1 \rightarrow 2$	<b>RR</b>
$1 \rightarrow 2 \rightarrow 3$	<b>RR</b>
$1 \rightarrow 2 \rightarrow 4$	<b>RR</b>
$2 \rightarrow 4 \rightarrow 1$	<b>RR</b>
<b><math>4 \rightarrow 1 \rightarrow 2</math></b>	<b>RR</b>



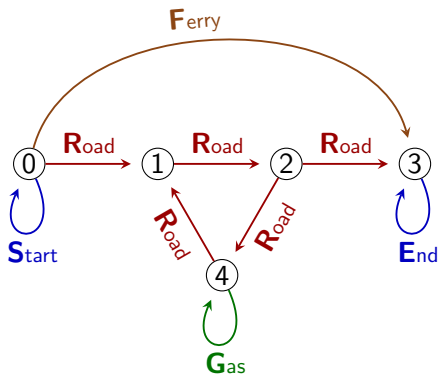
$$Q_4 = \mathbf{R} \cdot \mathbf{R}$$

$$L(Q_4) = \{\mathbf{RR}\}$$

Match for $Q_4$	Label
$0 \rightarrow 1 \rightarrow 2$	<b>RR</b>
$1 \rightarrow 2 \rightarrow 3$	<b>RR</b>
$1 \rightarrow 2 \rightarrow 4$	<b>RR</b>
$2 \rightarrow 4 \rightarrow 1$	<b>RR</b>
$4 \rightarrow 1 \rightarrow 2$	<b>RR</b>

$$\text{Matches for } Q_5 = \mathbf{S} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R}$$

$$L(Q_5) = \{\mathbf{SRRR}\}$$



$$Q_4 = \mathbf{R} \cdot \mathbf{R}$$

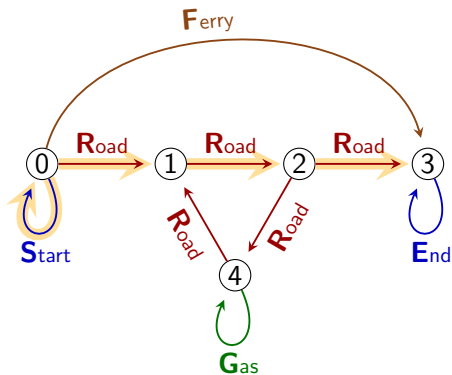
$$L(Q_4) = \{\mathbf{RR}\}$$

Match for $Q_4$	Label
$0 \rightarrow 1 \rightarrow 2$	<b>RR</b>
$1 \rightarrow 2 \rightarrow 3$	<b>RR</b>
$1 \rightarrow 2 \rightarrow 4$	<b>RR</b>
$2 \rightarrow 4 \rightarrow 1$	<b>RR</b>
$4 \rightarrow 1 \rightarrow 2$	<b>RR</b>

$$\text{Matches for } Q_5 = \mathbf{S} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R}$$

$$L(Q_5) = \{\mathbf{SRRR}\}$$

$0 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 3$	<b>SRRR</b>
$0 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 3$	<b>SRRR</b>



$$Q_4 = \mathbf{R} \cdot \mathbf{R}$$

$$L(Q_4) = \{\mathbf{RR}\}$$

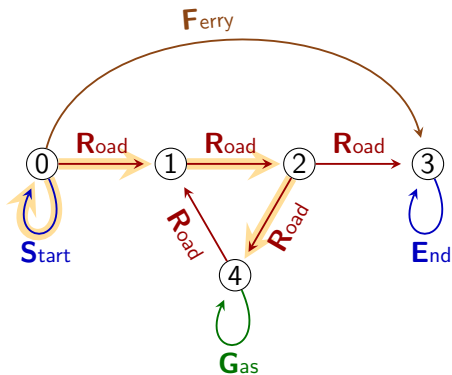
Match for $Q_4$	Label
0 → 1 → 2	<b>RR</b>
1 → 2 → 3	<b>RR</b>
1 → 2 → 4	<b>RR</b>
2 → 4 → 1	<b>RR</b>
4 → 1 → 2	<b>RR</b>

$$\text{Matches for } Q_5 = \mathbf{S} \cdot \mathbf{R} \cdot \mathbf{R} \cdot \mathbf{R}$$

$$L(Q_5) = \{\mathbf{SRRR}\}$$

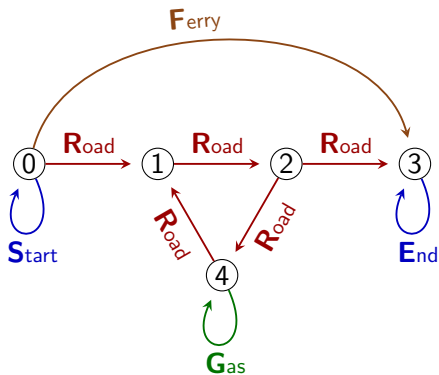
0 → 0 → 1 → 2 → 3    **SRRR**

0 → 0 → 1 → 2 → 3    **SRRR**



$$Q_6 = \mathbf{S} \cdot (\mathbf{R} + \mathbf{F})$$

$$L(Q_6) = \{\mathbf{SR}, \mathbf{SF}\}$$





$$Q_6 = S \cdot (R + F)$$

$$L(Q_6) = \{SR, SF\}$$

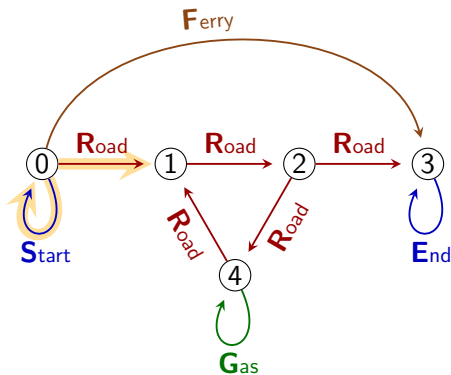
Match for  $Q_6$       Label

0 → 0 → 1

SR

0 → 0 → 3

SF



$$Q_6 = S \cdot (R + F)$$

$$L(Q_6) = \{SR, SF\}$$

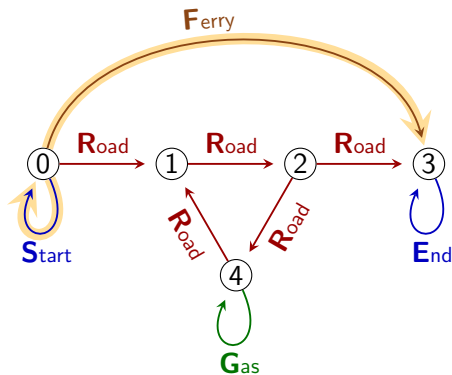
Match for  $Q_6$       Label

0 → 0 → 1

**SR**

0 → 0 → 3

**SF**



$$Q_6 = S \cdot (R + F)$$

$$L(Q_6) = \{SR, SF\}$$

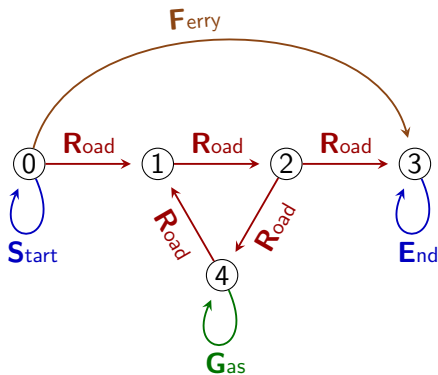
Match for  $Q_6$     Label

$0 \rightarrow 0 \rightarrow 1$     **SR**

$0 \rightarrow 0 \rightarrow 3$     **SF**

$$Q_7 = (S + R)(F + G)(E + R)$$

$$L(Q_7) =$$



$$Q_6 = \mathbf{S} \cdot (\mathbf{R} + \mathbf{F})$$

$$L(Q_6) = \{\mathbf{SR}, \mathbf{SF}\}$$

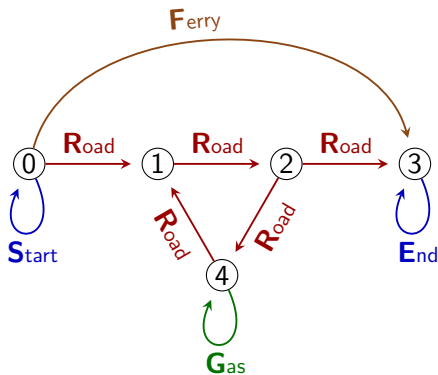
Match for  $Q_6$     Label

$0 \rightarrow 0 \rightarrow 1$     **SR**

$0 \rightarrow 0 \rightarrow 3$     **SF**

$$Q_7 = (\mathbf{S} + \mathbf{R})(\mathbf{F} + \mathbf{G})(\mathbf{E} + \mathbf{R})$$

$$L(Q_7) = \{\mathbf{SFE}, \mathbf{SFR}, \mathbf{SGE}, \mathbf{SGR}, \mathbf{RFE}, \mathbf{RFR}, \mathbf{RGE}, \mathbf{RGR}\}$$



$$Q_6 = S \cdot (R + F)$$

$$L(Q_6) = \{SR, SF\}$$

Match for  $Q_6$     Label

$0 \rightarrow 0 \rightarrow 1$     **SR**

$0 \rightarrow 0 \rightarrow 3$     **SF**

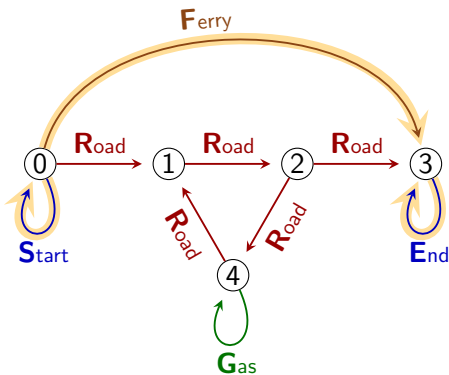
$$Q_7 = (S + R)(F + G)(E + R)$$

$$L(Q_7) = \{SFE, SFR, SGE, SGR, RFE, RFR, RGE, RGR\}$$

Match for  $Q_7$     Label

$0 \rightarrow 0 \rightarrow 3 \rightarrow 3$     **SFE**

$2 \rightarrow 4 \rightarrow 4 \rightarrow 1$     **RGR**



$$Q_6 = S \cdot (R + F)$$

$$L(Q_6) = \{SR, SF\}$$

Match for  $Q_6$       Label

0 → 0 → 1      **SR**

0 → 0 → 3      **SF**

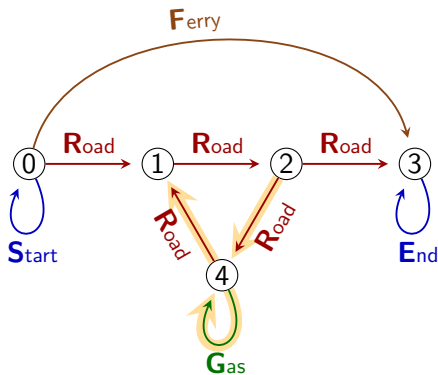
$$Q_7 = (S + R)(F + G)(E + R)$$

$$L(Q_7) = \{SFE, SFR, SGE, \\ SGR, RFE, RFR, RGE, RGR\}$$

Match for  $Q_7$       Label

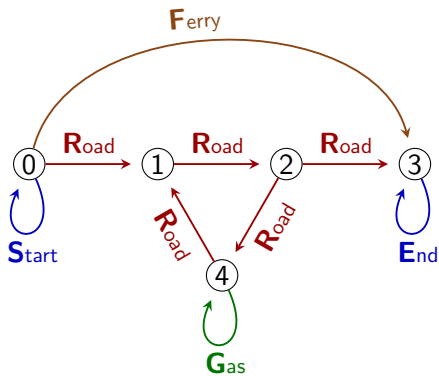
0 → 0 → 3 → 3      **SFE**

2 → 4 → 4 → 1      **RGR**



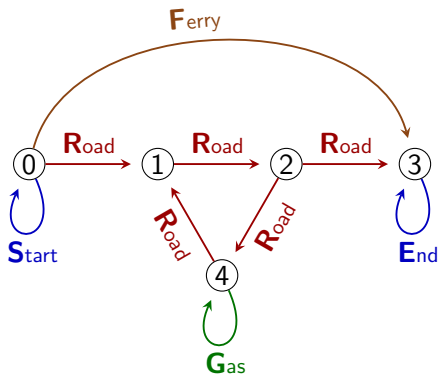
$$Q_8 = R^*$$

$$L(Q_8) = \{R, RR, RRR, RRRR, RRRRR, RRRRRR, \dots\}$$



$$Q_8 = R^*$$

$$L(Q_8) = \{R, RR, RRR, RRRR, RRRRR, RRRRRR, \dots\}$$



!  $L(Q_8)$  is infinite !



$$Q_8 = R^*$$

$$L(Q_8) = \{R, RR, RRR, RRRR, RRRRR, RRRRRR, \dots\}$$

Match for  $Q_8$ 

0 → 1

1 → 2

⋮

2 → 4 → 1

⋮

1 → 2 → 4 → 1

⋮

1 → 2 → 4 →

1 → 2 → 4 → 1

⋮

Label

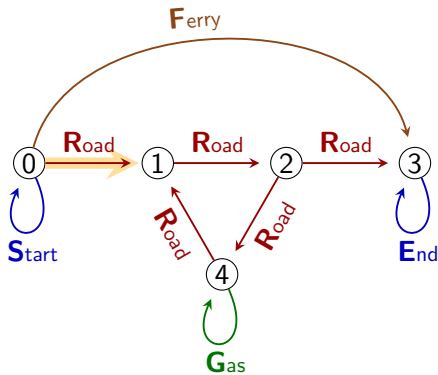
R

R

RR

RRR

RRRRRR



⚠  $L(Q_8)$  is infinite ⚠

$$Q_8 = R^*$$

$$L(Q_8) = \{R, RR, RRR, RRRR, RRRRR, RRRRRR, \dots\}$$

Match for  $Q_8$ 

0 → 1

1 → 2

⋮

2 → 4 → 1

⋮

1 → 2 → 4 → 1

⋮

1 → 2 → 4 →

1 → 2 → 4 → 1

⋮

Label

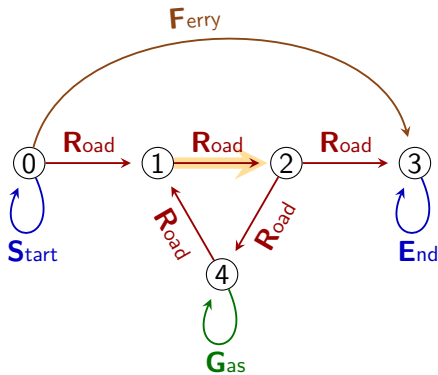
R

R

RR

RRR

RRRRRR

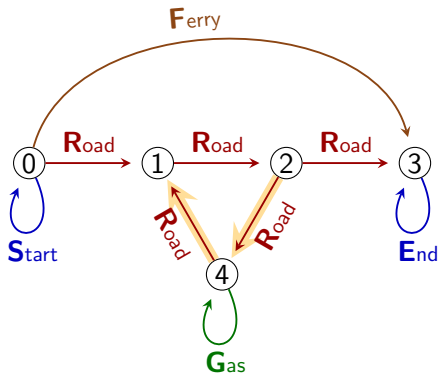


⚠  $L(Q_8)$  is infinite ⚠

$$Q_8 = R^*$$

$$L(Q_8) = \{R, RR, RRR, RRRR, RRRRR, RRRRRR, \dots\}$$

Match for $Q_8$	Label
0 → 1	R
1 → 2	R
⋮	
2 → 4 → 1	RR
⋮	
1 → 2 → 4 → 1	RRR
⋮	
1 → 2 → 4 → 1 → 2 → 4 → 1	RRRRRR
⋮	



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$$L(Q_8) = \{R, RR, \mathbf{RRR}, RRRR, RRRRR, RRRRRR, \dots\}$$

Match for  $Q_8$ 

0 → 1

1 → 2

⋮

2 → 4 → 1

⋮

1 → 2 → 4 → 1

⋮

1 → 2 → 4 →

1 → 2 → 4 → 1

⋮

Label

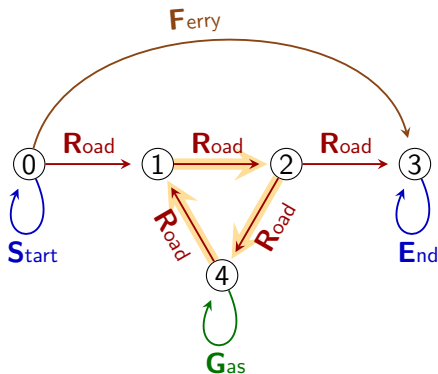
R

R

RR

RRR

RRRRRR



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$$Q_8 = R^*$$

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Match for  $Q_8$ 

0 → 1

1 → 2

⋮

2 → 4 → 1

⋮

1 → 2 → 4 → 1

⋮

1 → 2 → 4 →

1 → 2 → 4 → 1

⋮

Label

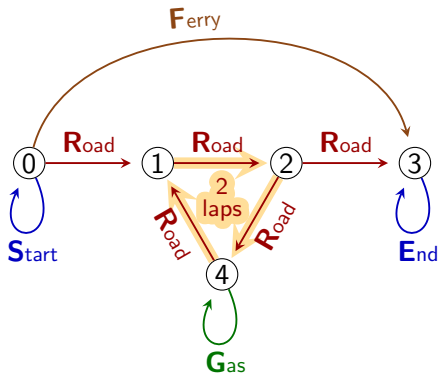
R

R

RR

RRR

RRRRRR

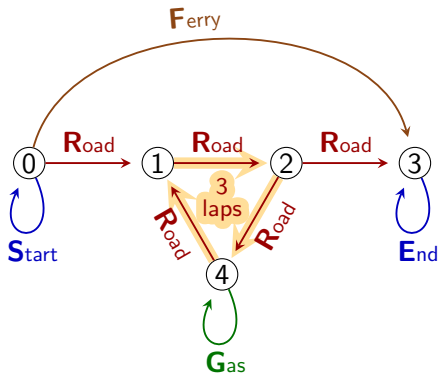


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0 → 1	R
1 → 2	R
⋮	
2 → 4 → 1	RR
⋮	
1 → 2 → 4 → 1	RRR
⋮	
1 → 2 → 4 → 1 → 2 → 4 → 1	RRRRRR
⋮	

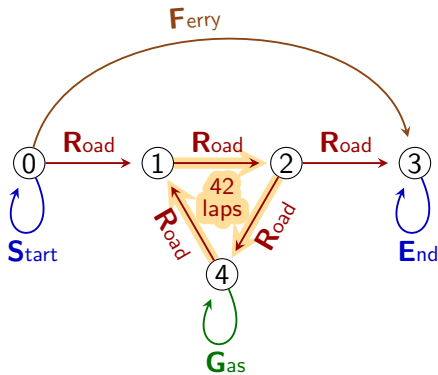


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Match for $Q_8$	Label
$0 \rightarrow 1$	<b>R</b>
$1 \rightarrow 2$	<b>R</b>
$\vdots$	
$2 \rightarrow 4 \rightarrow 1$	<b>RR</b>
$\vdots$	
$1 \rightarrow 2 \rightarrow 4 \rightarrow 1$	<b>RRR</b>
$\vdots$	
$1 \rightarrow 2 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 4 \rightarrow 1$	<b>RRRRRR</b>
$\vdots$	

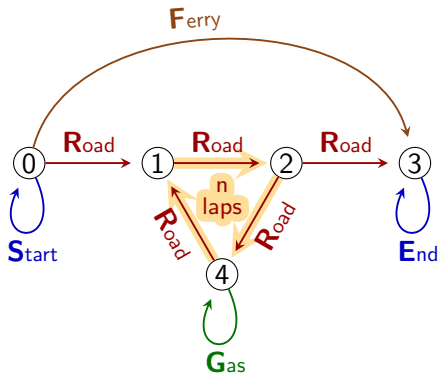


⚠  $L(Q_8)$  is infinite ⚠

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Match for $Q_8$	Label
0 → 1	R
1 → 2	R
⋮	
2 → 4 → 1	RR
⋮	
1 → 2 → 4 → 1	RRR
⋮	
1 → 2 → 4 → 1 → 2 → 4 → 1	RRRRRR
⋮	



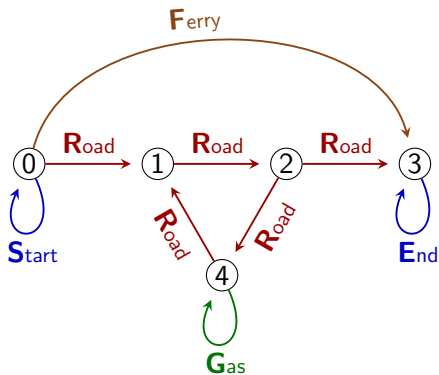
⚠  $L(Q_8)$  is infinite ⚠

⚠ Infinitely many matches ⚠



## Exercise

Compute the matches to query  $Q_9 = (\mathbf{R} + \mathbf{F})^* \mathbf{G} (\mathbf{R} + \mathbf{F})^*$  that start in 0 and end in 3.

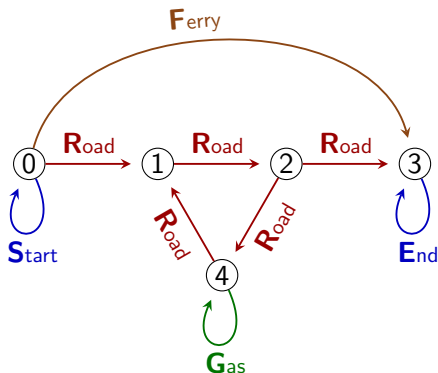


## Exercise

Compute the matches to query  $Q_9 = (\mathbf{R} + \mathbf{F})^* \mathbf{G} (\mathbf{R} + \mathbf{F})^*$  that start in 0 and end in 3.

## Answer

$$\begin{aligned}
 0 &\xrightarrow{\mathbf{R}} 1 \xrightarrow{\mathbf{R}} 2 \xrightarrow{\mathbf{R}} 4 \\
 &\left( \xrightarrow{\mathbf{R}} 1 \xrightarrow{\mathbf{R}} 2 \xrightarrow{\mathbf{R}} 4 \right)^* \\
 &\xrightarrow{\mathbf{G}} 4 \\
 &\left( \xrightarrow{\mathbf{R}} 1 \xrightarrow{\mathbf{R}} 2 \xrightarrow{\mathbf{R}} 4 \right)^* \\
 &\xrightarrow{\mathbf{R}} 1 \xrightarrow{\mathbf{R}} 2 \xrightarrow{\mathbf{R}} 3
 \end{aligned}$$



**Any idea an how to compute  
matches in general?**

## Glushkov Construction\*

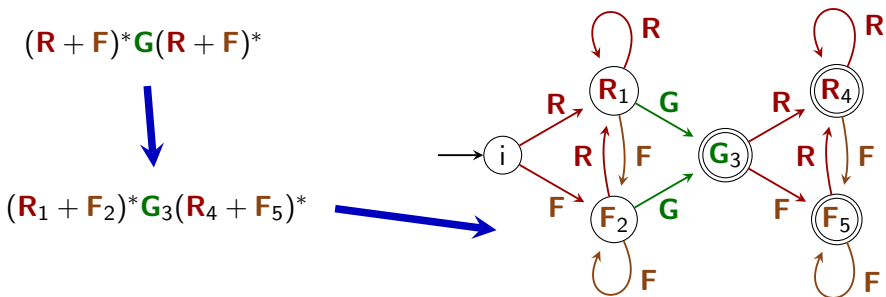
**Input** a regexp  $Q$

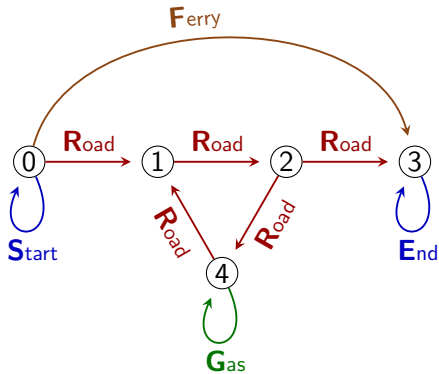
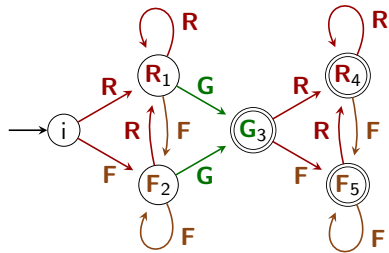
**Output** a nondeterministic automaton  $\mathcal{A}$  such that  $L(\mathcal{A}) = L(Q)$

**Properties of  $\mathcal{A}$**

- $\mathcal{D}$  is small: the number of state is in  $O(\text{size}(Q))$
- $\mathcal{D}$  is computed efficiently  $O(\text{size}(Q)^2)$
- $\mathcal{D}$  has no epsilon-transitions

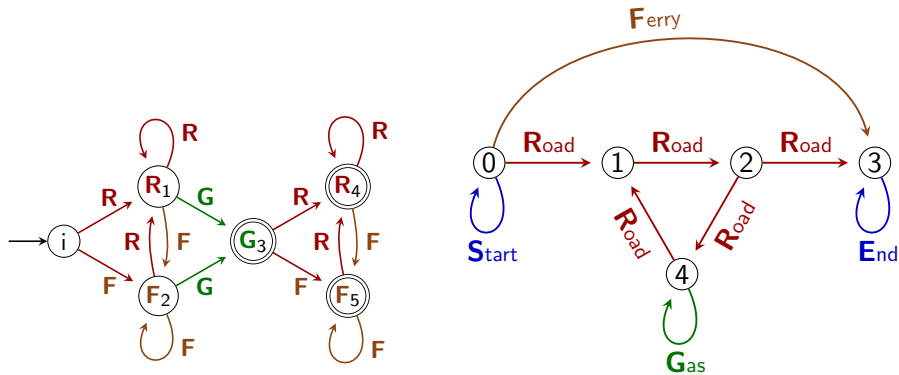
\*Other names: position automaton, standard automaton, Berry-Sethi construction





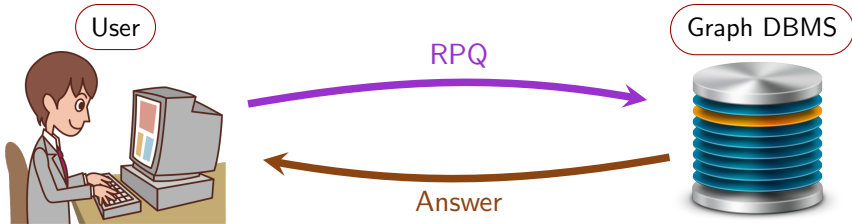
# A graph is essentially an automaton

Exercise: compute the product graph  $\times$  query



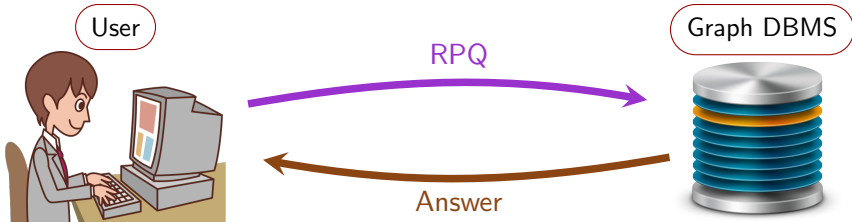
Part I: Theoretical foundations

## **4. The most common RPQ semantics**



⚠ **Infinitely** many matches but the user expects **finite** answer ⚠





⚠ **Infinitely** many matches but the user expects **finite** answer ⚠

## Different semantics for RPQs

- A **RPQ semantics** = a way to interpret RPQs
- The semantics defines the **correct answer**  
⇒ The same query has different answers under different semantics
- Goal of an RPQ semantics: ensure the answer to be **finite**, while remaining **meaningful** and **easy to compute**.

# Endpoint semantics (1)

Used by SparQL (RDF) and arguably GQL with keyword **ANY WALK**

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## Principles

- Returns a set of pairs of vertices (and not walks)
- Precisely, returns the endpoints (first and last vertex) of the matches

## Example

Matching walks	Projection on endpoints
$1 \rightarrow 0 \rightarrow 2 \rightarrow 2 \rightarrow 3$	(1,3)
$2 \rightarrow 2$	(2,2)
$0 \rightarrow 0 \rightarrow 2 \rightarrow 3 \rightarrow 0 \rightarrow 3$	(0,3)
$1 \rightarrow 0 \rightarrow 3$	(1,3)

Full answer is:  $\{(1, 3), (2, 2), (0, 3)\}$

Used by SparQL (RDF) and arguably GQL with keyword **ANY WALK**

## Principles

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- Precisely, returns the endpoints (first and last vertex) of the matches

## Example

Matching walks

1 → 0 → 2 → 2 → 3

2 → 2

0 → 0 → 2 → 3 → 0 → 3

1 → 0 → 3

Projection on endpoints

(1,3)

(2,2)

(0,3)

(1,3)

Full answer is:  $\{(1, 3), (2, 2), (0, 3)\}$

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Matching walks

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2 → 2

0 → 0 → 2 → 3 → 0 → 3

1 → 0 → 3

Projection on endpoints

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(0,3)

(1,3)

Full answer is:  $\{(1, 3), (2, 2), (0, 3)\}$

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<b><math>1 \rightarrow 0 \rightarrow 3</math></b>	<b>(1,3)</b>

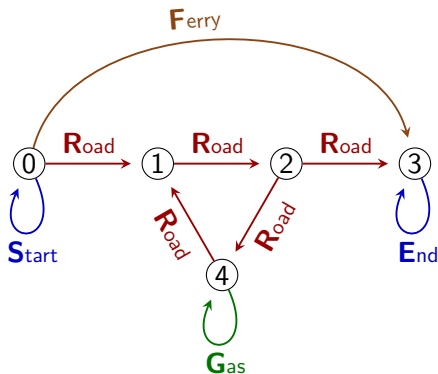
Full answer is:  **$\{(1, 3), (2, 2), (0, 3)\}$**

## Evaluating a reachability query

$$Q_{10} = \mathbf{GR}^*$$

Match	Endpoints
$4 \rightarrow 4$	(4,4)
$4 \rightarrow 4 \rightarrow 1$	(4,1)
$4 \rightarrow 4 \rightarrow 1 \rightarrow 2$	(4,2)
$4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 3$	(4,3)
$\vdots$	$\vdots$
$4 \rightarrow 4 \rightarrow 1 \rightarrow 2$	
$\rightarrow 4 \rightarrow 1 \rightarrow 2$	
$\rightarrow 3$	(4,3)
$\vdots$	$\vdots$

Other matches do not add new pairs to the answer



Answer to  $Q_{10}$  under endpoint sem.:  $\{(4, 4), (4, 1), (4, 2), (4, 3)\}$



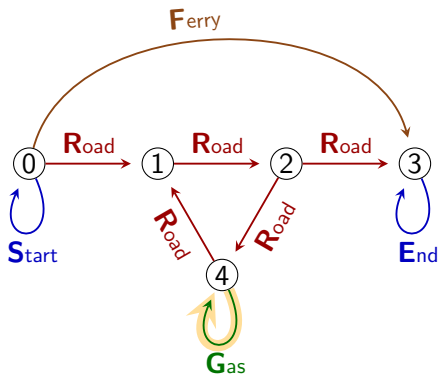
## Endpoint semantics (2)

### Evaluating a reachability query

$$Q_{10} = \mathbf{GR}^*$$

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$4 \rightarrow 4$	$(4,4)$
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$4 \rightarrow 4 \rightarrow 1 \rightarrow 2$	$(4,2)$
$4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 3$	$(4,3)$
$\vdots$	$\vdots$
$4 \rightarrow 4 \rightarrow 1 \rightarrow 2$ $\rightarrow 4 \rightarrow 1 \rightarrow 2$ $\rightarrow 3$	$(4,3)$
$\vdots$	$\vdots$

Other matches do not add new pairs to the answer



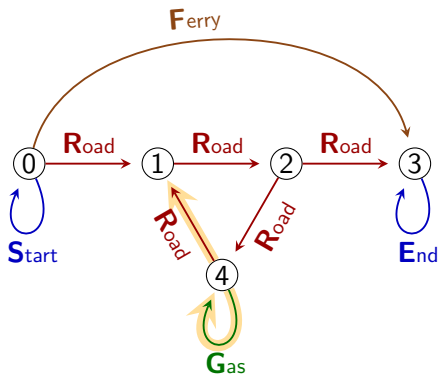
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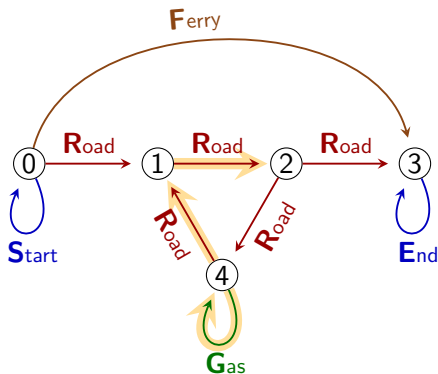
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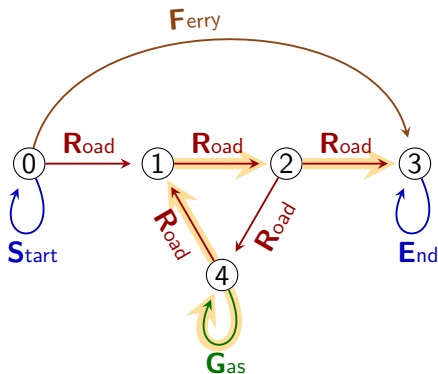
## Endpoint semantics (2)

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$$Q_{10} = \mathbf{GR}^*$$

Match	Endpoints
$4 \rightarrow 4$	(4,4)
$4 \rightarrow 4 \rightarrow 1$	(4,1)
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$\vdots$	$\vdots$
$4 \rightarrow 4 \rightarrow 1 \rightarrow 2$	
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Answer to  $Q_{10}$  under endpoint sem.:  $\{(4, 4), (4, 1), (4, 2), (4, 3)\}$

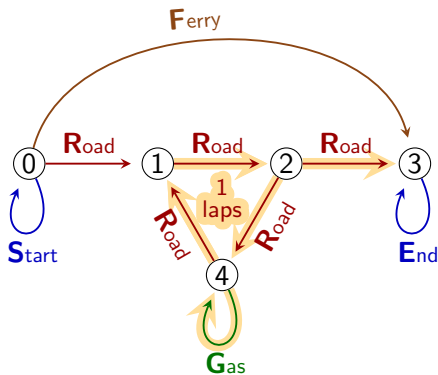
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$\vdots$	$\vdots$
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Other matches do not add new pairs to the answer



Answer to  $Q_{10}$  under endpoint sem.:  $\{(4,4), (4,1), (4,2), (4,3)\}$

## Pros and cons

### Pros

- Efficient algorithms
- Output is always small
- Well grounded theory

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- Efficient algorithms
- Output is always small
- Well grounded theory

### Cons

- Very limited information in the answer
  - User: *"I want to go from Paris to Lyon by car"*
  - Database: *"Yes you can"*

# Shortest semantics (1)

Used in GSQL (TigerGraph), PGQL (Oracle) and GQL with ALL SHORTEST



Used in GSQL (TigerGraph), PGQL (Oracle) and GQL with ALL SHORTEST

## Principles

- Return walks
- For each endpoints  $(s, t)$ , return the “best” match from  $s$  to  $t$
- Best = shortest = smallest number of edges

## Example

Match	Endpoints	Length	
$1 \rightarrow 0 \rightarrow 2 \rightarrow 3$	$(1, 3)$	3	Shortest for $(1, 3)$
$1 \rightarrow 0 \rightarrow 2 \rightarrow 2 \rightarrow 3$	$(1, 3)$	4	Not shortest for $(1, 3)$
$0 \rightarrow 2 \rightarrow 2 \rightarrow 3$	$(0, 3)$	3	Not shortest for $(0, 3)$
$0 \rightarrow 2 \rightarrow 3$	$(0, 3)$	2	Tied shortest for $(0, 3)$
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Full answer:  $\{1 \rightarrow 0 \rightarrow 2 \rightarrow 3, 0 \rightarrow 2 \rightarrow 3, 0 \rightarrow 0 \rightarrow 3\}$

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Full answer: {1 → 0 → 2 → 3, 0 → 2 → 3, 0 → 0 → 3}

Used in GSQL (TigerGraph), PGQL (Oracle) and GQL with ALL SHORTEST

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$0 \rightarrow 0 \rightarrow 3$	$(0, 3)$	2	Tied shortest for $(0, 3)$

Full answer:  $\{1 \rightarrow 0 \rightarrow 2 \rightarrow 3, 0 \rightarrow 2 \rightarrow 3, 0 \rightarrow 0 \rightarrow 3\}$

# Shortest semantics (1)

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Full answer:  $\{1 \rightarrow 0 \rightarrow 2 \rightarrow 3, 0 \rightarrow 2 \rightarrow 3, 0 \rightarrow 0 \rightarrow 3\}$

## Evaluating a reachability query

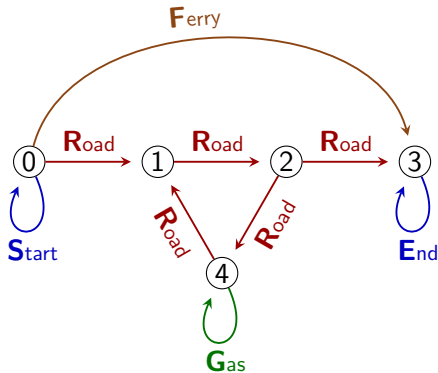
$$Q_{11} = \mathbf{GR}^*$$

Answer under shortest sem.

Walk	Shortest for
$4 \rightarrow 4$	$(4,4)$
$4 \rightarrow 4 \rightarrow 1$	$(4,1)$
$4 \rightarrow 4 \rightarrow 1 \rightarrow 2$	$(4,2)$
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Example of discarded match

$4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 4$  is not in the answer because it is longer than  $4 \rightarrow 4$



## Evaluating a reachability query

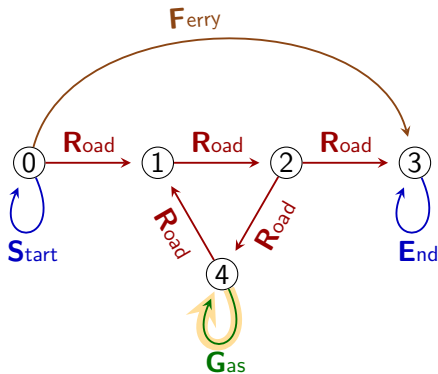
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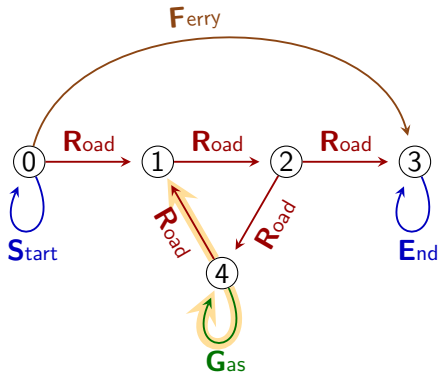
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## Shortest semantics (2)

Evaluating a reachability query

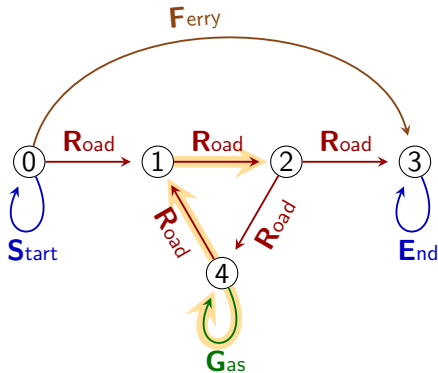
$$Q_{11} = \mathbf{GR}^*$$

Answer under shortest sem.

Walk	Shortest for
$4 \rightarrow 4$	$(4,4)$
$4 \rightarrow 4 \rightarrow 1$	$(4,1)$
$4 \rightarrow 4 \rightarrow 1 \rightarrow 2$	$(4,2)$
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Example of discarded match

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# Shortest semantics (2)

Evaluating a reachability query

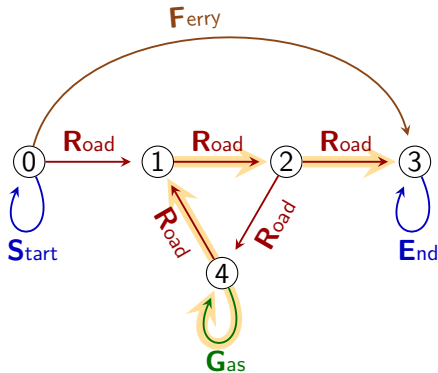
$$Q_{11} = \mathbf{GR}^*$$

Answer under shortest sem.

Walk	Shortest for
$4 \rightarrow 4$	$(4,4)$
$4 \rightarrow 4 \rightarrow 1$	$(4,1)$
$4 \rightarrow 4 \rightarrow 1 \rightarrow 2$	$(4,2)$
$4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 3$	$(4,3)$

Example of discarded match

$4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 4$  is not in the answer because it is longer than  $4 \rightarrow 4$



## Evaluating a reachability query

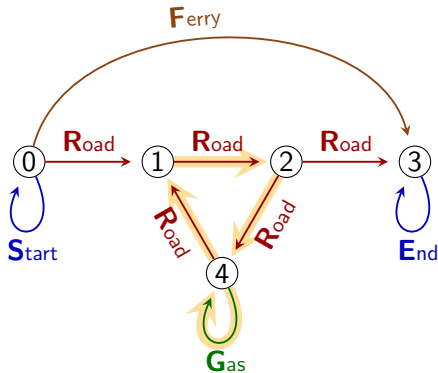
$$Q_{11} = \mathbf{GR}^*$$

Answer under shortest sem.

Walk	Shortest for
$4 \rightarrow 4$	(4,4)
$4 \rightarrow 4 \rightarrow 1$	(4,1)
$4 \rightarrow 4 \rightarrow 1 \rightarrow 2$	(4,2)
$4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 3$	(4,3)

Example of discarded match

$4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 4$  is not in the answer because it is longer than  $4 \rightarrow 4$



# Shortest semantics (3)

Exercise: evaluating some queries

$$Q_{12} = \mathbf{S(R+F)^*E}$$

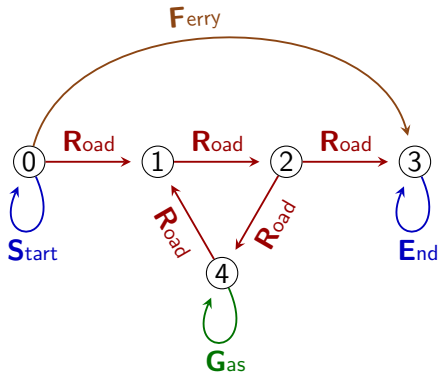
Answer to  $Q_{12}$ :

?

$$Q_{13} = \mathbf{S(R+F)^*G(R+F)^*E}$$

Answer to  $Q_{13}$ :

?



# Shortest semantics (3)

Exercise: evaluating some queries

$$Q_{12} = \mathbf{S(R+F)^*E}$$

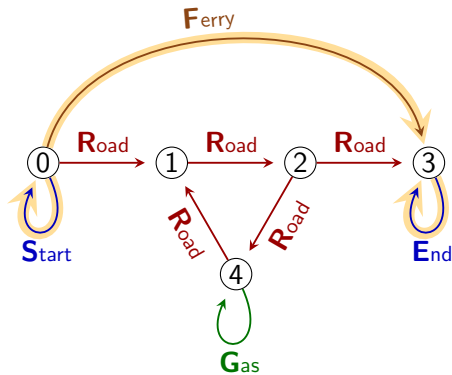
Answer to  $Q_{12}$ :

{ 0 → 0 → 3 → 3 }

$$Q_{13} = \mathbf{S(R+F)^*G(R+F)^*E}$$

Answer to  $Q_{13}$ :

?



# Shortest semantics (3)

Exercise: evaluating some queries

$$Q_{12} = \mathbf{S(R+F)^*E}$$

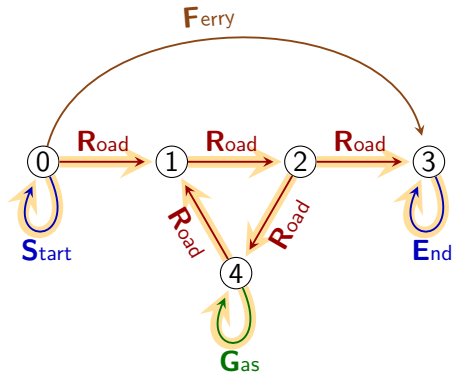
Answer to  $Q_{12}$ :

$$\{ 0 \rightarrow 0 \rightarrow 3 \rightarrow 3 \}$$

$$Q_{13} = \mathbf{S(R+F)^*G(R+F)^*E}$$

Answer to  $Q_{13}$ :

$$\{ 0 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 4 \\ \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 3 \}$$



## Pros and con

### Pros

- Returns walks
- Efficient algorithms (BFS in the product graph  $\times$  query)
- If there are matches from  $s$  to  $t$ , at least one of them is in the answer

## Pros and con

### Pros

- Returns walks
- Efficient algorithms (BFS in the product graph  $\times$  query)
- If there are matches from  $s$  to  $t$ , at least one of them is in the answer

### Cons

- The shortest walk is not always the “best”
  - *“Do we always want to take the ferry over the direct road?”*
  - (Real query languages allow to assign costs to edges/atoms)
- No vertical post-processing
  - Vertical = accross the walks with the same endpoints
  - *“What is the average time?”*
  - *“What is the connectedness level?”*



# Trail semantics (1)

Used by Cypher (Neo4j) and GQL with keyword **ALL TRAIL**

Used by Cypher (Neo4j) and GQL with keyword **ALL TRAIL**

## Principle

- Return a set of walks
- Apply a filter on the set of matching walks
- The filter is: each walk that repeats an edge is filtered out

## Examples

Match

1 → 0 → 2 → 2 → 3

1 → 0 → 2 → 3 → 0 → 2

Decision

No repetition ⇒ Kept in the answer

Repeated edges ⇒ Filtered out

Evaluating  $Q_{14}$

$$Q_{14} = \mathbf{S(R+F)^*E}$$

Applying the filter

Matches

Keep?

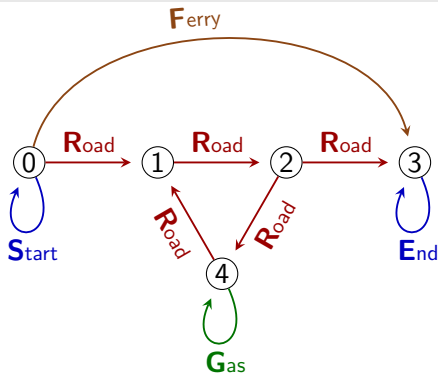
The ferry walk

The straight road

The road with 1 lap

The road with 2 laps

⋮



Answer of  $Q_1$  under trail semantics:

{

}

## Trail semantics (2)

52

Evaluating  $Q_{14}$

$$Q_{14} = \mathbf{S(R+F)^*E}$$

### Applying the filter

Matches

Keep?

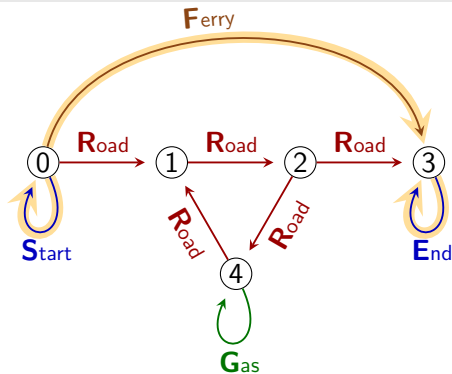
The ferry walk

The straight road

The road with 1 lap

The road with 2 laps

⋮



Answer of  $Q_1$  under trail semantics:

{

}

## Trail semantics (2)

52

Evaluating  $Q_{14}$

$$Q_{14} = \mathbf{S(R+F)^*E}$$

Applying the filter

Matches

Keep?

The ferry walk

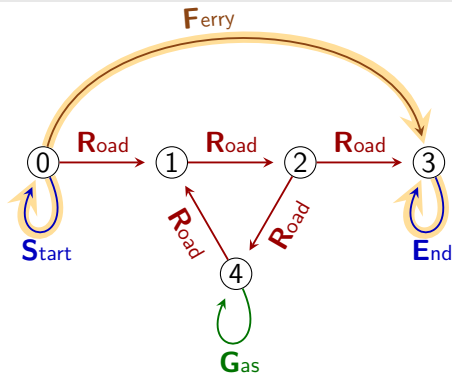
Yes

The straight road

The road with 1 lap

The road with 2 laps

⋮



Answer of  $Q_1$  under trail semantics:

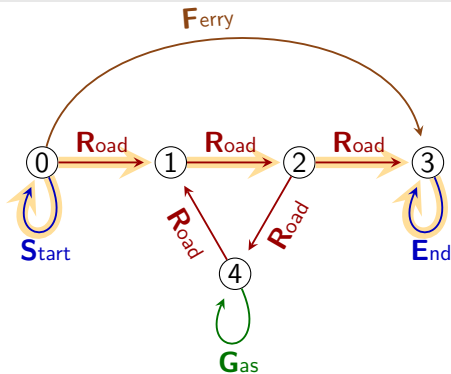
{  $0 \rightarrow 0 \rightarrow 3 \rightarrow 3$  }

Evaluating  $Q_{14}$ 

$$Q_{14} = \mathbf{S(R+F)^*E}$$

Applying the filter

Matches	Keep?
The ferry walk	Yes
The straight road	
The road with 1 lap	
The road with 2 laps	
⋮	

Answer of  $Q_1$  under trail semantics:

{  $0 \rightarrow 0 \rightarrow 3 \rightarrow 3$  }

# Trail semantics (2)

Evaluating  $Q_{14}$

$$Q_{14} = \mathbf{S(R+F)^*E}$$

Applying the filter

Matches	Keep?
---------	-------

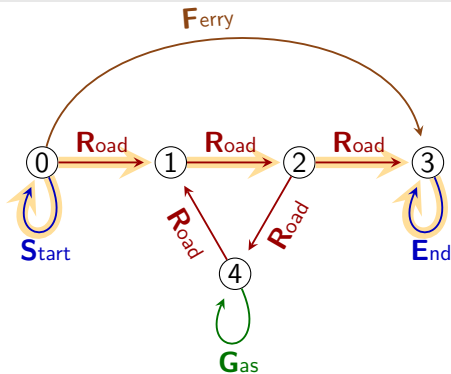
The ferry walk	Yes
----------------	-----

The straight road	Yes
-------------------	-----

The road with 1 lap	
---------------------	--

The road with 2 laps	
----------------------	--

⋮



Answer of  $Q_1$  under trail semantics:

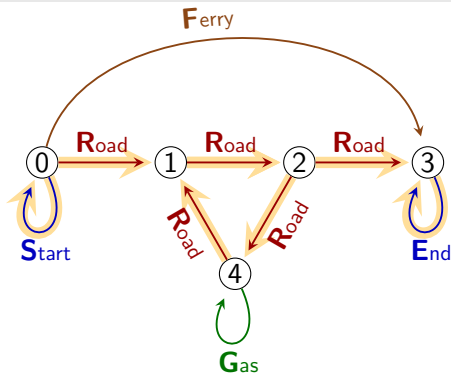
{  $0 \rightarrow 0 \rightarrow 3 \rightarrow 3$ ,  $0 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 3$  }

Evaluating  $Q_{14}$ 

$$Q_{14} = \mathbf{S}(\mathbf{R} + \mathbf{F})^* \mathbf{E}$$

Applying the filter

Matches	Keep?
The ferry walk	Yes
The straight road	Yes
The road with 1 lap	
The road with 2 laps	
⋮	

Answer of  $Q_1$  under trail semantics:
$$\{ \quad 0 \rightarrow 0 \rightarrow 3 \rightarrow 3, \quad 0 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 3 \quad \}$$



# Trail semantics (2)

Evaluating  $Q_{14}$

$$Q_{14} = \mathbf{S(R+F)^*E}$$

Applying the filter

Matches	Keep?
---------	-------

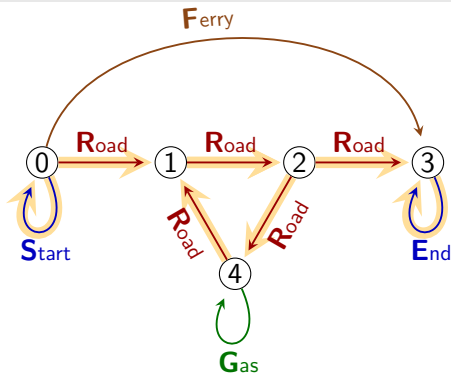
The ferry walk	Yes
----------------	-----

The straight road	Yes
-------------------	-----

The road with 1 lap	No
---------------------	----

The road with 2 laps	
----------------------	--

⋮



Answer of  $Q_1$  under trail semantics:

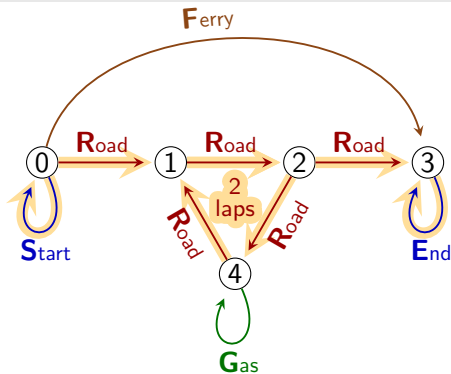
{  $0 \rightarrow 0 \rightarrow 3 \rightarrow 3$ ,  $0 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 3$  }

Evaluating  $Q_{14}$ 

$$Q_{14} = \mathbf{S(R+F)^*E}$$

Applying the filter

Matches	Keep?
The ferry walk	Yes
The straight road	Yes
The road with 1 lap	No
The road with 2 laps	Yes
⋮	

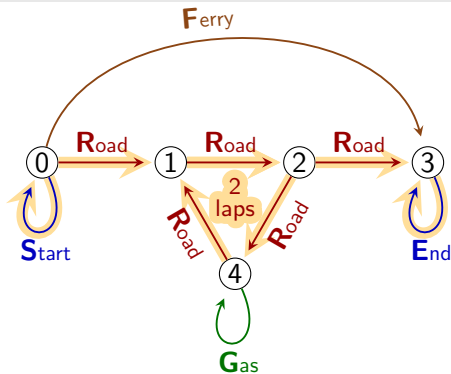
Answer of  $Q_1$  under trail semantics:
$$\{ \quad 0 \rightarrow 0 \rightarrow 3 \rightarrow 3, \quad 0 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 3 \quad \}$$

Evaluating  $Q_{14}$ 

$$Q_{14} = \mathbf{S(R+F)^*E}$$

Applying the filter

Matches	Keep?
The ferry walk	Yes
The straight road	Yes
The road with 1 lap	No
The road with 2 laps	No
⋮	

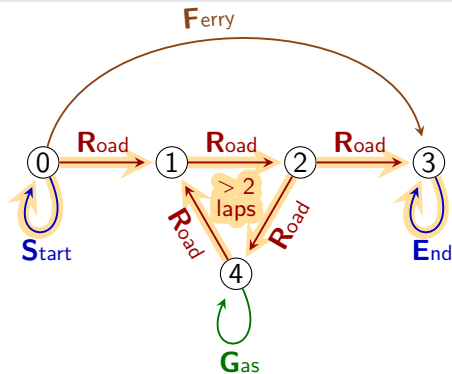
Answer of  $Q_1$  under trail semantics:
$$\{ \quad 0 \rightarrow 0 \rightarrow 3 \rightarrow 3, \quad 0 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 3 \quad \}$$

Evaluating  $Q_{14}$ 

$$Q_{14} = \mathbf{S(R+F)^*E}$$

Applying the filter

Matches	Keep?
The ferry walk	Yes
The straight road	Yes
The road with 1 lap	No
The road with 2 laps	No
⋮	No

Answer of  $Q_1$  under trail semantics:
$$\{ \quad 0 \rightarrow 0 \rightarrow 3 \rightarrow 3, \quad 0 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 3 \quad \}$$

Exercise: evaluating some queries

$$Q_{15} = \mathbf{GR}^*$$

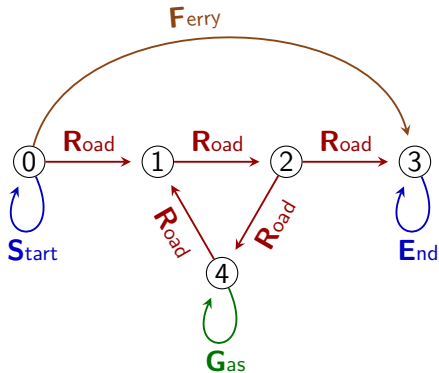
Answer to  $Q_{15}$ :

?

$$Q_{16} = \mathbf{S(R+F)^*G(R+F)^*E}$$

Answer to  $Q_{16}$ :

?

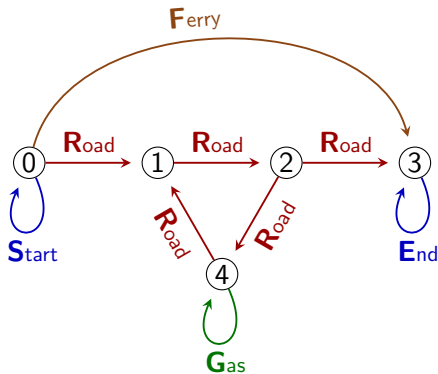


Exercise: evaluating some queries

$$Q_{15} = \mathbf{GR}^*$$

Answer to  $Q_{15}$ :

{  
   $4 \rightarrow 4$  ,  
   $4 \rightarrow 4 \rightarrow 1$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 3$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 4$  }  
}



$$Q_{16} = \mathbf{S(R+F)^*G(R+F)^*E}$$

Answer to  $Q_{16}$ :

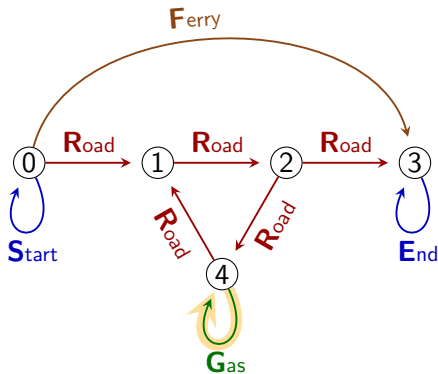
?

Exercise: evaluating some queries

$$Q_{15} = \mathbf{GR}^*$$

Answer to  $Q_{15}$ :

{  $4 \rightarrow 4$ ,  
 $4 \rightarrow 4 \rightarrow 1$ ,  
 $4 \rightarrow 4 \rightarrow 1 \rightarrow 2$ ,  
 $4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 3$ ,  
 $4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 4$  }



$$Q_{16} = \mathbf{S(R+F)^*G(R+F)^*E}$$

Answer to  $Q_{16}$ :

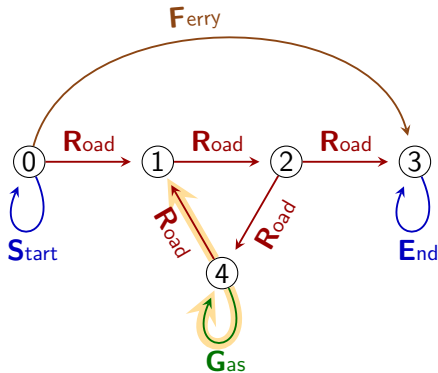
?

Exercise: evaluating some queries

$$Q_{15} = \mathbf{GR}^*$$

Answer to  $Q_{15}$ :

{  
   $4 \rightarrow 4$  ,  
   $4 \rightarrow 4 \rightarrow 1$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 3$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 4$  }  
}



$$Q_{16} = \mathbf{S(R+F)^*G(R+F)^*E}$$

Answer to  $Q_{16}$ :

?

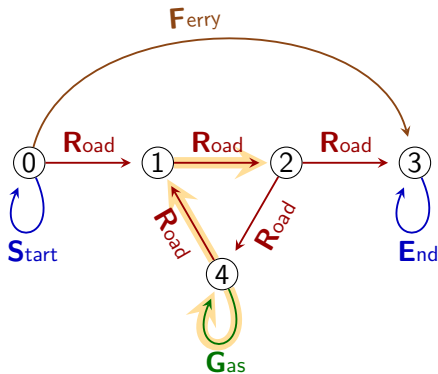


Exercise: evaluating some queries

$$Q_{15} = \mathbf{GR}^*$$

Answer to  $Q_{15}$ :

{  
   $4 \rightarrow 4$  ,  
   $4 \rightarrow 4 \rightarrow 1$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 3$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 4$  }  
}



$$Q_{16} = \mathbf{S(R+F)^*G(R+F)^*E}$$

Answer to  $Q_{16}$ :

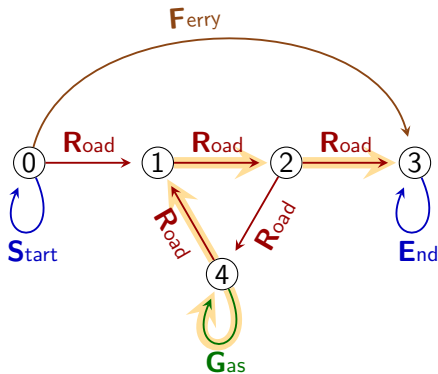
?

Exercise: evaluating some queries

$$Q_{15} = \mathbf{GR}^*$$

Answer to  $Q_{15}$ :

{  
   $4 \rightarrow 4$  ,  
   $4 \rightarrow 4 \rightarrow 1$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 3$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 4$  }  
}



$$Q_{16} = \mathbf{S(R+F)^*G(R+F)^*E}$$

Answer to  $Q_{16}$ :

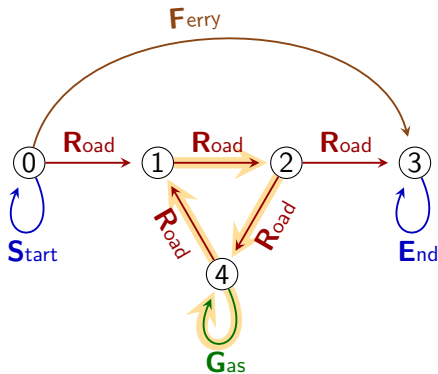
?

Exercise: evaluating some queries

$$Q_{15} = \mathbf{GR}^*$$

Answer to  $Q_{15}$ :

{  
   $4 \rightarrow 4$  ,  
   $4 \rightarrow 4 \rightarrow 1$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 3$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 4$  }  
}



$$Q_{16} = \mathbf{S(R+F)^*G(R+F)^*E}$$

Answer to  $Q_{16}$ :

?

Exercise: evaluating some queries

$$Q_{15} = \mathbf{GR}^*$$

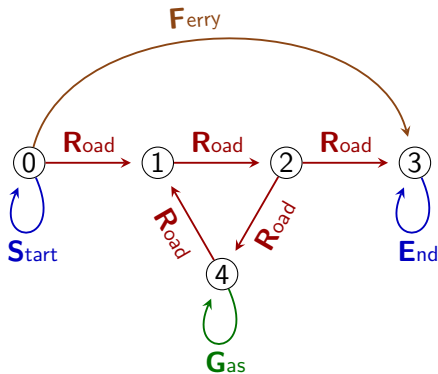
Answer to  $Q_{15}$ :

{ 4  $\rightarrow$  4 ,  
4  $\rightarrow$  4  $\rightarrow$  1 ,  
4  $\rightarrow$  4  $\rightarrow$  1  $\rightarrow$  2 ,  
4  $\rightarrow$  4  $\rightarrow$  1  $\rightarrow$  2  $\rightarrow$  3 ,  
4  $\rightarrow$  4  $\rightarrow$  1  $\rightarrow$  2  $\rightarrow$  4 } }

$$Q_{16} = \mathbf{S(R+F)^*G(R+F)^*E}$$

Answer to  $Q_{16}$ :

?

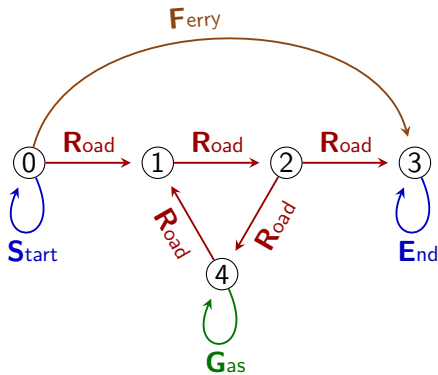


Exercise: evaluating some queries

$$Q_{15} = \mathbf{GR}^*$$

Answer to  $Q_{15}$ :

{  
   $4 \rightarrow 4$  ,  
   $4 \rightarrow 4 \rightarrow 1$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 3$  ,  
   $4 \rightarrow 4 \rightarrow 1 \rightarrow 2 \rightarrow 4$  }  
}



$$Q_{16} = \mathbf{S(R+F)^*G(R+F)^*E}$$

Answer to  $Q_{16}$ :

$\emptyset$

## Pros and cons

### Pros

- Returns walks
- Easy to explain
- Enable vertical post-processing
  - Vertical = across the walks with the same endpoints
  - *“What is the average time?”*
  - *“What is the connectedness level?”*

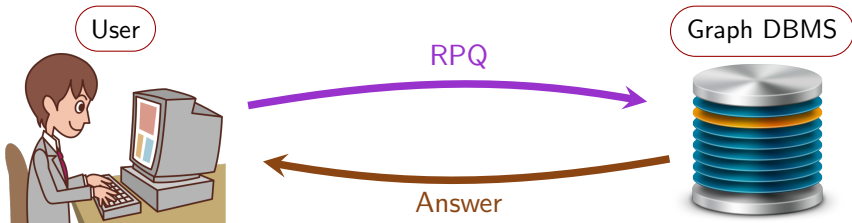
## Pros and cons

### Pros

- Returns walks
- Easy to explain
- Enable vertical post-processing
  - Vertical = accross the walks with the same endpoints
  - “*What is the average time?*”
  - “*What is the connectedness level?*”

### Cons

- **Inefficient** in bad cases.  
Ex: checking whether  $R^*GR^*$  returns anything is NP-hard
- “No repeated edge” is a filter that is sometimes **counterintuitive**  
Ex:  $S(R+F)^*G(R+F)^*E$  had matches but the answer is empty



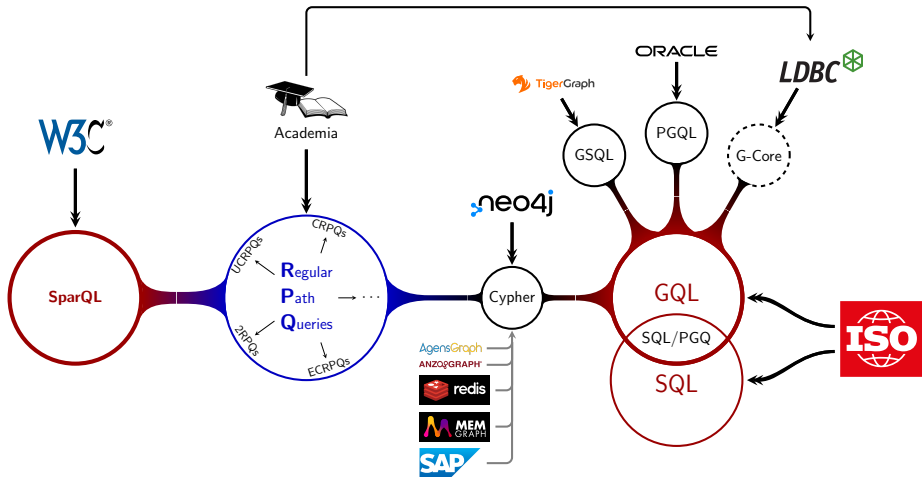
⚠ **Infinitely** many matches but the user expects **finite** answer ⚠

## Different semantics for RPQs

- Endpoint → Filters out all navigational information
- Shortest → No vertical postprocessing and arbitrary metrics
- Trail → Inefficient and sometimes discard meaningful matches

⇒ **No RPQ semantics is clearly superior**





- SparQL and most academic work on RPQs use endpoint semantics
- Cypher uses trail semantics
- GSQL, PGQL and G-Core uses shortest semantics (and variants)
- GQL and SQL/PGQ allow to switch between many RPQ semantics

## **Part II: Neo4j, Property graphs and Cypher**

Part II: Neo4j, Property graphs and Cypher

## 1. **Data model: Property graphs**

A **node** ( $\approx$ vertex) encodes a complex values.

It bears **labels** are for grouping.

Ex:  $t$  carries **Teacher**, **Person**  
 $c$  carries **Course**

A **Relation** ( $\approx$ edge) connects **nodes**

It bears one **type** ( $\approx$ label) provides the nature of the relation

Ex:  $e = t \xrightarrow{\text{TEACHES}} c$

A **property** describes an aspect of a **node** or an **relation**

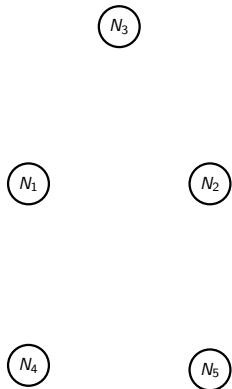
It maps

- a **key** (described aspect)
- to a **pure value** (description)

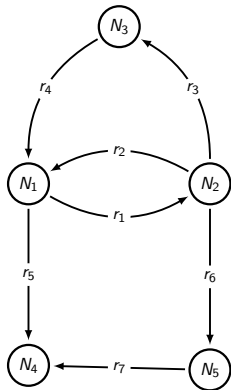
Ex:  $t$  has **name**:"Victor"  
 $e$  has **since**:2023

A **pure value** (int, string, etc) contains all the information about itself.

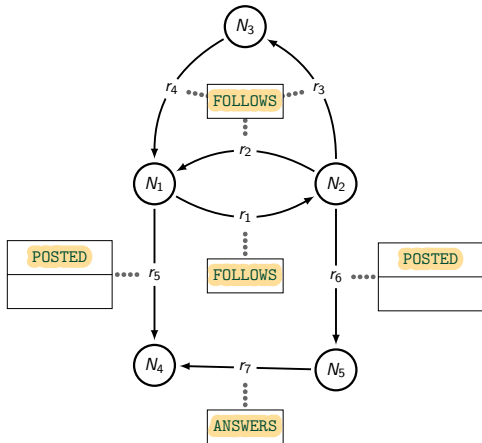
Ex: "Victor" has 6 letters



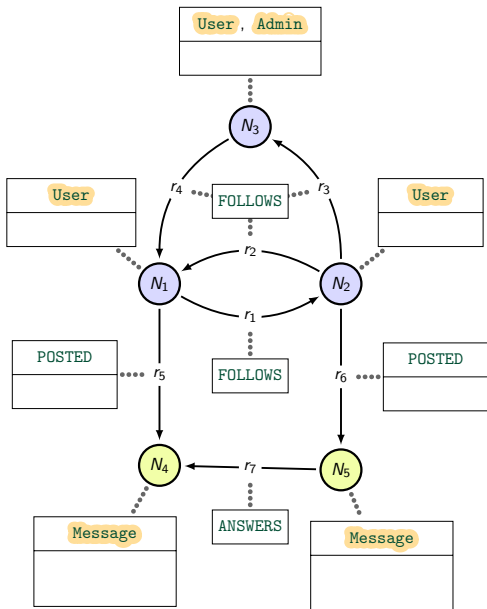
- Nodes :  $N_1, N_2, \dots, N_5$



- Nodes :  $N_1, N_2, \dots, N_5$
- Relations :  $r_1, r_2, \dots, r_7$



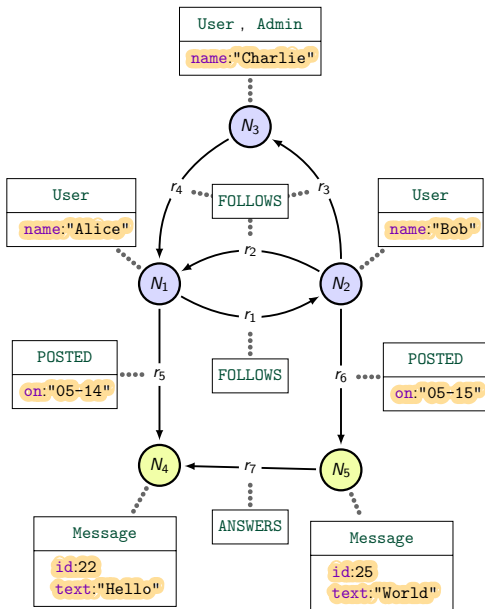
- Nodes :  $N_1, N_2, \dots, N_5$
- Relations :  $r_1, r_2, \dots, r_7$
- Types: follows, posted, answers



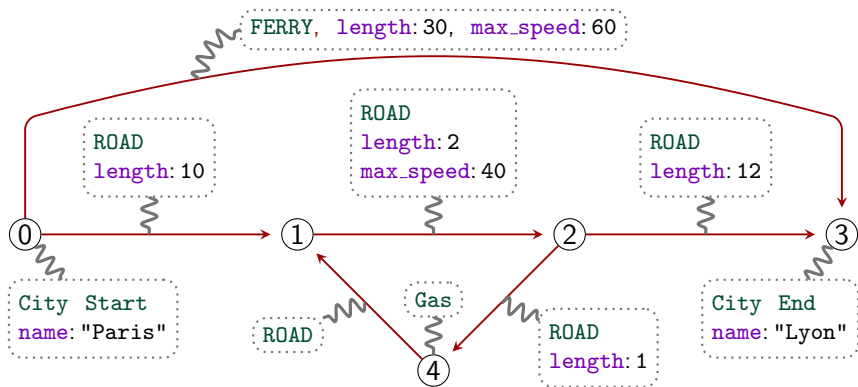
- Nodes :  $N_1, N_2, \dots, N_5$
- Relations :  $r_1, r_2, \dots, r_7$
- Types: follows, posted, answers
- Labels: User, Admin, Message



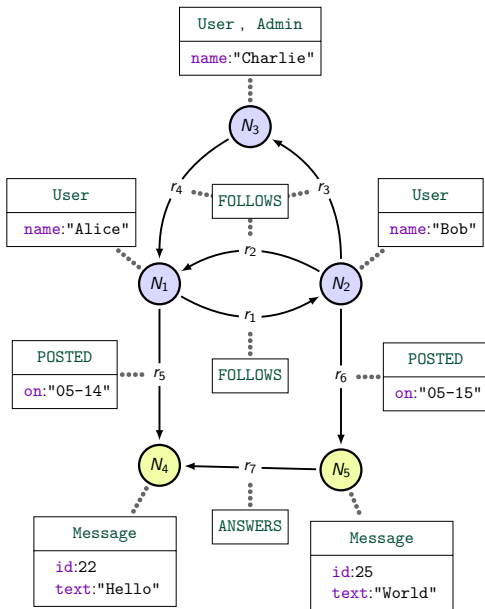
# First example of a property graph



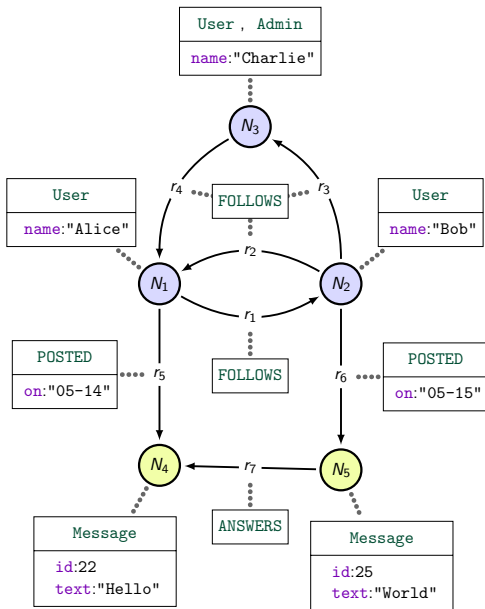
- Nodes :  $N_1, N_2, \dots, N_5$
- Relations :  $r_1, r_2, \dots, r_7$
- Types: follows, posted, answers
- Labels: User, Admin, Message
- Properties, that is Key-Value pairs:
  - name: "Alice"
  - id: 22
  - text: "Hello"etc.



# Storing graph 1 in tables

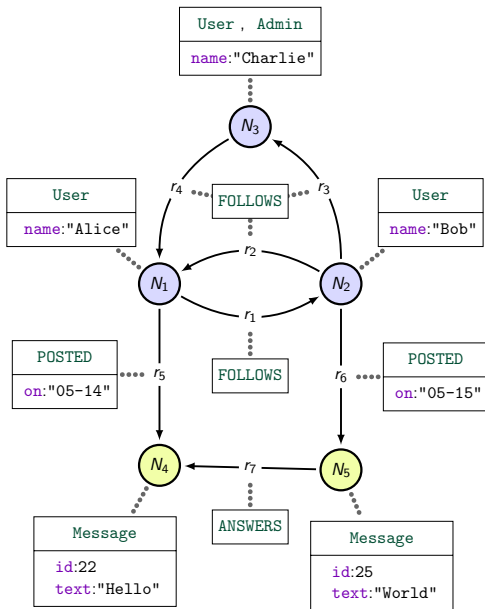


# Storing graph 1 in tables



Node	Relation		
<u>id</u>	<u>id</u>	#src	#tgt
1	1	1	2
2	2	2	1
⋮	⋮	⋮	⋮

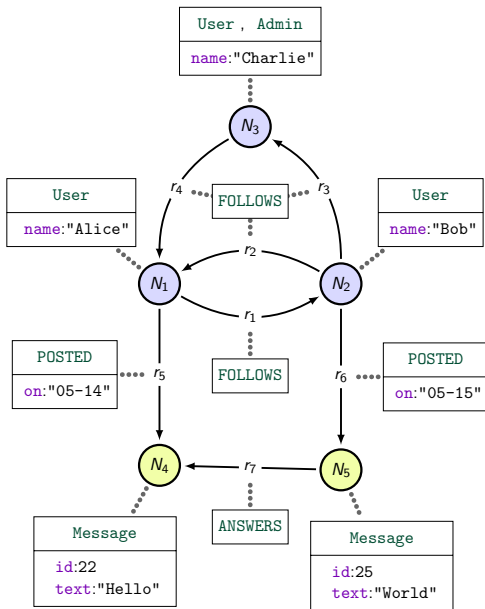
# Storing graph 1 in tables



<u>Node</u>	<u>Relation</u>		
<u>id</u>	<u>id</u>	<u>#src</u>	<u>#tgt</u>
1	1	1	2
2	2	2	1
⋮	⋮	⋮	⋮

<u>Posted</u>	<u>Message</u>
<u>#eid</u>	<u>#vid</u>
5	4
6	5

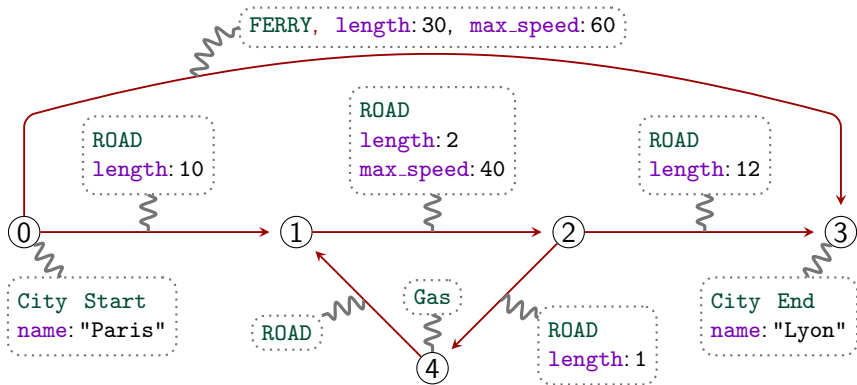
# Why so many tables?



<u>Node</u>	<u>Relation</u>		
id	id	#src	#tgt
1	1	1	2
2	2	2	1
⋮	⋮	⋮	⋮

<u>Posted</u>	<u>Message</u>
#eid	#vid
5	4
6	5

<u>On</u>		<u>Id</u>	
#eid	val	#vid	val
5	"05-14"	4	22
6	"05-15"	5	25



- Relations with the same type may have different property keys
- Nodes may have any number of labels and property keys

## Native storage

- Similar to "adjacency lists" : each node knows its adjacent relations
- Query answering is based on graph algorithms and not on joins  
Ex:  $S(R+F)^2$ ,  $S(R+F)^3$ ,  $S(R+F)^*$
- Allows flexible schemas or a schema-less approach

NB: some graph DBMS do not use native storage



## Native storage

- Similar to "adjacency lists" : each node knows its adjacent relations
- Query answering is based on graph algorithms and not on joins  
Ex:  $S(R+F)^2$ ,  $S(R+F)^3$ ,  $S(R+F)^*$
- Allows flexible schemas or a schema-less approach

NB: some graph DBMS do not use native storage

## Specialized algorithms and languages

Restriction on the DM increases the liberty in the query language

- Graph notions in the core of the language (path as values)
- Graph algorithm directly available

"We never have to treat the case of non-binary relations"

NB: Relational DBMS require a graph-view (SQL/PGQ)

Part II: Neo4j, Property graphs and Cypher

## **2. General presentation of Cypher**

## A Cypher query

- queries a property graph
- returns a table

## Example of Cypher query:

```
MATCH (u1)-[p1:POSTED]->(m1)
WHERE p1.id = 22
RETURN u1.name AS uname,
       p1.on AS date,
       m1.text AS msg
```

## Example Returned table

uname	date	msg
"Alice"	"05-14"	"Hello"

## A Cypher query

- queries a property graph
- returns a table
- Is a sequence of **clauses**  
(3 clauses on the right)
- Last clause is always **RETURN**

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## Example Returned table

uname	date	msg
"Alice"	"05-14"	"Hello"

## A Cypher query

- queries a property graph
- returns a table
- Is a sequence of **clauses**  
(3 clauses on the right)
- Last clause is always **RETURN**
- manipulates a working table
- uses **variables**, which refer to column names

## Example of Cypher query:

```
MATCH (u1)-[p1:POSTED]->(m1)
WHERE p1.id = 22
RETURN u1.name AS uname,
       p1.on AS date,
       m1.text AS msg
```

## Example Returned table

uname	date	msg
"Alice"	"05-14"	"Hello"

- **Values** are the elements that may appear in tables
- **Pure values** are the values with no reference to the graph
- **Property** is a key to pure values

## Values are

- Base values Ex: true, 42, "NoSQL"
- Graph elements Ex: nodes, relations
- Paths (alternate lists of nodes and relations)
- List of values Ex: [1, "Hello", true, "World, n<sub>1</sub>]
- Property dictionary Ex: {name: "Victor", age: 35}

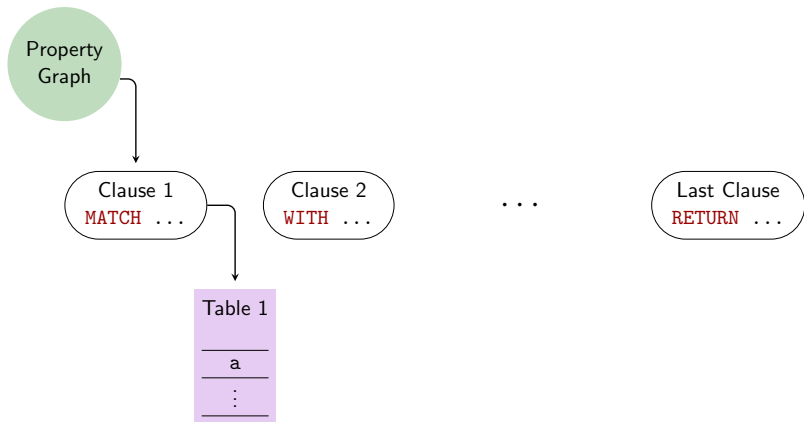
Property  
Graph

Clause 1  
**MATCH** ...

Clause 2  
**WITH** ...

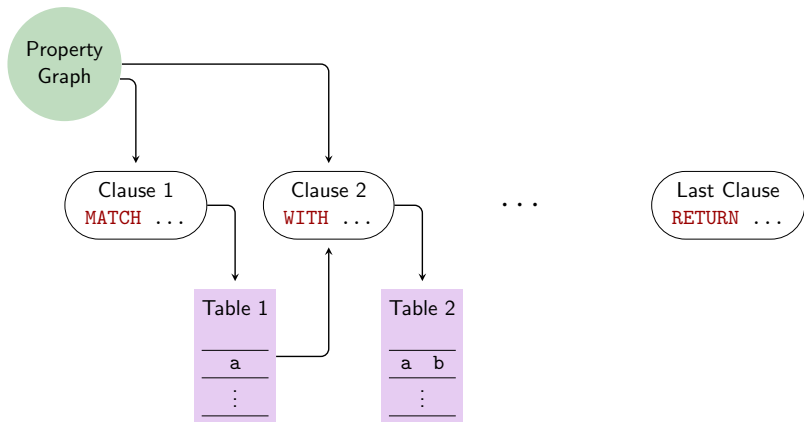
...

Last Clause  
**RETURN** ...

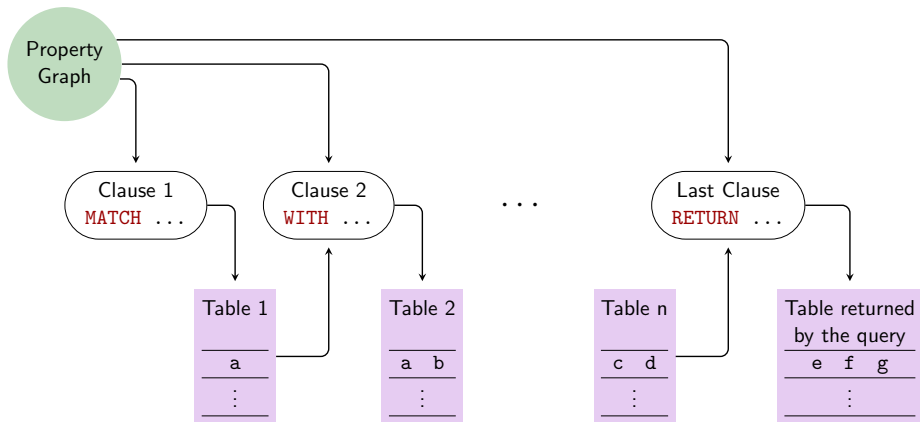


- The first clause produces a table from the property graph





- The first clause produces a table from the property graph
- Subsequent clauses produces a new table from the property graph **and the prior table**



- The first clause produces a table from the property graph
- Subsequent clauses produces a new table from the property graph **and the prior table**
- Until we reach the last clause, which produces the table to return

**MATCH** is for pattern matching

- RPQ-like
- Trail semantics
- Projects paths into a table
- Inner join with the input table
- The variant **OPTIONAL MATCH** does an outer join instead

**WHERE** filters rows

- Subclause of **WITH** and **MATCH**

**UNWIND** splits rows for each element in a list

**WITH** is for:

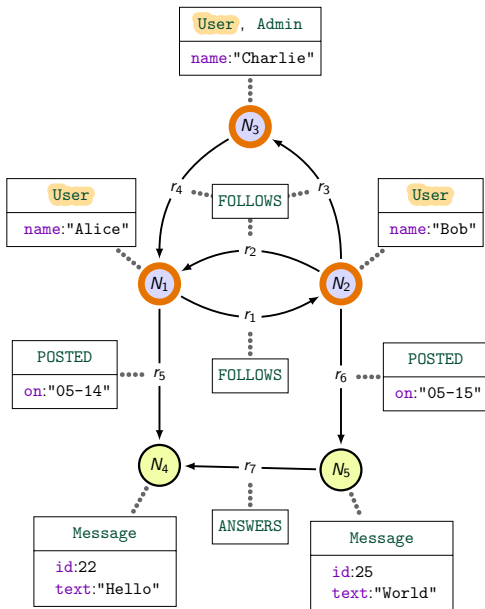
- Column manipulation (add, remove, rename, etc.)
- Aggregation
  - Vertical
  - Horizontal (**reduce**)
- Order and limit output size (**ORDER BY**, **SKIP** and **LIMIT**)

**RETURN** is a mandatory **WITH** at the end of the query

**UNION** and **UNION ALL** are for set and bag union.

Part II: Neo4j, Property graphs and Cypher

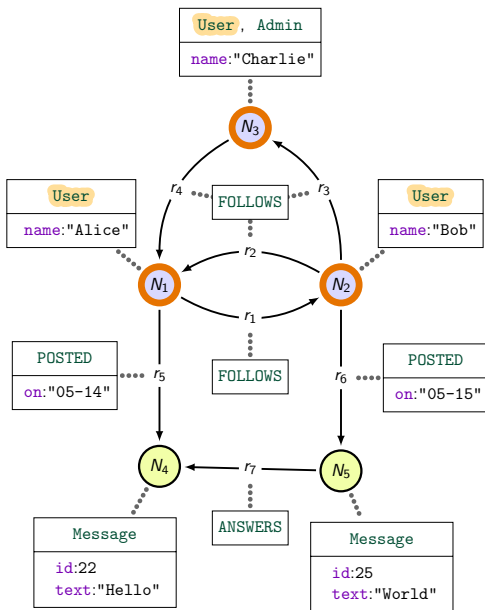
### 3. Pattern matching with **MATCH**



Query:

**MATCH** (u1:User)

# Matching nodes (1)

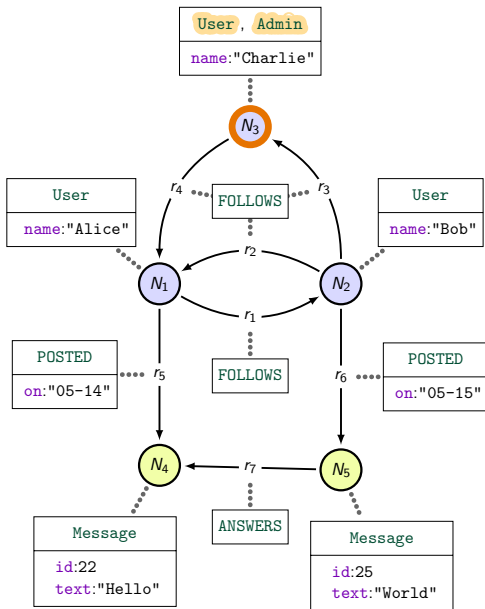


Query:

**MATCH** (u1:User)

Result:

u1
$N_1$
$N_2$
$N_3$



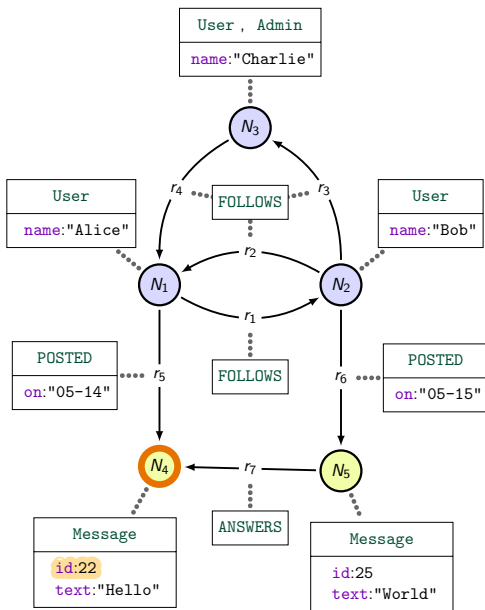
Query:

**MATCH** (u1:User:Admin)

Result:

u1

$N_3$



Query:

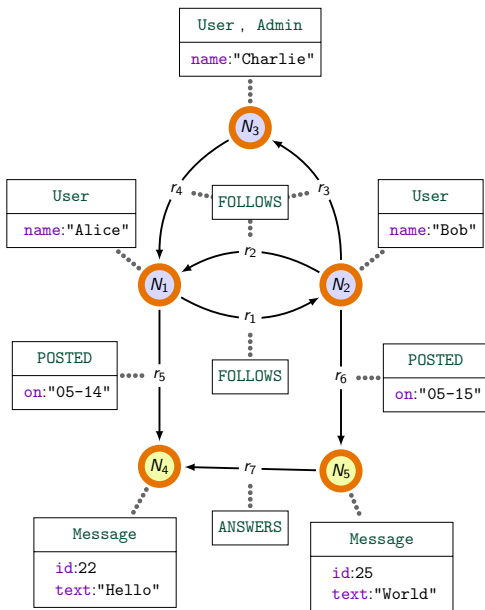
`MATCH (u1{id:22})`

Result:

u1

N4



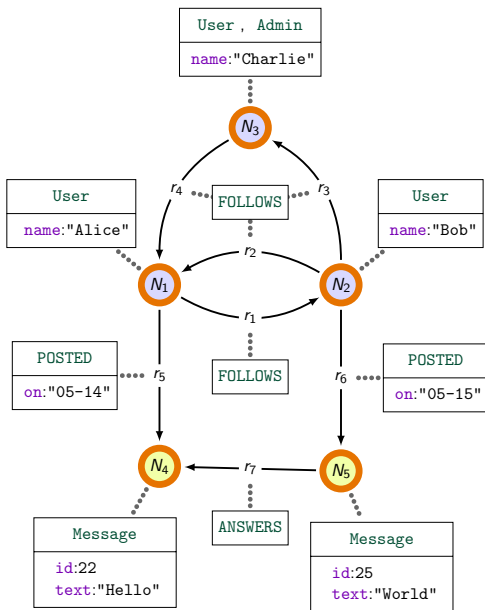


Query:

**MATCH** (u1)

Result:

# Matching nodes (4)

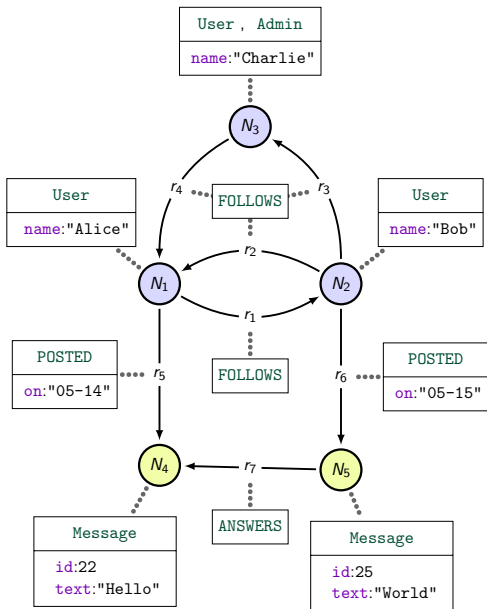


Query:

**MATCH** (u1)

Result:

u1  
 $N_1$   
 $N_2$   
 $N_3$   
 $N_4$   
 $N_5$

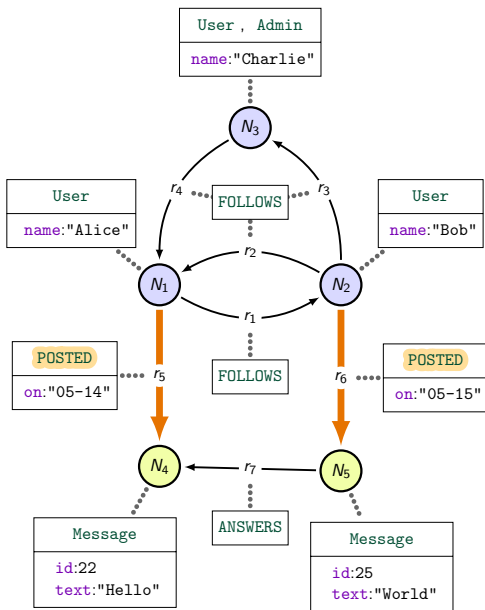


Query:

**MATCH** ()-[p1]->()

Result:

          
p1  
          
r1  
r2  
r3  
r4  
r5  
r6  
r7

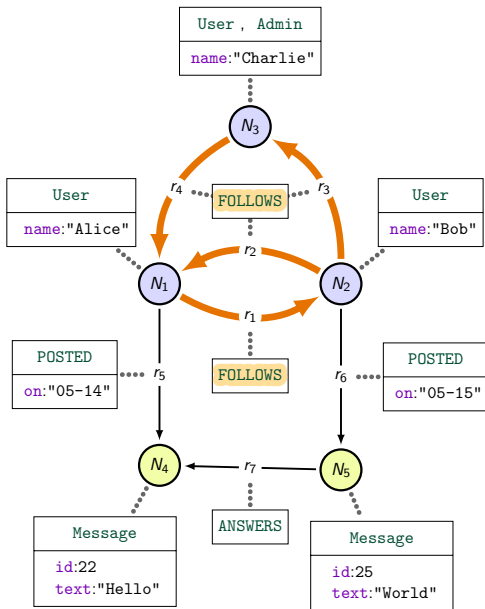


Query:

**MATCH** (u1) - [p1:POSTED] -> (m1)

Result:

u1	p1	m1
$N_1$	$r_5$	$N_4$
$N_2$	$r_6$	$N_5$



Query:

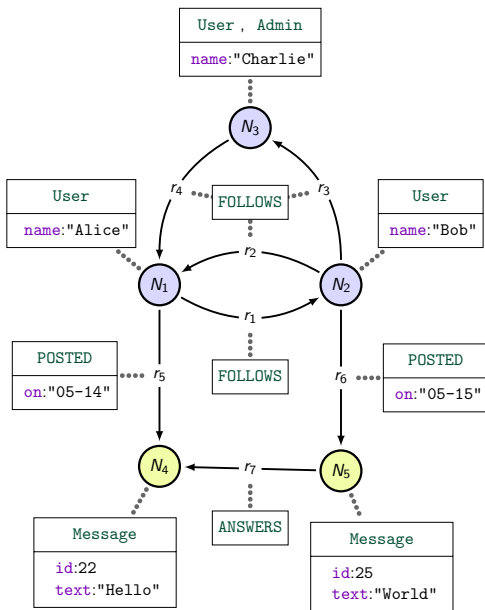
`MATCH (u1)-[:FOLLOWS]->()`

Result:

u1
$N_1$
$N_2$
$N_2$
$N_3$

Cypher has bag semantics:  
 $N_2$  has two outgoing `follows` relations  $\Rightarrow$  two lines  $N_2$

# Matching joined relations (1)



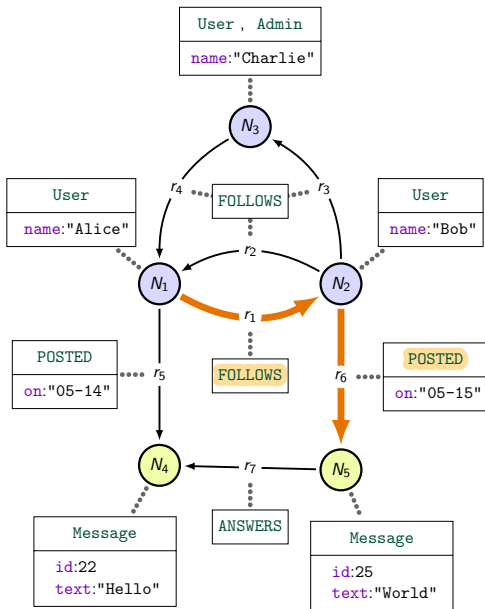
Query:

```
MATCH (u1)-[:FOLLOWS]->()  
      -[:POSTED]->(m1)
```

Result:

u1	m1
$N_1$	$N_5$
$N_2$	$N_4$
$N_3$	$N_5$

# Matching joined relations (1)



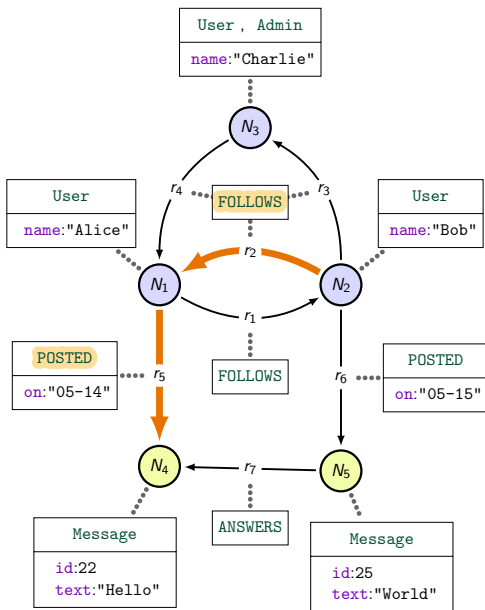
Query:

```
MATCH (u1)-[:FOLLOWS]->()  
      -[:POSTED]->(m1)
```

Result:

u1	m1
$N_1$	$N_5$
$N_2$	$N_4$
$N_3$	$N_5$

# Matching joined relations (1)



Query:

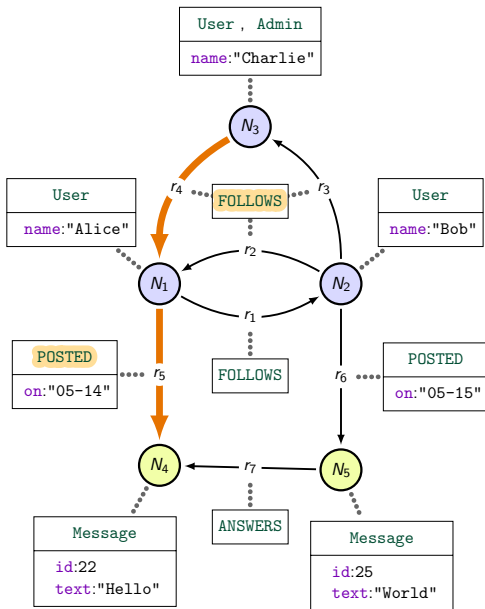
```
MATCH (u1)-[:FOLLOWS]->()  
      -[:POSTED]->(m1)
```

Result:

u1	m1
$N_1$	$N_5$
$N_2$	$N_4$
$N_3$	$N_5$



# Matching joined relations (1)



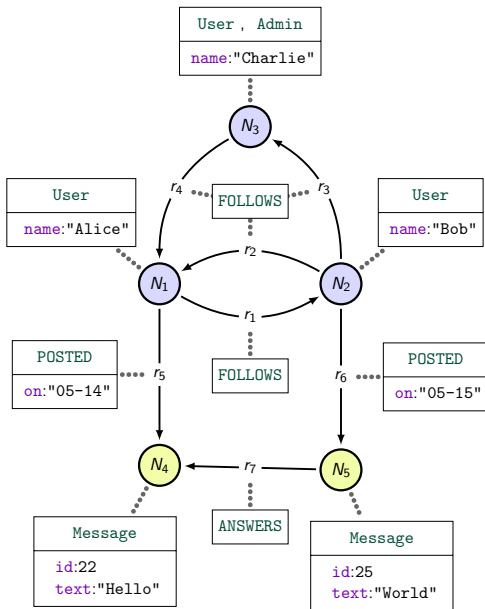
Query:

```
MATCH (u1)-[:FOLLOWS]->()  
      -[:POSTED]->(m1)
```

Result:

u1	m1
$N_1$	$N_5$
$N_2$	$N_4$
$N_3$	$N_5$

## Matching joined relations (2)



Query:

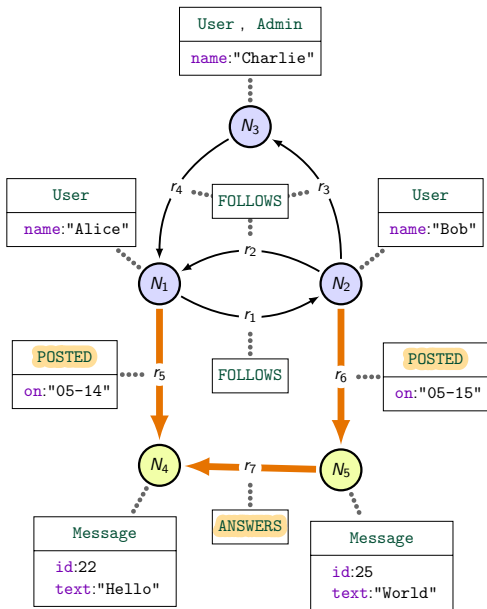
```
MATCH (u1)-[:POSTED]->()  
<-[:ANSWERS]- (m2)  
<-[:POSTED]- (u2)
```

Result:

u1	m2	m2
$N_1$	$N_5$	$N_2$



Cypher allows backward relations



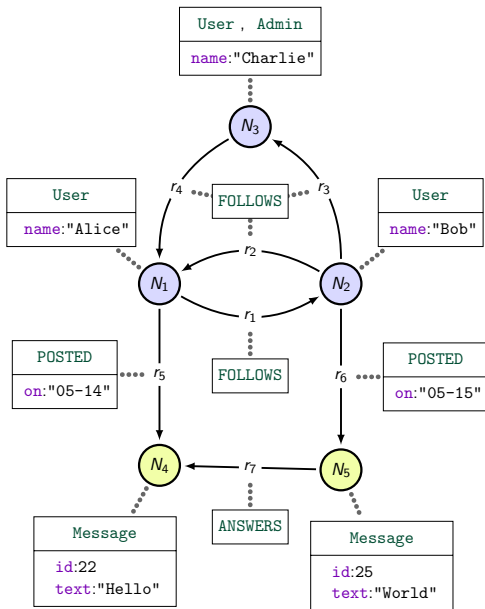
Query:

```
MATCH (u1)-[:POSTED]->()
      <-[:ANSWERS]- (m2)
      <-[:POSTED]- (u2)
```

Result:

u1	m2	m2
$N_1$	$N_5$	$N_2$

⚠️ Cypher allows backward relations

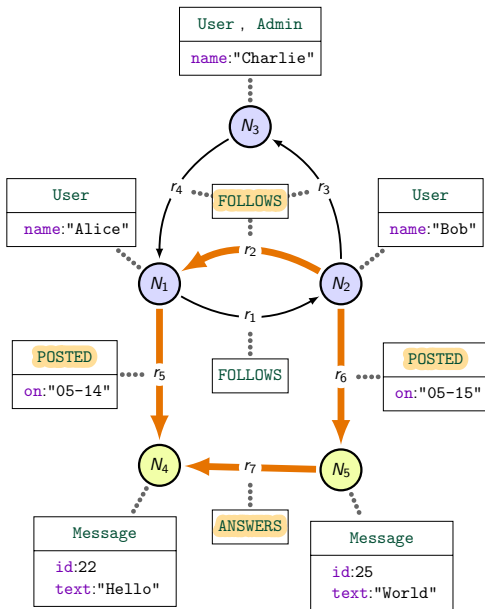


Query:

```
MATCH (u1)-[:POSTED]->()
      <-[:ANSWERS]-(m2)
      <-[:POSTED]-(u2)
      -[:FOLLOWS]->(u1)
```

Result:

u1	u2	m2
$N_1$	$N_5$	$N_2$

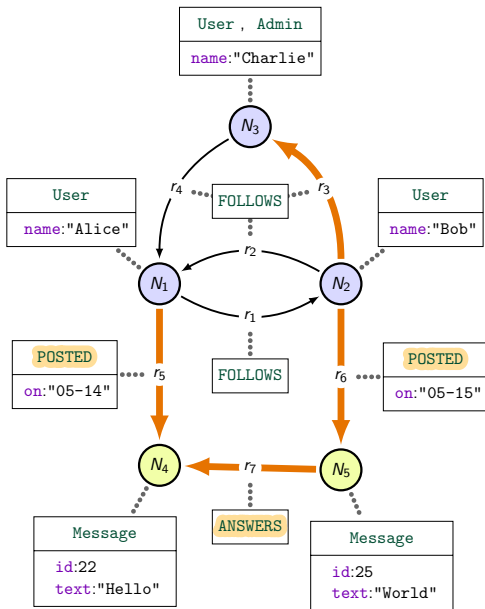


Query:

```
MATCH (u1)-[:POSTED]->()
      <-[:ANSWERS]-(m2)
      <-[:POSTED]-(u2)
      -[:FOLLOWS]->(u1)
```

Result:

u1	u2	m2
$N_1$	$N_5$	$N_2$

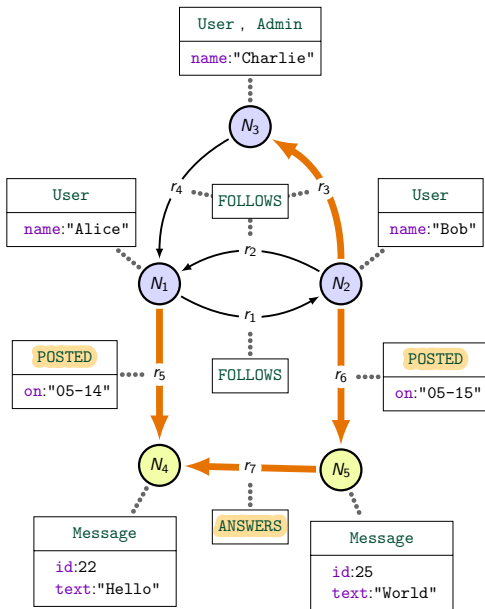


Query:

```
MATCH (u1)-[:POSTED]->()
      <-[:ANSWERS]-(m2)
      <-[:POSTED]-(u2)
      -[:FOLLOWS]->(u1)
```

Result:

u1	u2	m2
$N_1$	$N_5$	$N_2$



Query:

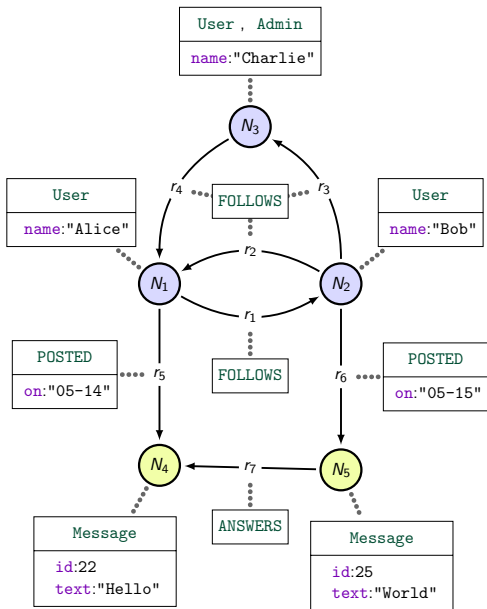
```
MATCH (u1)-[:POSTED]->()
      <-[:ANSWERS]-(m2)
      <-[:POSTED]-(u2)
      -[:FOLLOWS]->(u1)
```

Result:

u1	u2	m2
$N_1$	$N_5$	$N_2$

This path is invalid: two different nodes for u1.

Variable reuse  $\implies$  equality



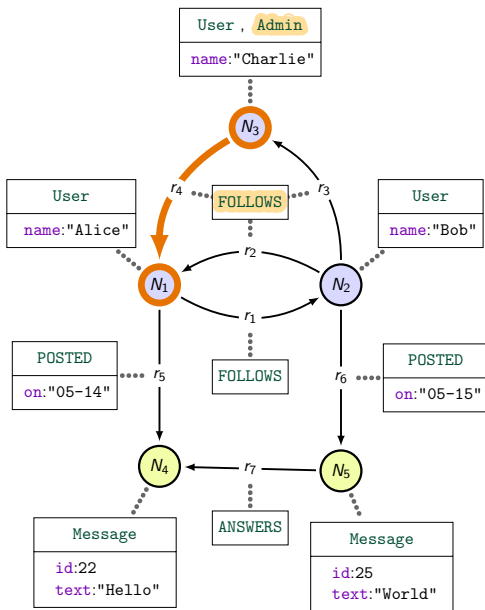
Query:

**MATCH** (u1:Admin)  
 -[l1:FOLLOWS\*]->(m1)

Result:

u1	l1	m1
$N_3$	$[r_4]$	$N_1$
$N_3$	$[r_4, r_1]$	$N_2$
$N_3$	$[r_4, r_1, r_2]$	$N_1$
$N_3$	$[r_4, r_1, r_3]$	$N_3$



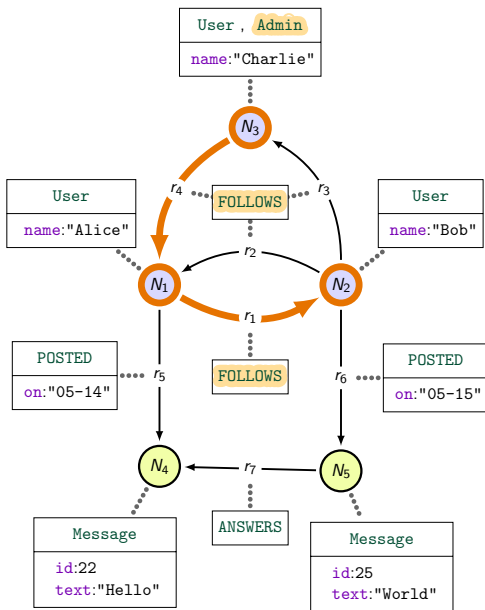


Query:

**MATCH** (u1:Admin)  
 -[l1:FOLLOWS\*]->(m1)

Result:

u1	l1	m1
<i>N<sub>3</sub></i>	[ <i>r<sub>4</sub></i> ]	<i>N<sub>1</sub></i>
<i>N<sub>3</sub></i>	[ <i>r<sub>4</sub>, r<sub>1</sub></i> ]	<i>N<sub>2</sub></i>
<i>N<sub>3</sub></i>	[ <i>r<sub>4</sub>, r<sub>1</sub>, r<sub>2</sub></i> ]	<i>N<sub>1</sub></i>
<i>N<sub>3</sub></i>	[ <i>r<sub>4</sub>, r<sub>1</sub>, r<sub>3</sub></i> ]	<i>N<sub>3</sub></i>



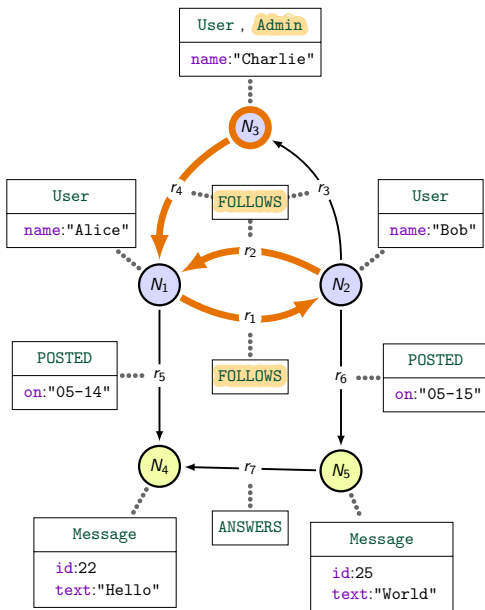
Query:

**MATCH** (u1:Admin)  
 -[l1:FOLLOWS\*]->(m1)

Result:

u1	l1	m1
$N_3$	$[r_4]$	$N_1$
$N_3$	$[r_4, r_1]$	$N_2$
$N_3$	$[r_4, r_1, r_2]$	$N_1$
$N_3$	$[r_4, r_1, r_3]$	$N_3$

# Matching paths (1)

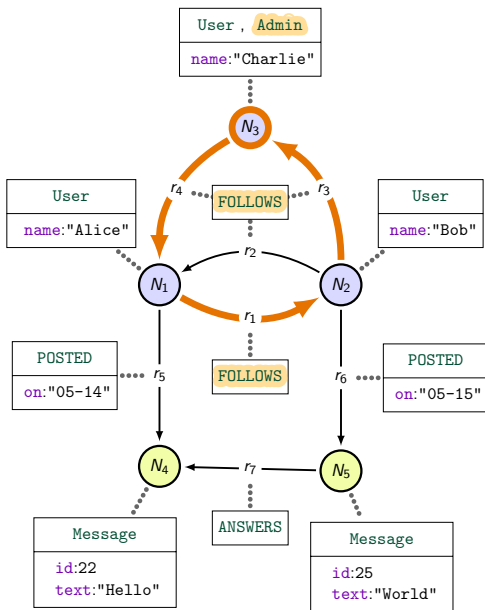


Query:

```
MATCH (u1:Admin)
      -[l1:FOLLOWS*]->(m1)
```

Result:

u1	l1	m1
$N_3$	$[r_4]$	$N_1$
$N_3$	$[r_4, r_1]$	$N_2$
$N_3$	$[r_4, r_1, r_2]$	$N_1$
$N_3$	$[r_4, r_1, r_3]$	$N_3$

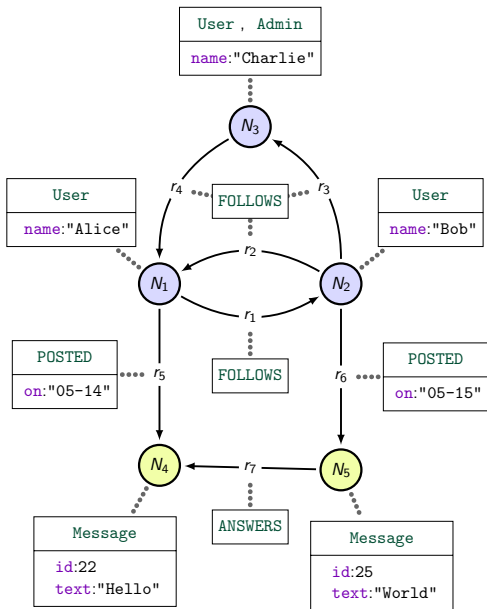


Query:

**MATCH** (u1:Admin)  
 -[l1:FOLLOWS\*]->(m1)

Result:

u1	l1	m1
$N_3$	[ $r_4$ ]	$N_1$
$N_3$	[ $r_4, r_1$ ]	$N_2$
$N_3$	[ $r_4, r_1, r_2$ ]	$N_1$
$N_3$	[ $r_4, r_1, r_3$ ]	$N_3$



Query:

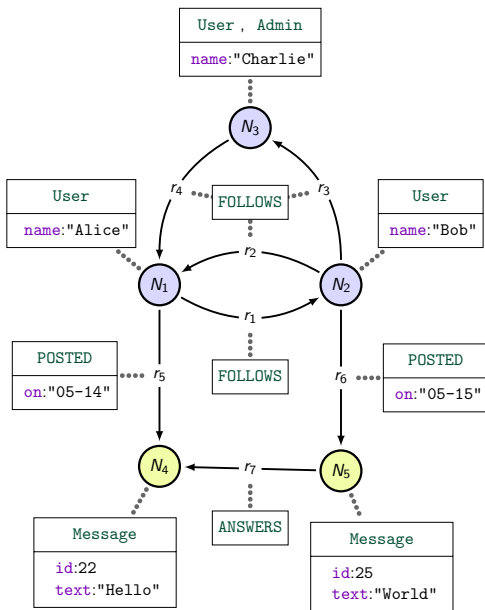
```
MATCH (u1:Admin)
      -[l1:FOLLOWS*]->(m1)
```

Result:

u1	l1	m1
$N_3$	$[r_4]$	$N_1$
$N_3$	$[r_4, r_1]$	$N_2$
$N_3$	$[r_4, r_1, r_2]$	$N_1$
$N_3$	$[r_4, r_1, r_3]$	$N_3$

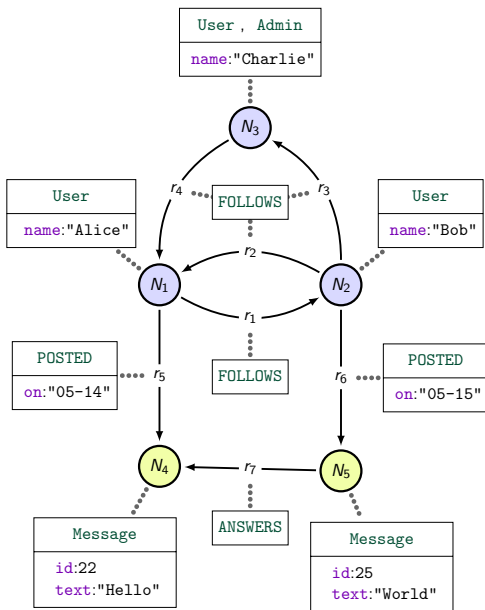
Cypher uses **trail semantics**.

In Cypher the Kleene star means **one or more**.



Query:

```
MATCH (u2:)-[:FOLLOWS]->(u1)
      <-[:FOLLOWS]-(u3)
```



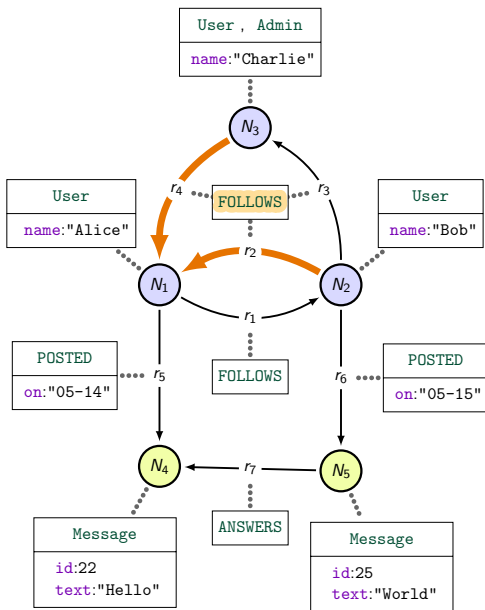
Query:

**MATCH** (u2:)-[:FOLLOWS]->(u1)  
 <-[:FOLLOWS]-(u3)

Result:

u2	u1	u3
$N_3$	$N_1$	$N_2$
$N_2$	$N_1$	$N_3$

- Line 1:  $N_3 \xrightarrow{r_4} N_1 \xleftarrow{r_2} N_2$
- Line 2:  $N_2 \xrightarrow{r_2} N_1 \xleftarrow{r_4} N_3$
- No  $(N_3, N_1, N_3)$  due to trail semantics



Query:

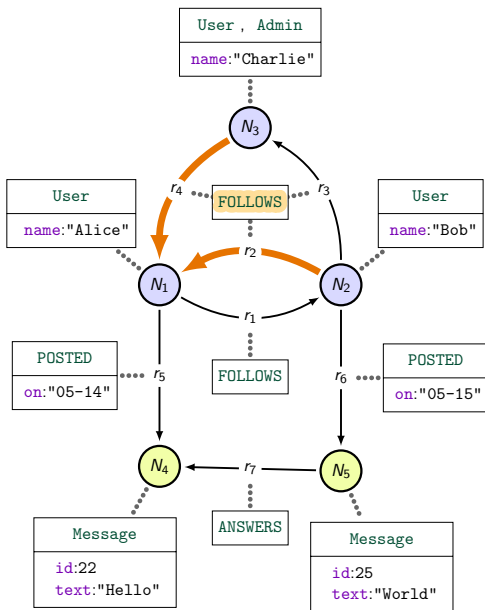
**MATCH** (u2:)-[:FOLLOWS]->(u1)  
 <-[:FOLLOWS]-(u3)

Result:

u2	u1	u3
$N_3$	$N_1$	$N_2$
$N_2$	$N_1$	$N_3$

- Line 1:  $N_3 \xrightarrow{r_4} N_1 \xleftarrow{r_2} N_2$
- Line 2:  $N_2 \xrightarrow{r_2} N_1 \xleftarrow{r_4} N_3$
- No  $(N_3, N_1, N_3)$  due to trail semantics





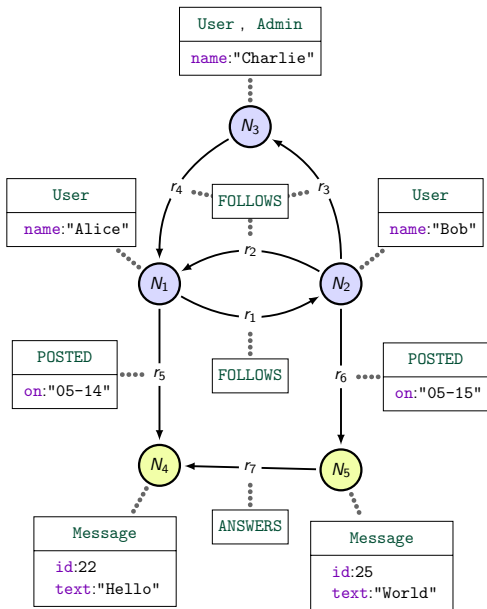
Query:

**MATCH** (u2:)-[:FOLLOWS]->(u1)  
 <-[:FOLLOWS]-(u3)

Result:

u2	u1	u3
$N_3$	$N_1$	$N_2$
$N_2$	$N_1$	$N_3$

- Line 1:  $N_3 \xrightarrow{r_4} N_1 \xleftarrow{r_2} N_2$
- Line 2:  $N_2 \xrightarrow{r_2} N_1 \xleftarrow{r_4} N_3$
- No  $(N_3, N_1, N_3)$  due to trail semantics



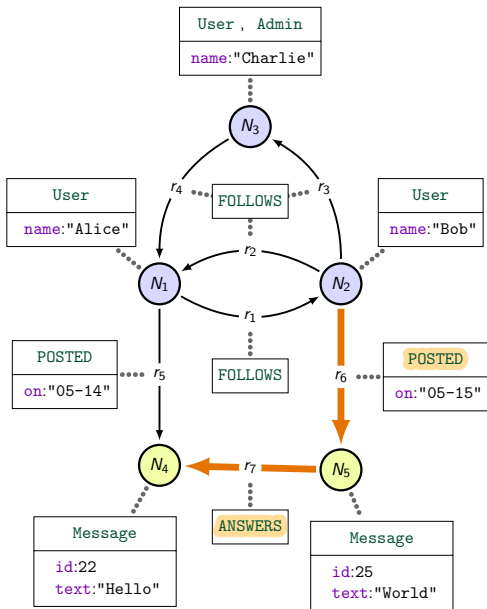
Query:

**MATCH** (u1)

- [l1:POSTED | ANSWERS \*]->(m1)

Result:

u1	l1	m1
$N_2$	$[r_6, r_7]$	$N_4$
$N_5$	$[r_7]$	$N_4$
$N_2$	$[r_6]$	$N_5$

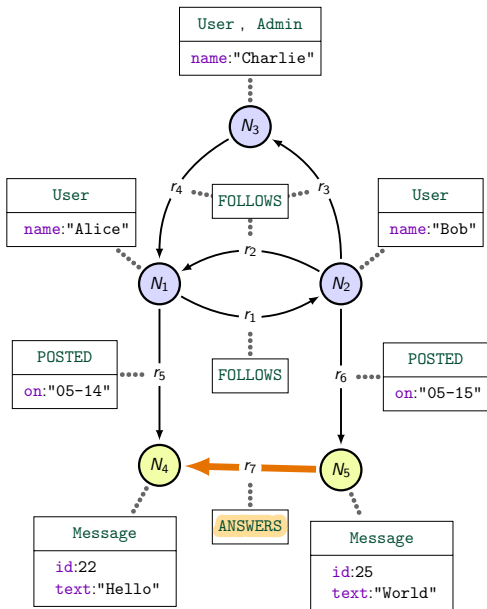


Query:

**MATCH** (u1)  
 - [l1:POSTED | ANSWERS \*]->(m1)

Result:

u1	l1	m1
$N_2$	$[r_6, r_7]$	$N_4$
$N_5$	$[r_7]$	$N_4$
$N_2$	$[r_6]$	$N_5$

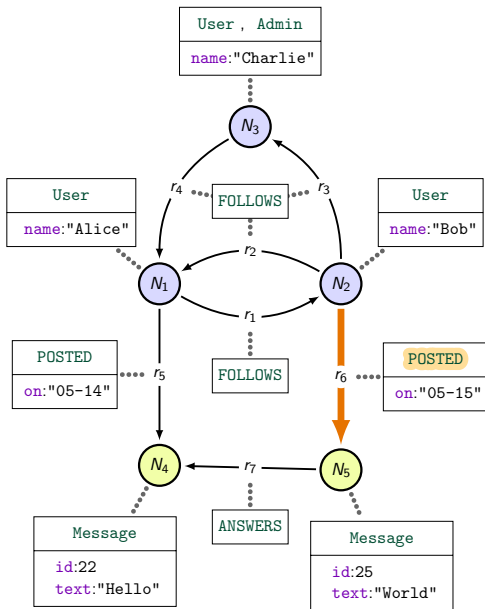


Query:

**MATCH** (u1)  
 - [l1:POSTED | ANSWERS \*]->(m1)

Result:

u1	l1	m1
$N_2$	$[r_6, r_7]$	$N_4$
$N_5$	$[r_7]$	$N_4$
$N_2$	$[r_6]$	$N_5$



Query:

**MATCH** (u1)  
 - [l1:POSTED | ANSWERS \*]->(m1)

Result:

u1	l1	m1
$N_2$	$[r_6, r_7]$	$N_4$
$N_5$	$[r_7]$	$N_4$
$N_2$	$[r_6]$	$N_5$

## MATCH allows RPQ-like pattern-matching

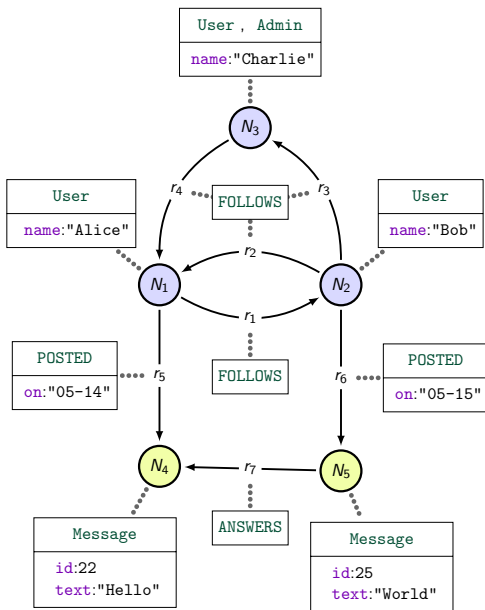
- Computes paths
- Uses trail semantics to keep the output finite
- Project paths on variables

## MATCH does not allow the full extent of regular expressions

- Only disjunction of atoms under star Ex:  $(R^*G)^*$  and  $(RR)^*$
- Disjunction only for atoms Ex:  $R^* + F$  and  $RR + FF$

## MATCH goes beyond RPQs

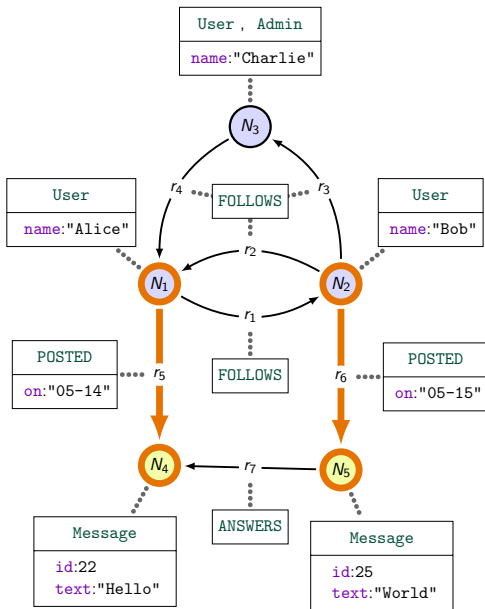
- Matching against properties Ex: `MATCH ({id:22})`
- Taking relation backward Ex: `MATCH ()<-[e]-()`
- Implicit join on variable reuse Ex: `MATCH (a)<-[*]-(a)`



Query:

**MATCH** (u1)-[:**POSTED**]->(m1)

**MATCH** (u2)<-[:**FOLLOWS**]->(u1)  
 -[:**FOLLOWS**]->(u3)



Query:

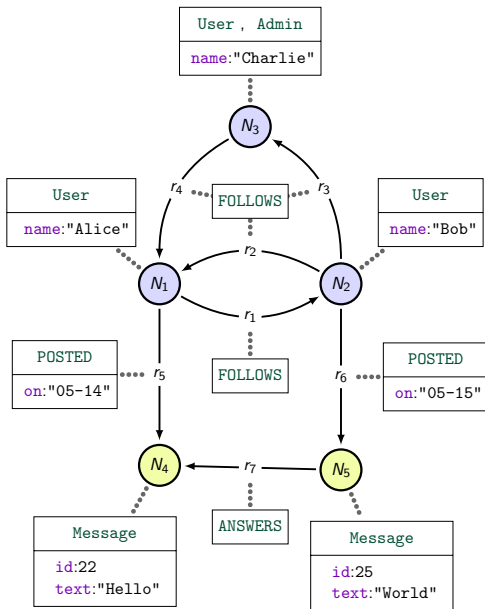
**MATCH** (u1)-[:POSTED]->(m1)

**MATCH** (u2)<-[:FOLLOWS]-(u1)  
-[:FOLLOWS]->(u3)

Table after first **MATCH**:

u1	m1
$N_1$	$N_4$
$N_2$	$N_5$





Query:

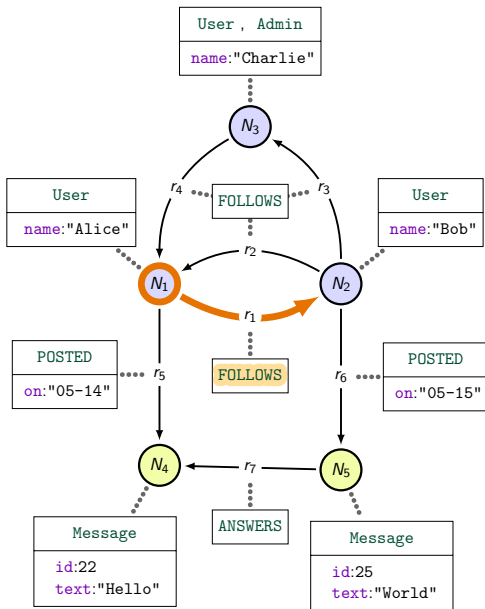
```
MATCH (u1)-[:POSTED]->(m1)
MATCH (u2)-[:FOLLOWS]->(u1)
      -[:FOLLOWS]->(u3)
```

Table after first MATCH:

u1	m1
N <sub>1</sub>	N <sub>4</sub>
N <sub>2</sub>	N <sub>5</sub>

Table after second MATCH:

u1	m1	u2	u3
N <sub>1</sub>	N <sub>4</sub>	.	.
N <sub>2</sub>	N <sub>5</sub>	.	.



Query:

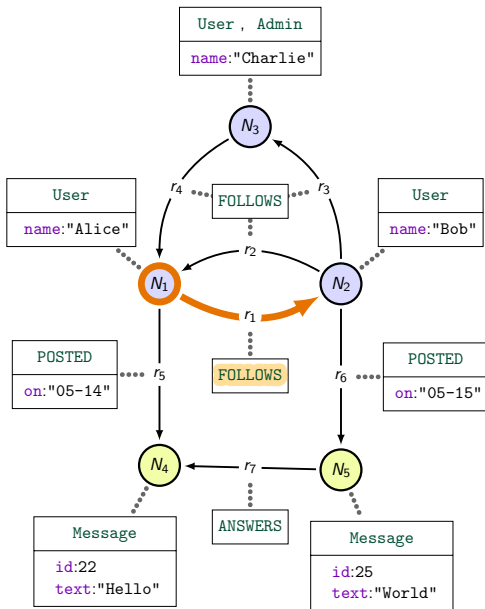
```
MATCH (u1)-[:POSTED]->(m1)
MATCH (u2)-[:FOLLOWS]->(u1)
      -[:FOLLOWS]->(u3)
```

Table after first MATCH:

u1	m1
N <sub>1</sub>	N <sub>4</sub>
N <sub>2</sub>	N <sub>5</sub>

Table after second MATCH:

u1	m1	u2	u3
N <sub>1</sub>	N <sub>4</sub>	.	.
N <sub>2</sub>	N <sub>5</sub>	.	.



Query:

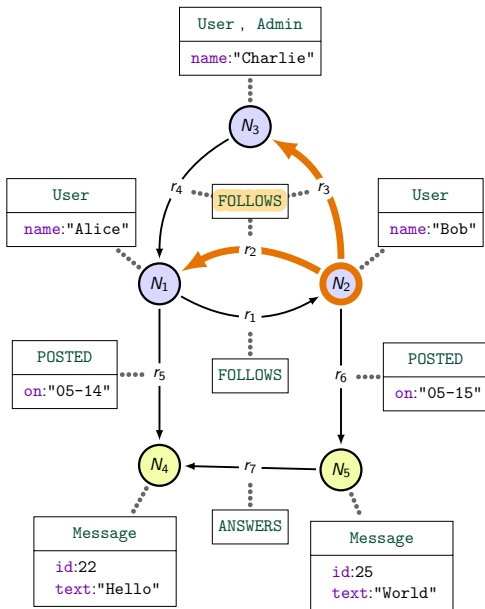
```
MATCH (u1)-[:POSTED]->(m1)
MATCH (u2)-[:FOLLOWS]->(u1)
      -[:FOLLOWS]->(u3)
```

Table after first MATCH:

u1	m1
$N_1$	$N_4$
$N_2$	$N_5$

Table after second MATCH:

u1	m1	u2	u3
<del><math>N_1</math></del>	<del><math>N_4</math></del>		
$N_2$	$N_5$	.	.



Query:

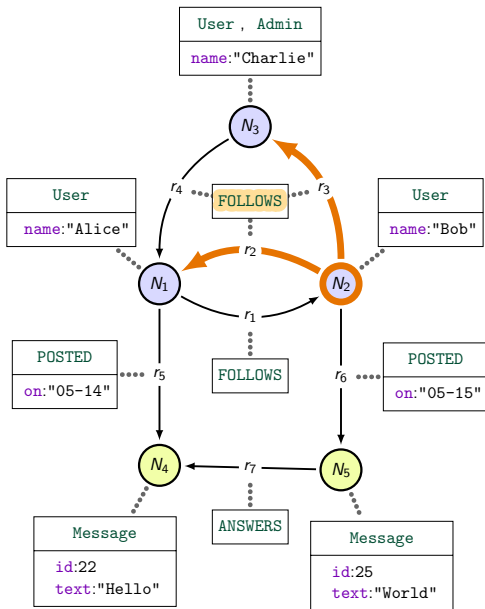
```
MATCH (u1)-[:POSTED]->(m1)
MATCH (u2)-[:FOLLOWS]->(u1)
      -[:FOLLOWS]->(u3)
```

Table after first MATCH:

u1	m1
$N_1$	$N_4$
$N_2$	$N_5$

Table after second MATCH:

u1	m1	u2	u3
<del><math>N_1</math></del>	<del><math>N_4</math></del>		
$N_2$	$N_5$	.	.



Query:

**MATCH** (u1)-[:POSTED]->(m1)

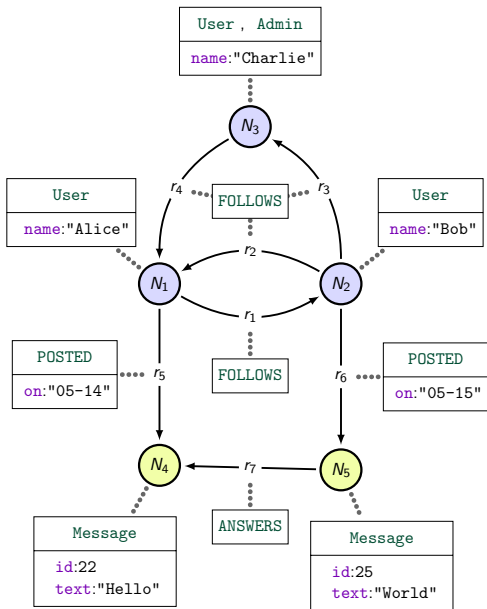
**MATCH** (u2)<-[:FOLLOWS]-(u1)  
-[:FOLLOWS]->(u3)

Table after first **MATCH**:

u1	m1
$N_1$	$N_4$
$N_2$	$N_5$

Table after second **MATCH**:

u1	m1	u2	u3
$N_2$	$N_5$	$N_1$	$N_3$
$N_2$	$N_5$	$N_3$	$N_1$



The two following queries compute similar things:

**MATCH** (a)  $\langle pat_1 \rangle$  (b)  $\langle pat_2 \rangle$  (c)

**MATCH** (a)  $\langle pat_1 \rangle$  (b)

**MATCH** (b)  $\langle pat_2 \rangle$  (c)

**1** Compute their answer for

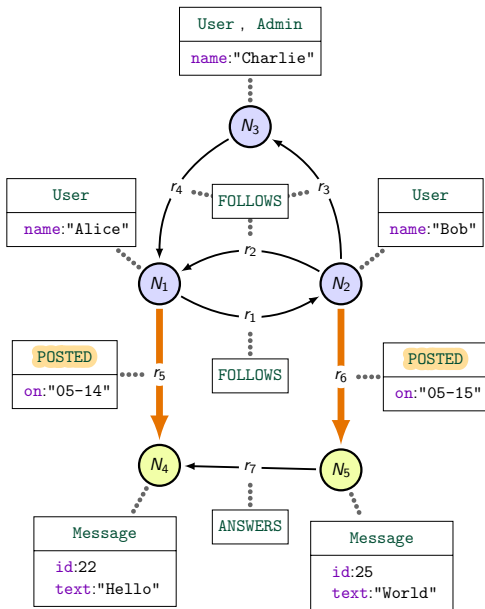
$\langle pat_1 \rangle = -[:FOLLOWS]->$

$\langle pat_1 \rangle = -[:POSTED]->$

**2** Can you find patterns  $\langle pat_1 \rangle$  and  $\langle pat_2 \rangle$  for which their answer is different?

Part II: Neo4j, Property graphs and Cypher

## 4. Usage of **WITH** (or **RETURN**)



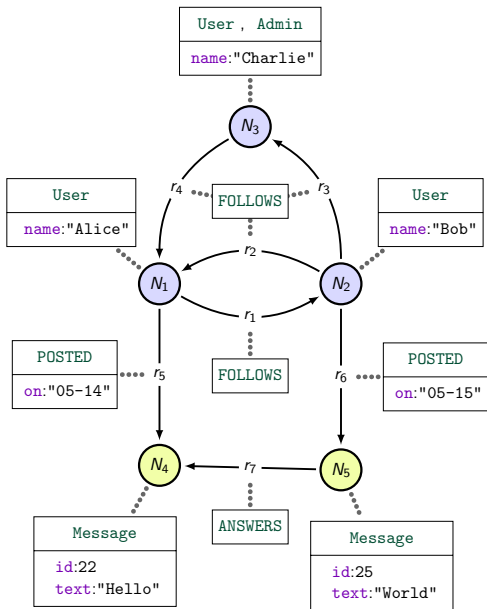
Query:

```
MATCH (u1)-[p1:POSTED]->(m1)
WITH u1, p1, m1.text AS t1
```

After the MATCH clause

u1	p1	m1
$N_1$	$r_5$	$N_4$
$N_2$	$r_6$	$N_5$





Query:

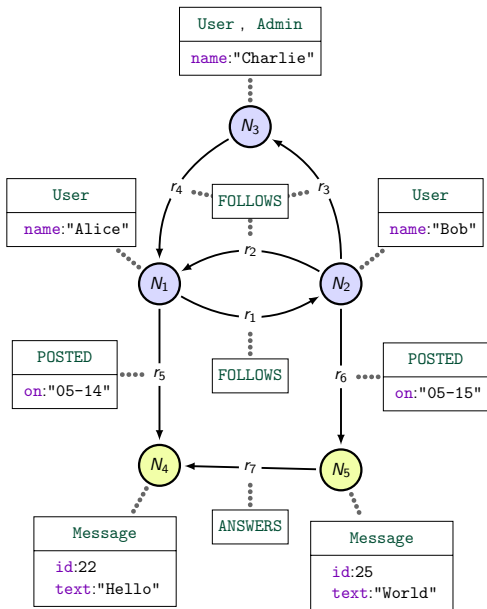
```
MATCH (u1)-[p1:POSTED]->(m1)
WITH u1, p1, m1.text AS t1
```

After the MATCH clause

u1	p1	m1
N1	r5	N4
N2	r6	N5

Execution of the WITH clause

u1	p1	t1
N1	r5	
N2	r6	



Query:

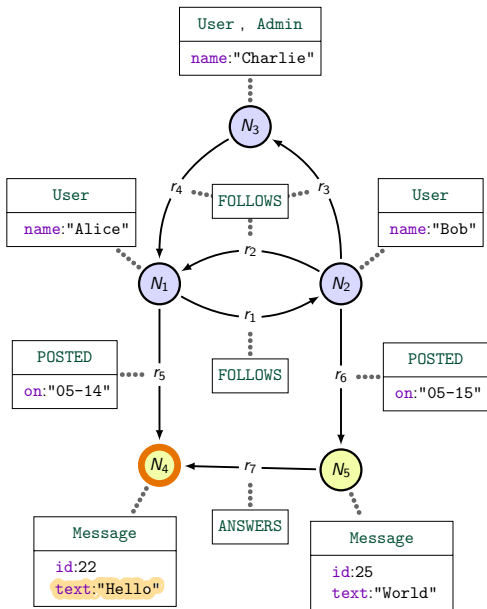
**MATCH** (u1)-[p1:POSTED]->(m1)  
**WITH** u1, p1, m1.text AS t1

After the **MATCH** clause

u1	p1	m1
N <sub>1</sub>	r <sub>5</sub>	N <sub>4</sub>
N <sub>2</sub>	r <sub>6</sub>	N <sub>5</sub>

Execution of the **WITH** clause

u1	p1	t1
N <sub>1</sub>	r <sub>5</sub>	
N <sub>2</sub>	r <sub>6</sub>	



Query:

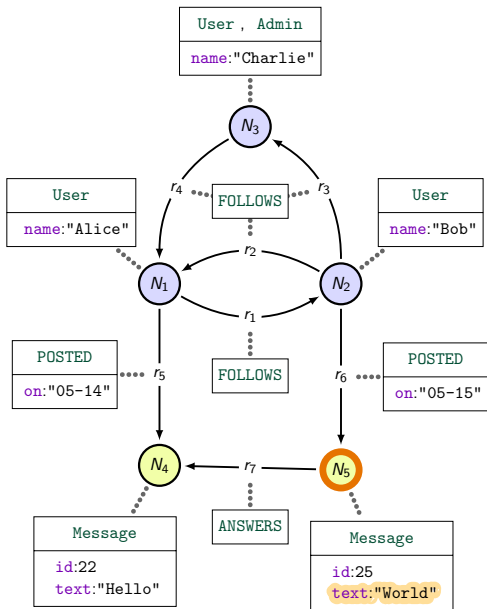
**MATCH** (u1)-[p1:POSTED]->(m1)  
**WITH** u1, p1, m1.text AS t1

After the **MATCH** clause

u1	p1	m1
$N_1$	$r_5$	$N_4$
$N_2$	$r_6$	$N_5$

Execution of the **WITH** clause

u1	p1	t1
$N_1$	$r_5$	"Hello"
$N_2$	$r_6$	



Query:

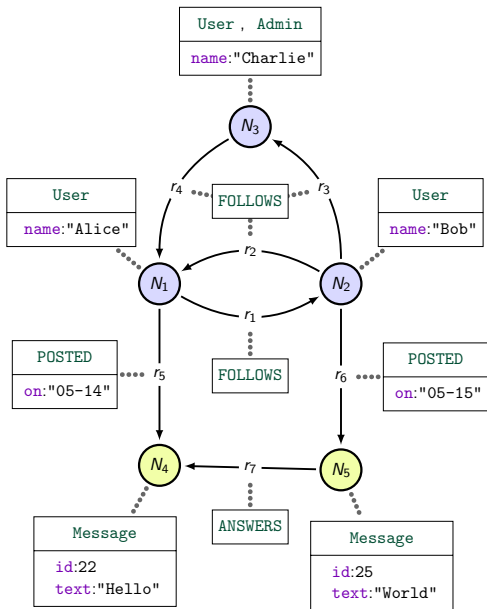
```
MATCH (u1)-[p1:POSTED]->(m1)
WITH u1, p1, m1.text AS t1
```

After the **MATCH** clause

u1	p1	m1
$N_1$	$r_5$	$N_4$
$N_2$	$r_6$	$N_5$

Execution of the **WITH** clause

u1	p1	t1
$N_1$	$r_5$	"Hello"
$N_2$	$r_6$	"World"



Query:

```

MATCH (u1)-[p1:POSTED]->(m1)
WITH u1, p1, m1.text AS t1
    
```

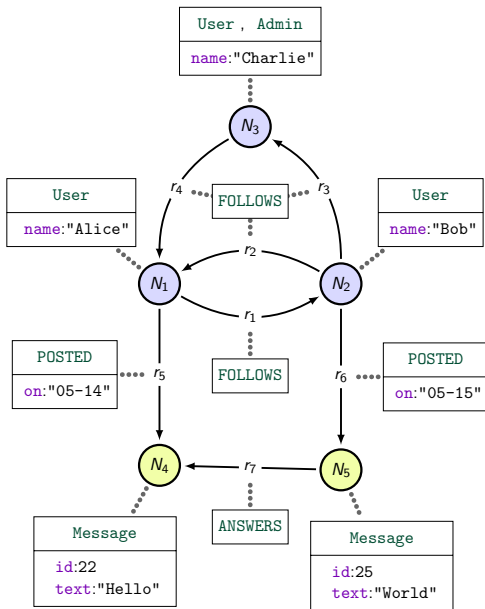
After the **MATCH** clause

u1	p1	m1
N1	r5	N4
N2	r6	N5

Final result

u1	p1	t1
N1	r5	"Hello"
N2	r6	"World"

# Elimination of duplicate rows



Query:

```
MATCH (u1)-[:FOLLOWS]->()  
WITH DISTINCT u1
```

After MATCH:

u1  
 $N_1$   
 $N_2$   
 $N_2$   
 $N_3$

After WITH:

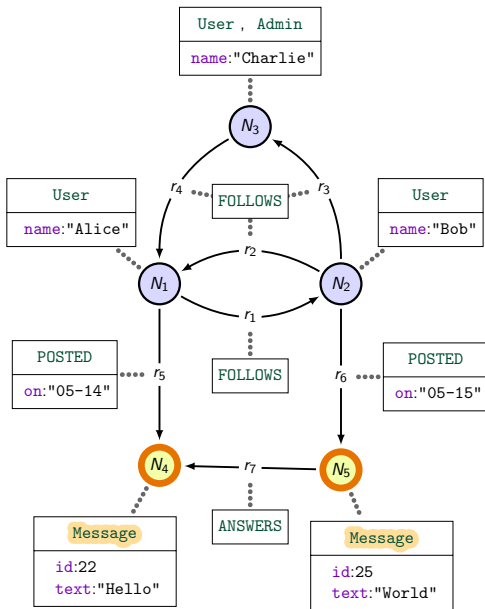
u1  
 $N_1$   
 $N_2$   
 $N_3$

**WITH** *<columns>*, *<aggr>*(*<expr>*)

**Grouping is implicit:** every variable used in *<columns>* is used for grouping

*<aggr>* in a built-in **aggregation function**, that is, a function from list to a single value.

Example: **count**, **sum**, **min**, **collect**, etc.



Query:

```
MATCH (m1:Message)
WITH count(m1) AS c
```

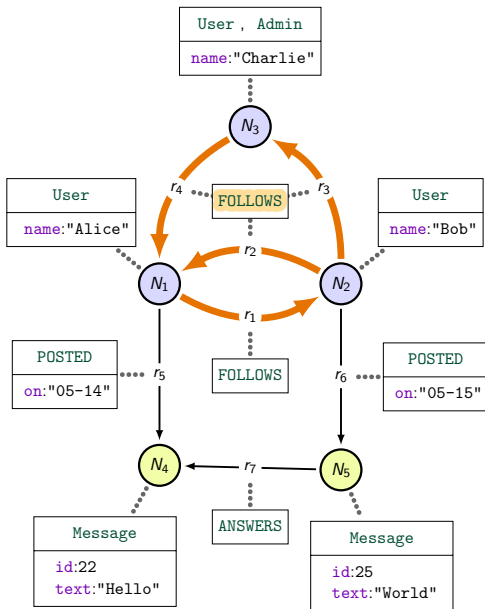
After MATCH:

<u>m1</u>
$N_4$
$N_5$

After WITH:

<u>c</u>
2





Query:

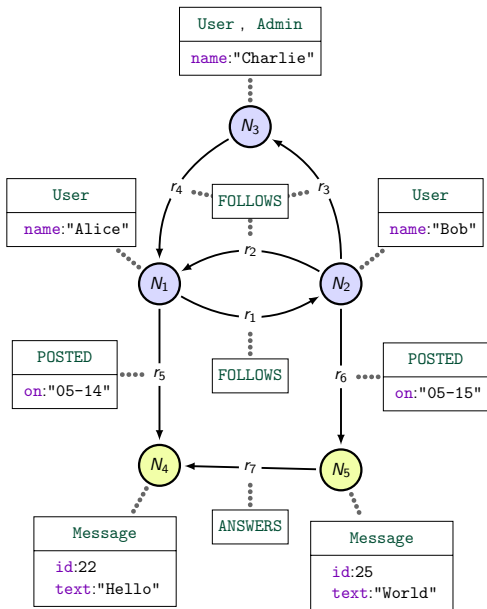
```
MATCH (u1)-[:FOLLOWS]-(u2)
WITH u1, collect(u2.name) AS n
```

Result after WITH:

u1	n
$N_1$	["Bob", "Charlie"]
$N_2$	["Alice"]
$N_3$	["Bob"]

⚠ Grouping by u1

# Exercice: what does this compute?



Query:

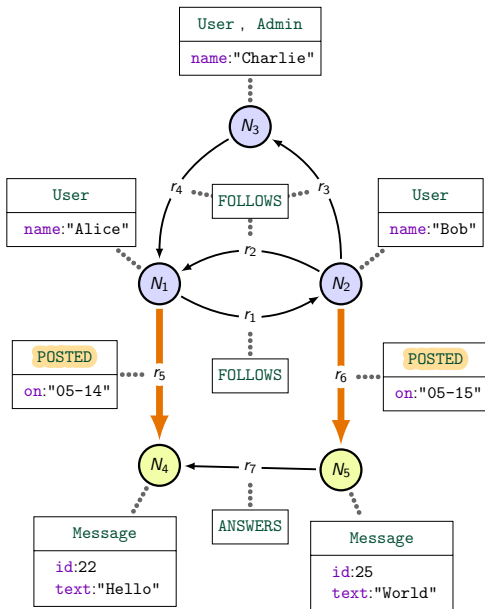
```
MATCH ()-[e:POSTED]->()
```

```
WITH max(e.on) AS d
```

```
MATCH ()-[:POSTED  
{on:d}]->(m1)
```

```
WITH m1.text as txt
```

# Exercice: what does this compute?



Query:

```
MATCH ()-[e:POSTED]->()
```

```
WITH max(e.on) AS d
```

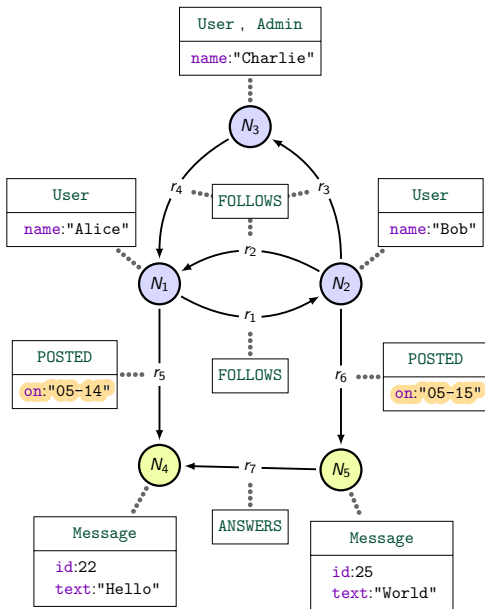
```
MATCH ()-[:POSTED
```

```
{on:d}]->(m1)
```

```
WITH m1.text as txt
```

            
e  
            
r<sub>5</sub>  
            
r<sub>6</sub>

# Exercice: what does this compute?



Query:

```
MATCH ()-[e:POSTED]->()
```

```
WITH max(e.on) AS d
```

```
MATCH ()-[:POSTED  
{on:d}]->(m1)
```

```
WITH m1.text as txt
```

e

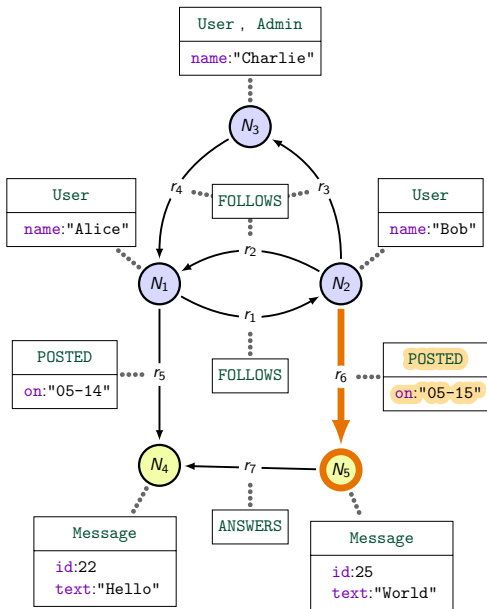
$r_5$

$r_6$

d

"05-15"

# Exercise: what does this compute?



Query:

```
MATCH ()-[e:POSTED]->()
```

```
WITH max(e.on) AS d
```

```
MATCH ()-[ :POSTED  
                {on:d}] ->(m1)
```

```
WITH m1.text as txt
```

e
---

$r_5$
-------

$r_6$
-------

d
---

"05-15"
---------

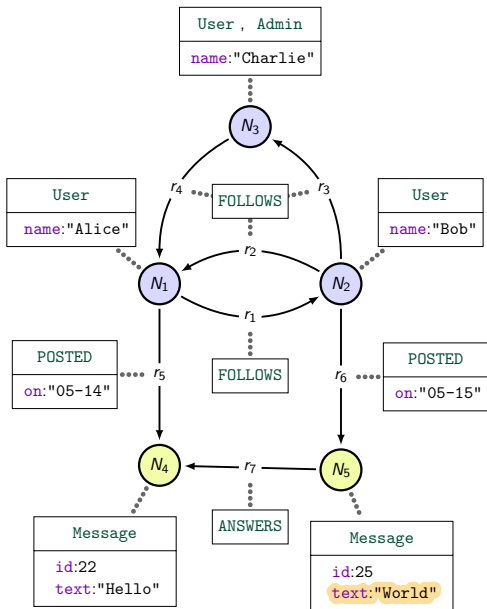
d
---

m1
----

"05-15"
---------

$N_5$
-------

# Exercise: what does this compute?



Query:

```
MATCH ()-[e:POSTED]->()
```

```
WITH max(e.on) AS d
```

```
MATCH ()-[:POSTED
```

```
{on:d}]->(m1)
```

```
WITH m1.text as txt
```

e

r<sub>5</sub>

r<sub>6</sub>

d

"05-15"

d

m1

"05-15"

N<sub>5</sub>

txt

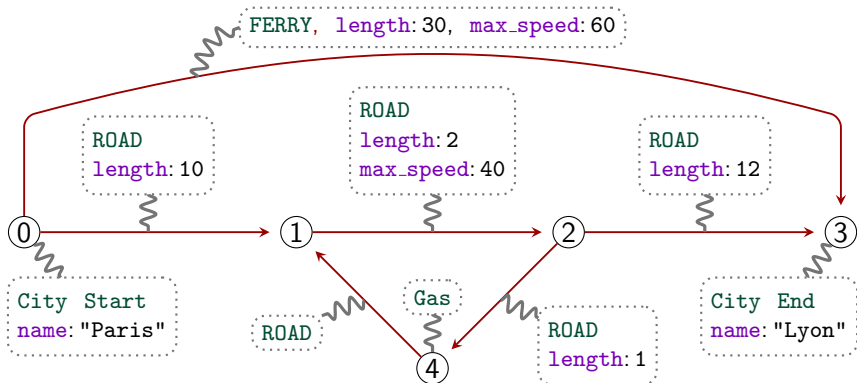
"World"

## Syntax

```
reduce( $\langle acc \rangle = \langle init \rangle, \langle var \rangle$  IN  $\langle list \rangle$  |  $\langle update \rangle$ )
```

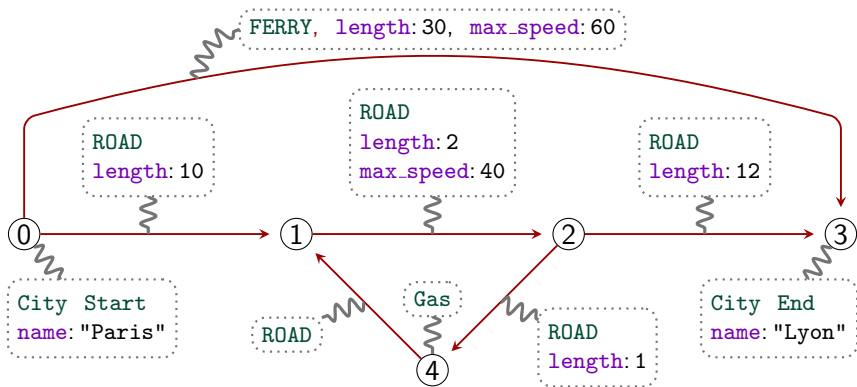
Equivalent to the following pseudo code

```
 $\langle acc \rangle := \langle init \rangle$   
for  $\langle var \rangle$  in  $\langle list \rangle$ :  
     $\langle acc \rangle := \langle update \rangle$ 
```



```
MATCH (:Start)-[e:ROAD|FERRY*]->(:End)
WITH reduce(acc=0, x IN e | acc+x.length) AS l
```





```
MATCH (:Start)-[e:ROAD|FERRY*]->(:End)
WITH reduce(acc = 0, x IN e
            | acc + x.length*coalesce(x.max_speed,80)) AS d
```

# Why “horizontal” and “vertical” aggregation

Part II: Neo4j, Property graphs and Cypher

## 5. Subclauses of **MATCH** and/or **WITH**

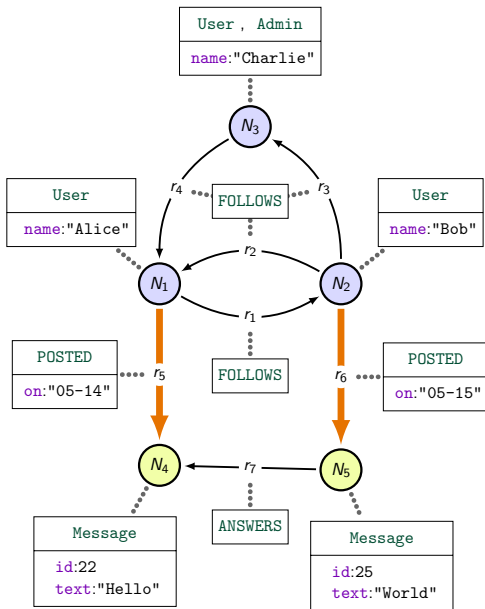
## Syntax

**MATCH** ... **WHERE** *<condition>*

or

**WITH** ... **WHERE** *<condition>*

Remove from the table computed by **MATCH** or **WHERE** the row that make *<condition>* false



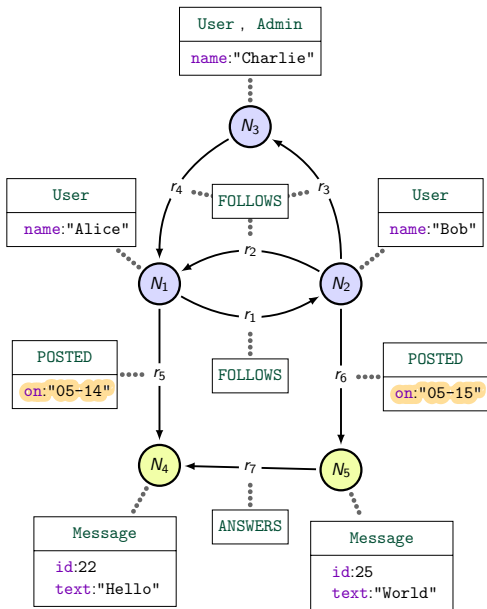
Query:

**MATCH** (u1)-[p1:POSTED]->(m1)  
**WHERE** p1.on > "05-14"

After the **WITH** clause

u1	p1	m1
$N_1$	$r_5$	$N_4$
$N_2$	$r_6$	$N_5$

# Filtering rows with WHERE (2)



Query:

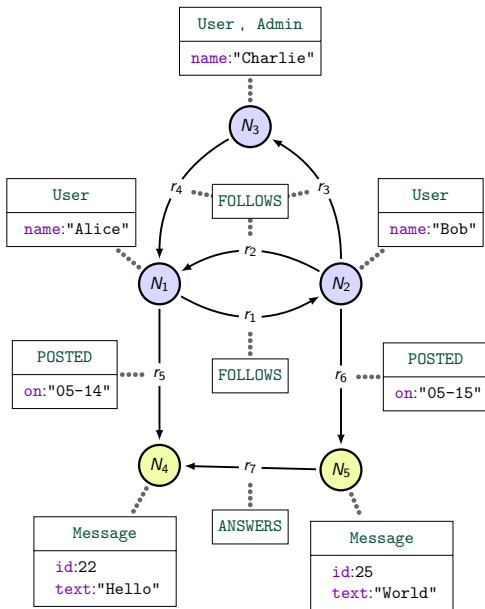
**MATCH** (u1)-[p1:POSTED]->(m1)  
**WHERE** p1.on > "05-14"

After the **WITH** clause

u1	p1	m1
$N_1$	$r_5$	$N_4$
$N_2$	$r_6$	$N_5$

Execution of the **WHERE** clause

u1	p1	m1
$N_1$	$r_5$	$N_4$
<del><math>N_2</math></del>	<del><math>r_6</math></del>	<del><math>N_5</math></del>



Query:

**MATCH** (u1)-[p1:POSTED]->(m1)  
**WHERE** p1.on > "05-14"

After the **WITH** clause

u1	p1	m1
$N_1$	$r_5$	$N_4$
$N_2$	$r_6$	$N_5$


Final result

u1	p1	m1
$N_1$	$r_5$	$N_4$

## Syntax

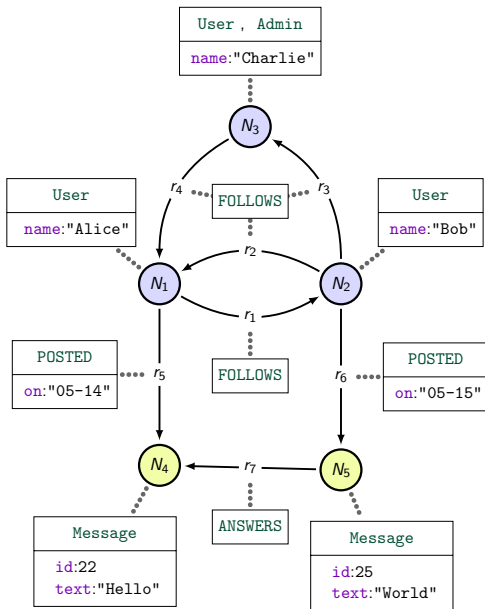
WITH ... ORDER BY  $\langle oexpr_1 \rangle$  <sup>optional</sup> DESC, ... SKIP  $\langle sexpr \rangle$  LIMIT  $\langle lexr \rangle$

optional                      optional                      optional

- Order the table by  $\langle oexpr_1 \rangle$ 
  - Ties are broken by the value of  $\langle oexpr_2 \rangle$ , remaining ties are broken by  $\langle oexpr_3 \rangle$ , etc
  - **DESC** means the order is descending.
  -  We might end up with ties → Nondeterminism
- Then, remove the first  $\langle sexpr \rangle$  rows
- Then, keep the first  $\langle lexr \rangle$  rows, at most



# Compute the User with the most followers

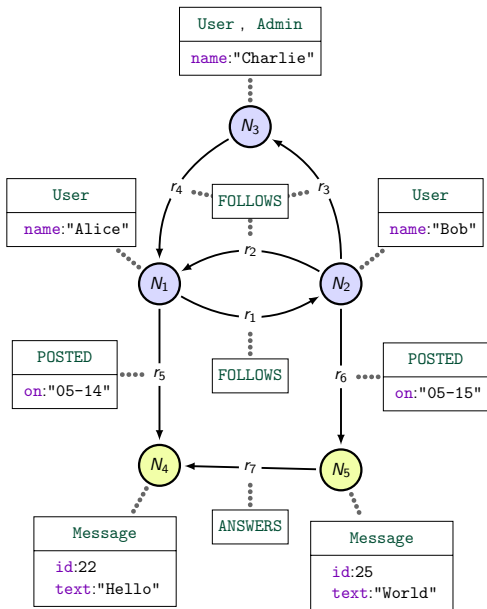


Query:

```
MATCH (u1)-[:FOLLOWS]-(u2)
WITH u1, count(b) AS c
ORDER BY c
LIMIT 1 DESC
```

u1	c
$N_1$	2

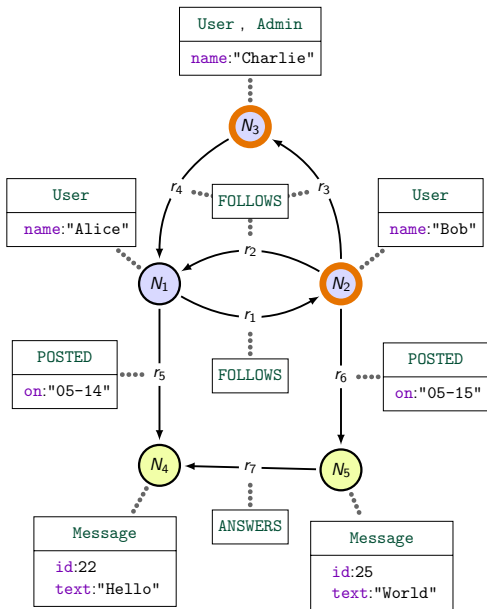
# Compute the two User with the most followers



Query:

```
MATCH (u1)-[:FOLLOWS]-(u2)
WITH u1, count(b) AS c
ORDER BY c DESC
LIMIT 2
```

# Compute the two User with the most followers



Query:

```
MATCH (u1)-[:FOLLOWS]-(u2)
WITH u1, count(b) AS c
ORDER BY c DESC
LIMIT 2
```

Since Charlie and Bob both have 1 follower, the final table is either:

u1	c
$N_1$	2
$N_2$	1

u1	c
$N_1$	2
$N_3$	1

Part II: Neo4j, Property graphs and Cypher

## **6. Updating the property graph**

Property  
Graph

Clause 1

**MATCH** ...

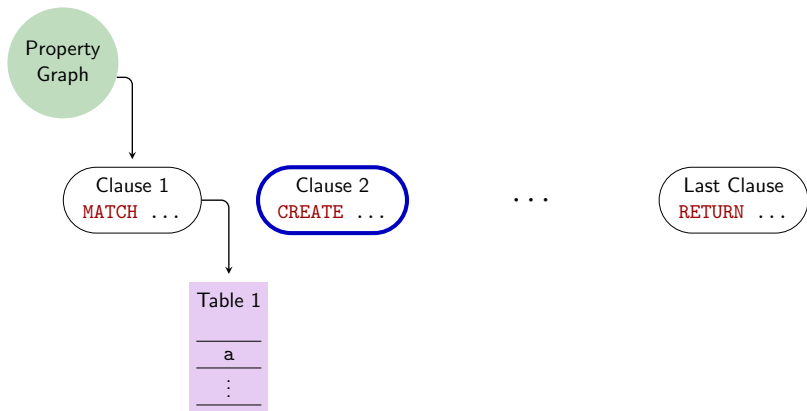
Clause 2

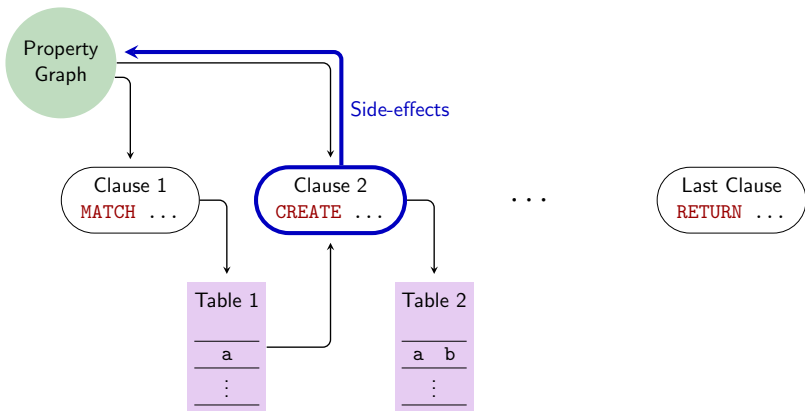
**CREATE** ...

...

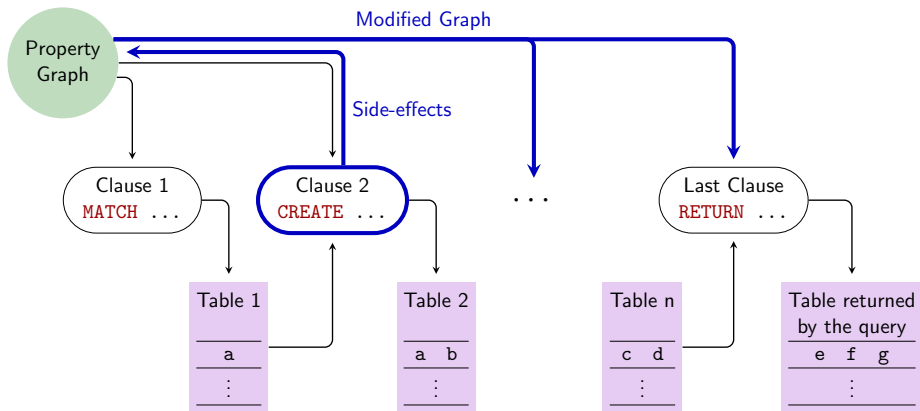
Last Clause

**RETURN** ...

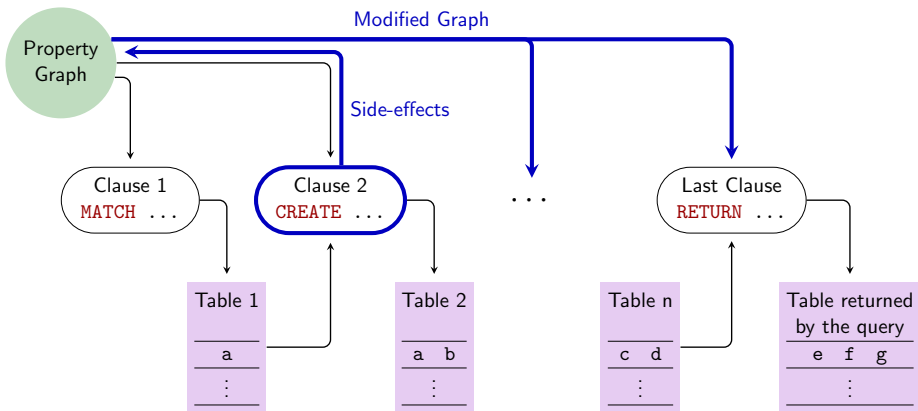




# How evaluation works with update clauses





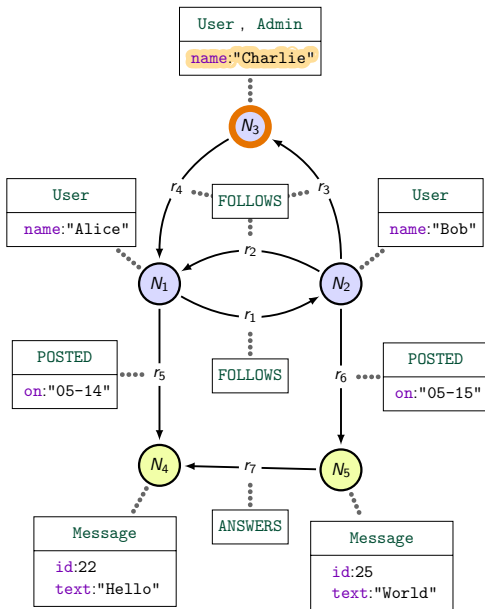


## Neo4j complies to ACID

- A  $\implies$  Modifications are **undone** if evaluation fails
- C  $\implies$  The PG must comply to IC **at the end** of evaluation only
- I  $\implies$  Modifications are **invisible** to concurrent queries

- **CREATE** (a:User {name:"Alice"})
  - Creates a new node
  - Stores it in column a
- **CREATE** (a)-[e:POSTED {on:"12-07"}]->(b)
  - Creates a new relation from a to b
  - If a the input table has no column named a, creates a new node
  - Idem for b
  - Stores the new relation in column e

## Create nodes and relations (2)



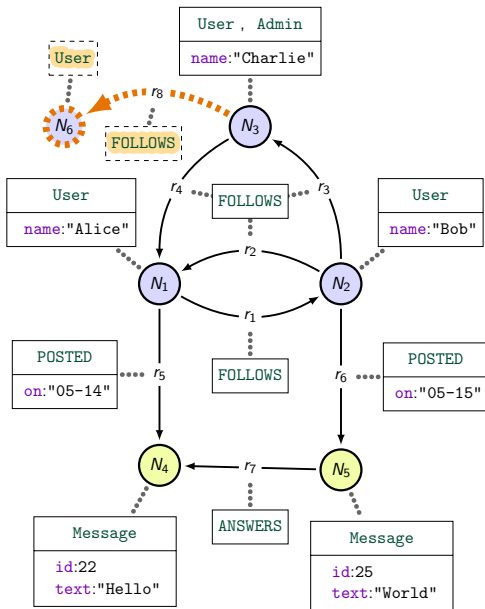
Query:

```
MATCH (a {name:"Charlie"})  
CREATE (a)-[:FOLLOWS]->  
      (b:User)
```

Table after **MATCH** clause:

a
$N_3$

## Create nodes and relations (2)



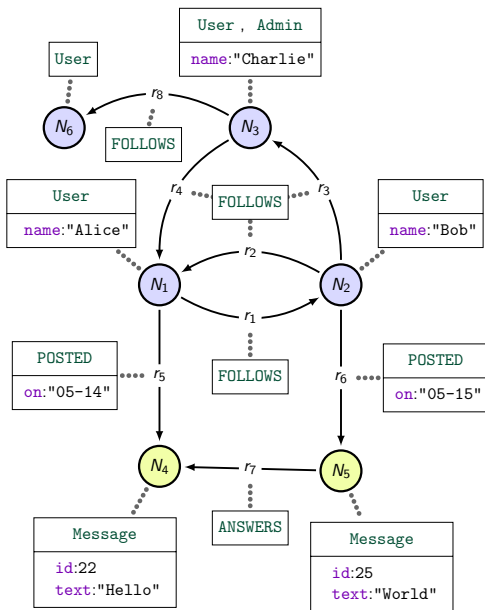
Query:

```
MATCH (a {name:"Charlie"})  
CREATE (a)-[:FOLLOWS]->  
      (b:User)
```

Table after **MATCH** clause:

a
N <sub>3</sub>

## Create nodes and relations (2)



Query:

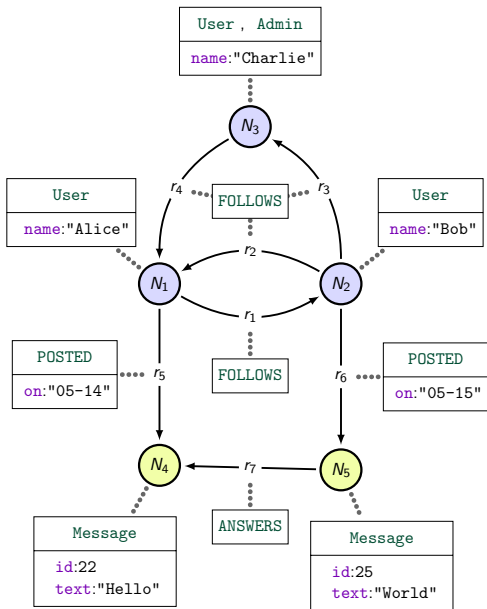
```
MATCH (a {name:"Charlie"})  
CREATE (a)-[:FOLLOWS]->  
      (b:User)
```

Table after **MATCH** clause:

a
N3

Table after **CREATE** clause:

a	b
N3	N6



Query:

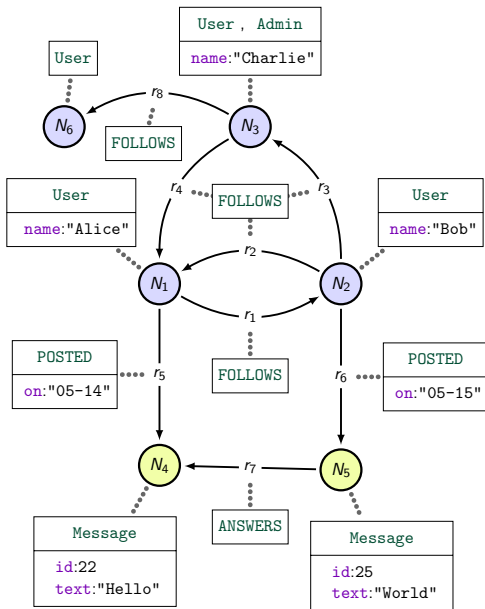
CREATE

```
(n1:User{name:"Alice"}),
(n2:User{name:"Bob"}),
(n3:User:Admin
  {name:"Charlie"}),
(n4:Message {id:22,
  text:"Hello"}),
(n5:Message {id:25,
  text:"World"})
```

CREATE

```
(n1)-[:FOLLOWS]->(n2),
(n1)-[:POSTED
  {on:"05-04"}]->(n4),
(n2)-[:FOLLOWS]->(n1),
(n2)-[:FOLLOWS]->(n3),
(n2)-[:POSTED
  {on:"05-04"}]->(n5),
(n3)-[:FOLLOWS]->(n1),
(n5)-[:ANSWERS]->(n4),
```

- **DELETE** a
  - If column a contains relations, delete them
  - If column a contains node:
    - if none of them has adjacent relation, delete them
    - otherwise the query fails.
- **DETACH DELETE** a
  - If column a contains relations, delete them
  - If column a contains nodes, delete them as well as every adjacent relations.



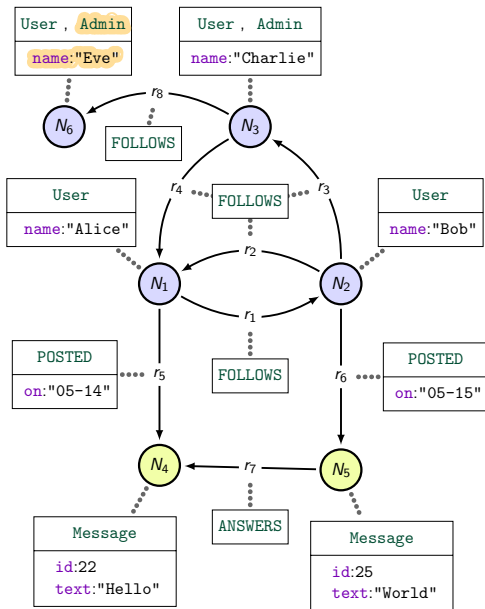
Query:

```
MATCH (a{name:"Charlie"})
CREATE (a)-[:FOLLOWS]->
    (b:User)
SET b:Admin, b.name="Eve"
```

Table after CREATE clause:

a	b
$N_3$	$N_6$



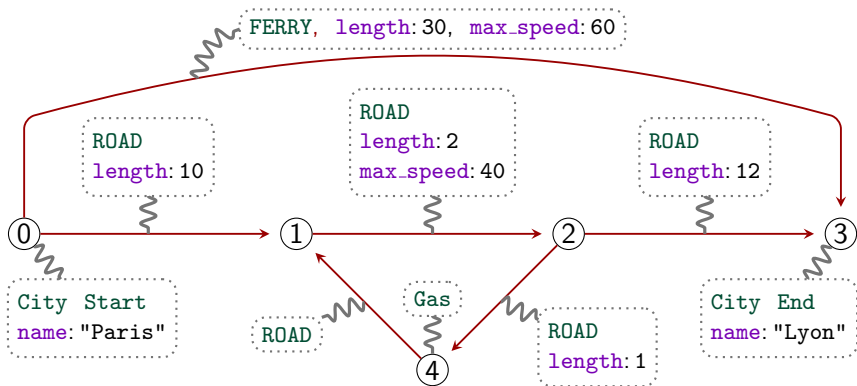


Query:

```
MATCH (a{name:"Charlie"})
CREATE (a)-[:FOLLOWS]->
      (b:User)
SET b:Admin, b.name="Eve"
```

Table after CREATE clause:

a	b
$N_3$	$N_6$



```
MATCH ()-[e:ROAD]->()
  WHERE e.max_speed IS NULL
  SET e.max_speed=80
```

⇒ Adds the property `max_speed:80` to all `ROAD` that do not have one.

# Appendix

## Introduction

- About this PDF
- Overview of query answering
- Property graphs vs Relational
- History of query languages for PG's
- Outline

## Part I: Theoretical foundations

- Reminder about sets and bag

### 1 Data model: labeled graphs

- Definition
- Limits to our data model

### 2 Graph DM vs Relational DM

- Translation: Graph to Tables
- Translation: Tables to Graph
- Having non-binary relations in graphs

### 3 Regular Path Queries

- Reminders about regular expressions
- RPQs matching
- Matching RPQs
- Computing matches

### 4 The most common RPQ semantics

- Endpoint semantics
- Shortest semantics
- Trail semantics

## Part II: Neo4j, Property graphs and Cypher

### 1 Data model: Property graphs

### 2 General presentation of Cypher

- Generalities
- Values in Cypher
- How evaluation works
- Overview of read-only Cypher

### 3 Pattern matching with **MATCH**

- Matching nodes
- Matching relations
- Matching joined relations
- Implicit join on variable reuse
- Matching paths
- Recap of pattern matching

- Sequence of **MATCH** clauses

### 4 Usage of **WITH** (or **RETURN**)

- Column manipulation
- Elimination of duplicate rows
- Vertical Aggregation
- Horizontal aggregation

### 5 Subclauses of **MATCH** and/or **WITH**

- Filtering rows with **WHERE**
- Controlling order and size of the output

### 6 Updating the property graph

- Create nodes and relations
- Delete nodes and relations
- Modifying labels and properties
- Cypher allows flexible bulk updates

<b>English</b>	<b>French</b>
Acyclic	Acyclique, Acircuitique
Bag, multiset	Multi-ensemble
Data model (DM)	Modèle de données
Edge	Arête, Arc
Endpoints	Extrémités
Key	Clef
Label	Etiquette
Match	
Node	Noeud
Path	Chemin
Pattern matching	Recherche de motif
Property, Attribute	Propriété, Attribut

Property Graph (PG)	Graphe à propriétés, Graphe de propriété, Graphe attribué
Regular Path Query (RPQ)	
Semantics	Semantique
Set	Ensemble
Source	Source
Target	Destination
Trail	
Type	Type
Value	Valeur
Vertex	Sommet
Walk	Chemin, Marche