Formal semantics of Cypher:
towards a standard language for querying property graphs

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Outline

1 Introduction

2 Property graphs

3 Regular Path Queries

4 Cypher by example

5 Principles of the semantics

6 Towards a standard language for querying property graphs

Representing data as graphs

Most databases use the relational model

- Relational algebra in theory
- The language SQL in practice

Representing data as graphs

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- Relational algebra in theory
- The language SQL in practice

Some data have intrinsically the structure of graphs:

- Semantic web
- Social Networks
- Bioinformatic networks

Native representation of data as graphs allows:

- Efficient algorithms on graphs
- Pattern matching
- Optimisations

More and more ways to manipulate data as graphs

Data model

Property graphs, RDF

Query languages

Cypher, Gremlin, PGQL, SPARQL, G-Core

Engines

JanusGraph, Jena, Neo4j, Virtuoso

Domain

Fraud detection, Investigative journalism

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Cypher

- Language for querying and updating *property graphs*

- Invented by Neo Technology
- Originally, part of engine Neo4j $\longrightarrow$ commercial success
- Now, in multiple datagraph engines (e.g., SAP HANA Graph, Redis Graph, Agens Graph)

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The openCypher project

- Since 2015
- Seeks to standardise Cypher (SQL for property graphs?)
  - Community-led evolution
  - Complete specification

Short term goal

Full denotational semantics for the language Cypher.

- Industrial partnership Neo4j/University of Edinburgh
- Reverse engineering and formalisation from Neo4j
- Done semantics of the “core fragment’ [Francis et al’18]
- Soon: semantics of the “update clauses

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Long term goal

Design a standard language for querying property graphs: GQL.

- Merging Cypher, PGQL, G-Core.
- Involvement of Neo Technology, Oracle and LDBC.

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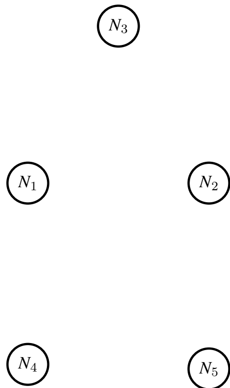
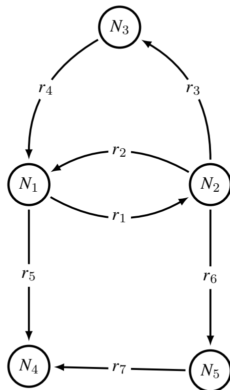
Property graph



- Nodes : $N_1, N_2, \cdots, N_5$

Property graph



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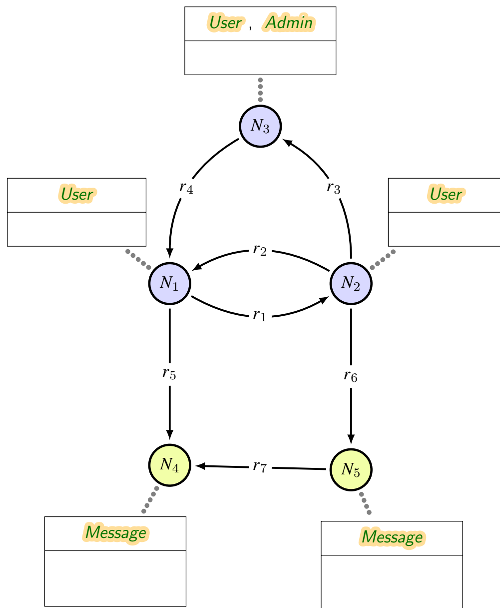
- Relationships : $r_1, r_2, \cdots, r_7$

Property graph



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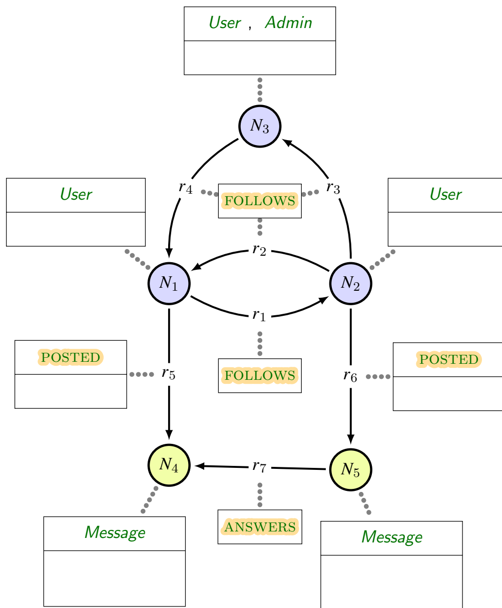
- Labels (de nœuds) :
  - *User*
  - *Message*

Property graph



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- Types (of relationships) :
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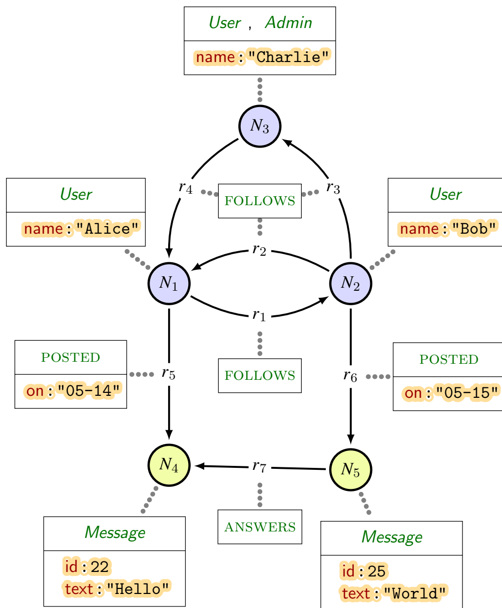
  - POSTED
  - ANSWERS

Property graph



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- Labels (de nœuds) :
  - *User*
  - *Message*
- Types (of relationships) :
  - FOLLOWS
  - POSTED
  - ANSWERS
- Properties (*i.e.* Key/Value pairs) :
  - name : "Alice"
  - id : 22
  - text : "Hello"

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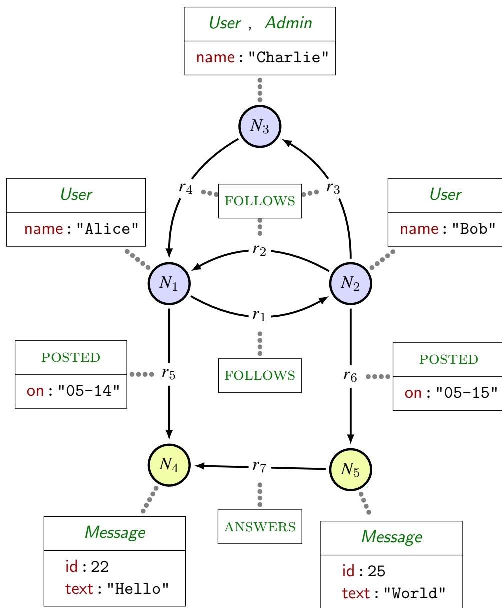
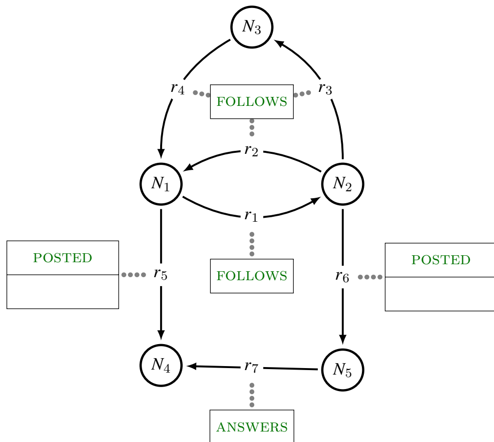
Data model for theoretical query languages



Graph database

- Relation bear types
- Node do not bear types (could be simulated)
- **Neither bears property**

Data model for theoretical query languages



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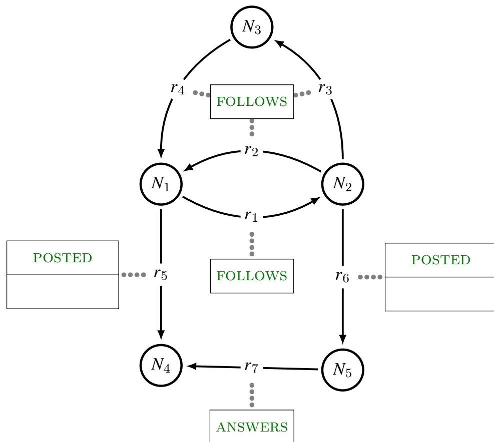

Graph database

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Finite number of relation types (and node types)
→ "Dataless graph"

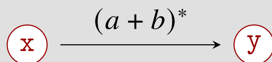
A: finite alphabet (of relation types)
N: nodes in the graph

Definition (RPQ R):

$$R = (x, E, y)$$

$$\begin{cases} E : & \text{regular expr. over } A \\ x, y : & \text{two variables} \end{cases}$$

Example:

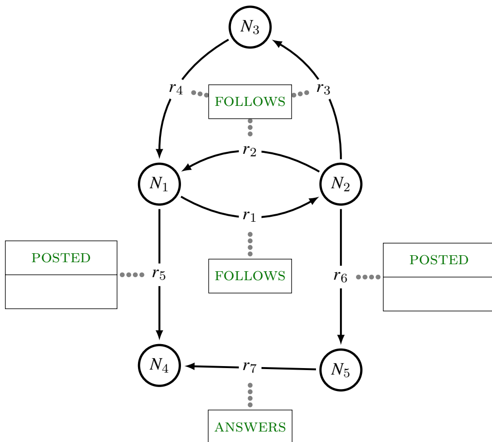


Definition (Answer to R):

Set of the functions

$$F : \{x, y\} \longrightarrow N \ , \quad \exists u \in E,\ F(x) \xrightarrow{u} F(y)$$

Regular Path Query (RPQ) – Example 1



Query:

Answers:

$$F_1 : \quad x \mapsto N_1 \quad y \mapsto N_4$$
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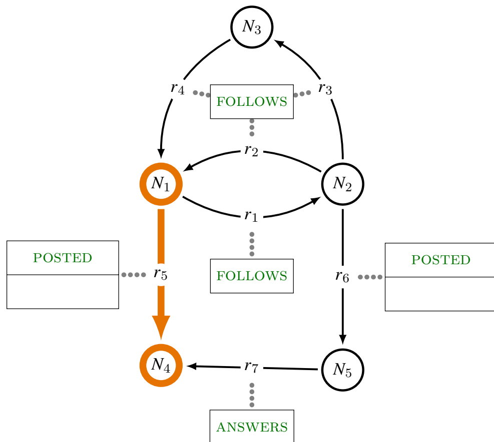

Query:

$x \xrightarrow{\text{POSTED.(ANSWERS)}^*} y$

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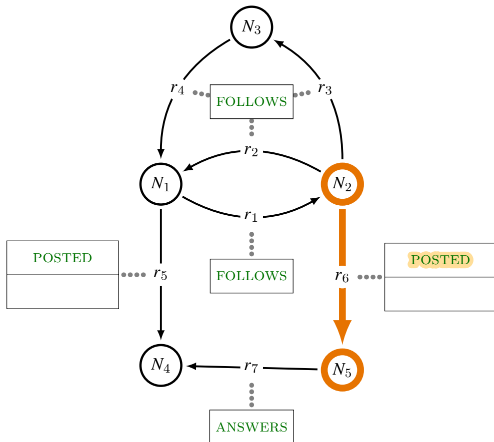

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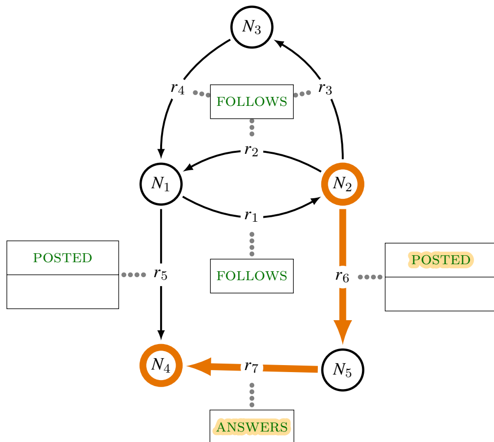

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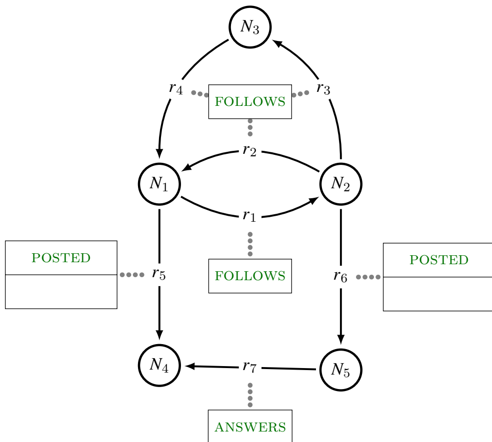

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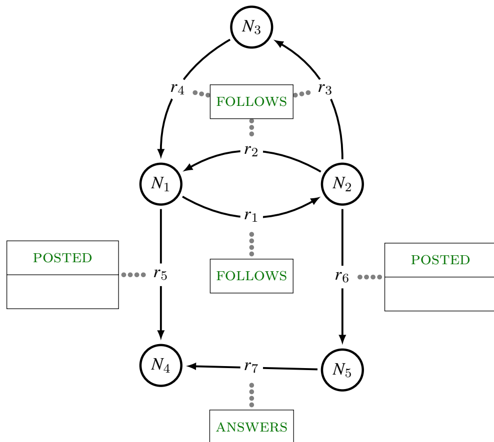

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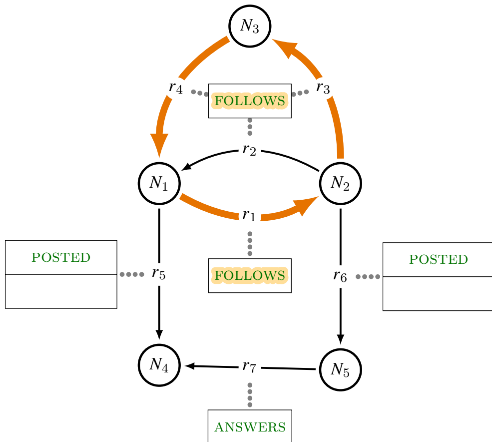

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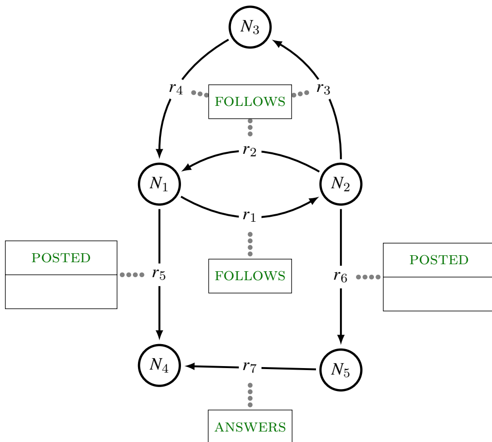

Query:

Answers:

$$
\begin{array}{llll}
F_1: & x \mapsto N_1 & y \mapsto N_1 \\
F_2: & x \mapsto N_1 & y \mapsto N_2 \\
F_3: & x \mapsto N_1 & y \mapsto N_3 \\
\vdots & \vdots & \vdots \\
F_9: & x \mapsto N_3 & y \mapsto N_3
\end{array}
$$

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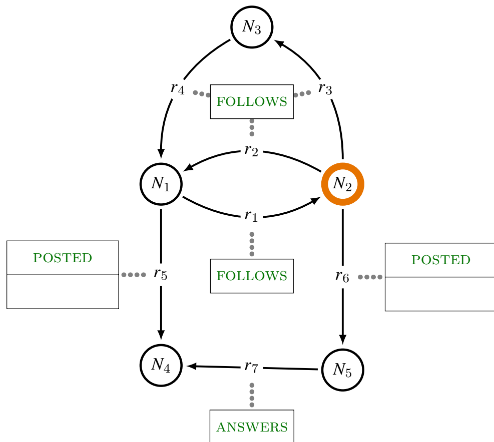

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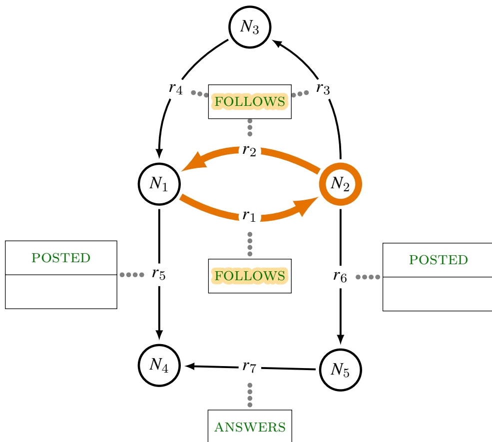

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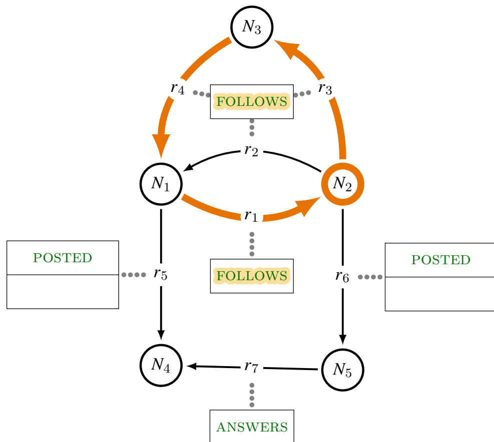

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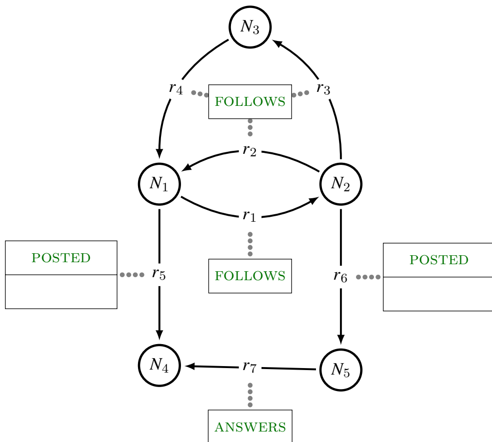

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F_2 :	$x \mapsto N_1$	$y \mapsto N_2$
F_3 :	$x \mapsto N_1$	$y \mapsto N_3$
$\vdots$	$\vdots$	$\vdots$
F_9 :	$x \mapsto N_3$	$y \mapsto N_3$

$\longrightarrow$ ∞-many path realise F_1

$\longrightarrow$ RPQ follows set-semantics

Conjunction of RPQs (CRPQ) – Definition

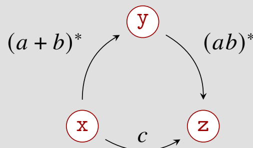
A: alphabet (of relation types)
N: nodes in the graph

Definition (a CRPQ C):

$$C = (R_1 \wedge R_2 \wedge \cdots \wedge R_n)$$

where $R_1, , \ldots, R_n$ are RPQs

Example:

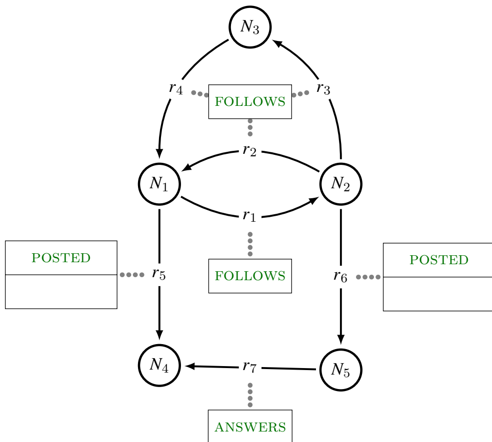


Answer to C:

Set of the functions $F : \mathsf{var}(C) \longrightarrow N$

such that $\forall i,\ F_{|\mathsf{var}(R_i)}$ is an answer to R_i

Conjunction of RPQs (CRPQ) – Example



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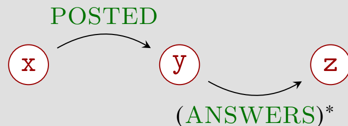

Answers:

	x	y	z
	↧	↧	↧
F_1 :	N_1	N_4	N_4
F_2 :	N_2	N_5	N_4
F_3 :	N_2	N_5	N_5

A: alphabet (of relation types)
N: nodes in the graph

Definition (UCRPQ Q)

$$Q = (C_1 \cup C_2 \cup \cdots \cup C_n)$$

where C_1, C_2, $\ldots$, C_n are CRPQs

Answer to Q:

Set of partial functions: $\big(\mathrm{var}(C_1) \cup \cdots \cup \mathrm{var}(C_n)\big) \to N$

$$\bigcup_{i=1}^{n} F_i \ ,$$

where,$\forall i$, F_i is the answer to C_i.

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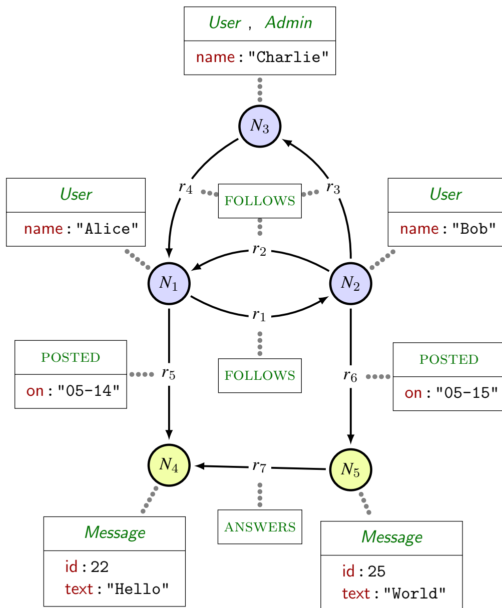
Evaluating a Cypher query



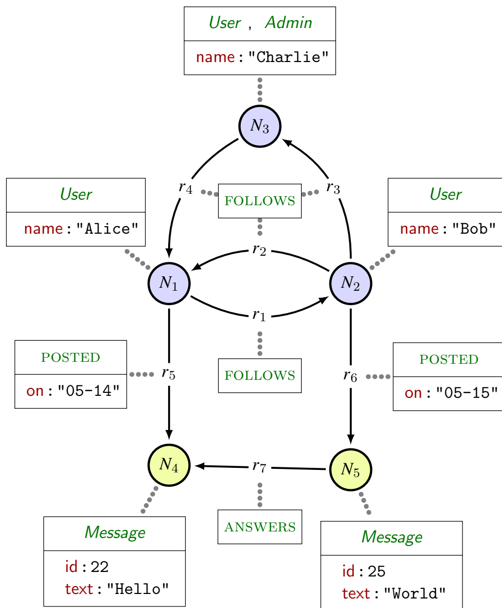
Example of Cypher query:

```
MATCH (u1)-[p1:POSTED]->(m1)
WITH u1, p1, m1.text AS t1
WHERE t1 = "Hello"
```

A Cypher statement

- is a sequence of *clauses*

Evaluating a Cypher query



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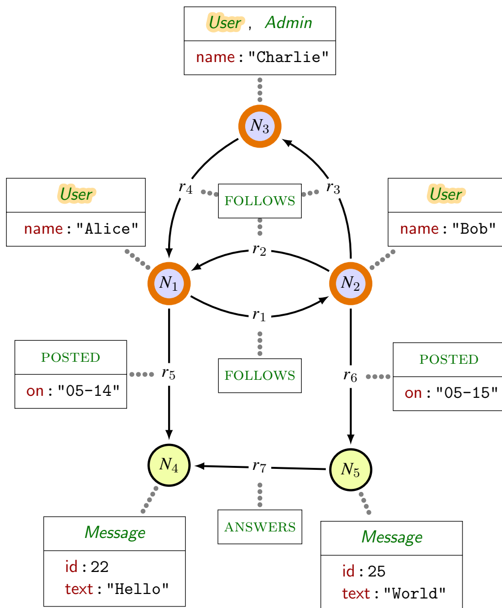
A Cypher statement

- is a sequence of *clauses*
- queries a graph
- returns a table

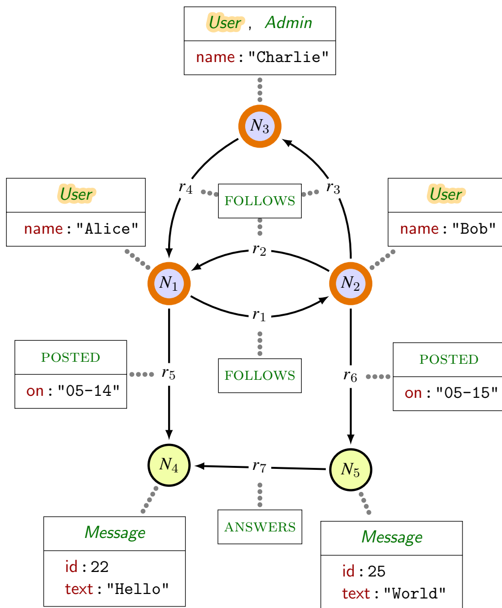
Matching nodes (1)



Query:

```
MATCH (u1:User)
```

Matching nodes (1)



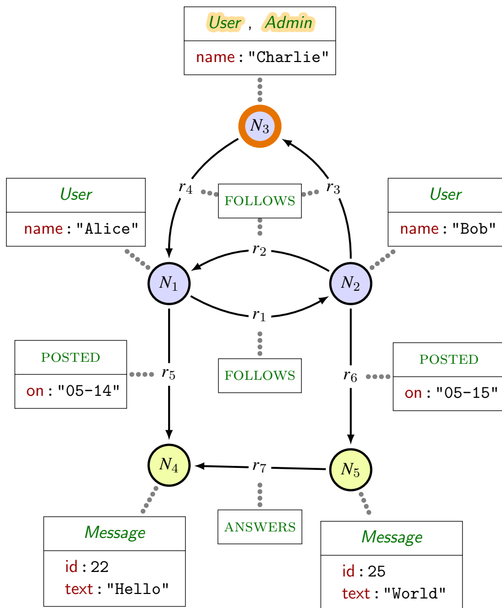
Query:

```
MATCH (u1:User)
```

Result:

u1
N_1
N_2
N_3

Matching nodes (2)



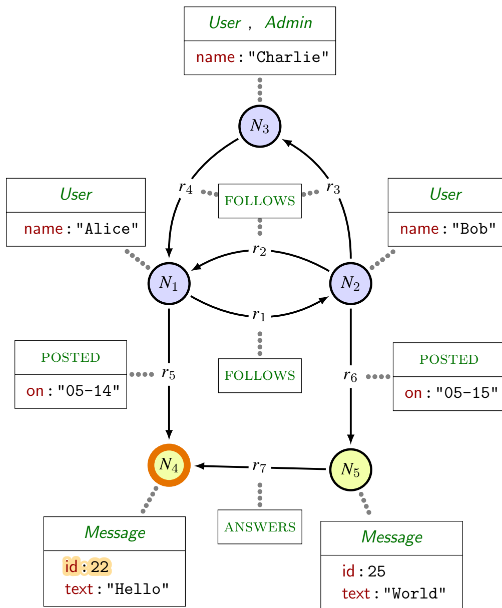
Query:

```
MATCH (u1:User:Admin)
```

Result:

u1
N_3

Matching nodes (3)



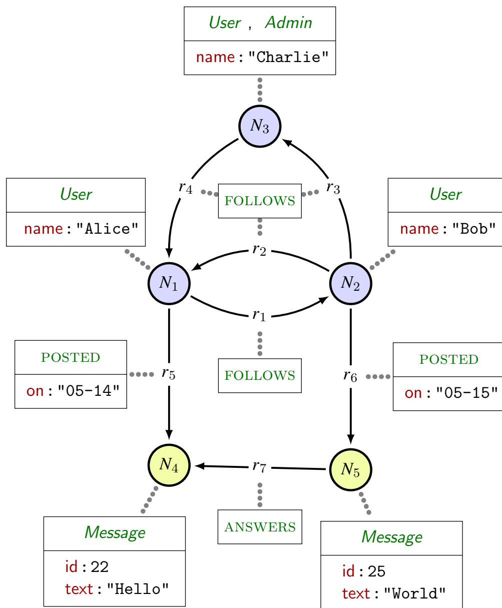
Query:

```
MATCH (u1{id:22})
```

Result:

u1
N_4

Matching relations (1)



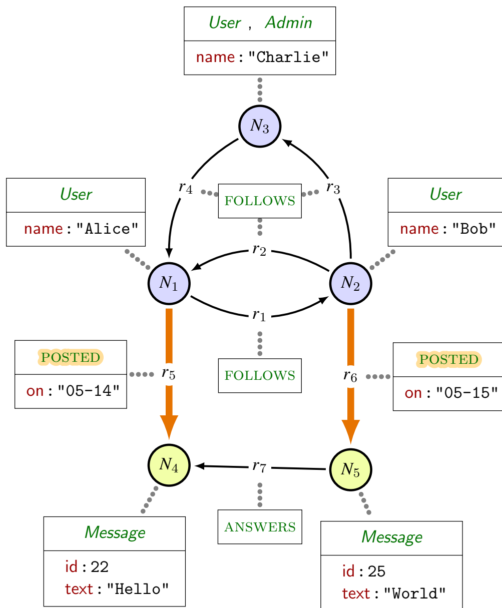
Query:

```
MATCH ()-[p1]->()
```

Result:

p1
r_1
r_2
r_3
r_4
r_5
r_6
r_7

Matching relation (2)



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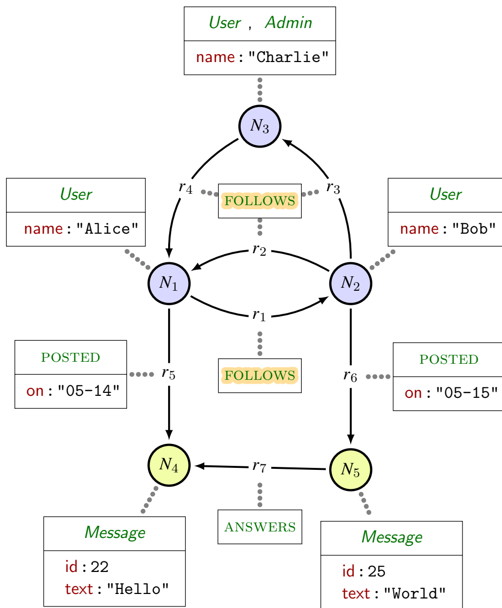
Result:

u1	p1	m1
N_1	r_5	N_4
N_2	r_6	N_5

Matching relationships (3)



Query:

```
MATCH (u1)-[:FOLLOWS]->()
```

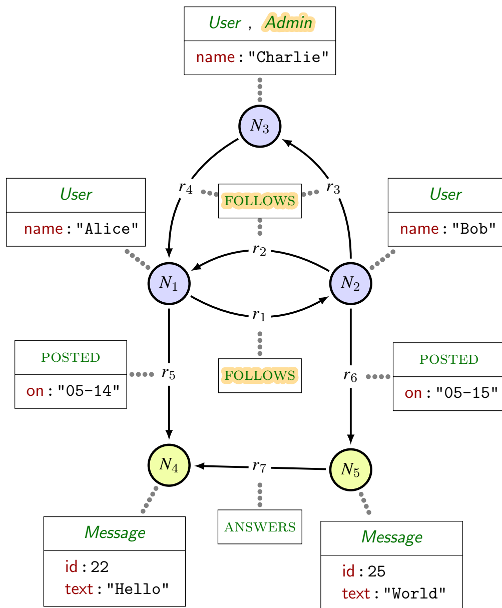
Result:

u1
N_1
N_2
N_2
N_3

Cypher has bag semantics:

N_2 has two outgoing FOLLOWS relations $\Rightarrow$ two lines N_2

Matching paths (1)



Query:

```
MATCH (u1:Admin)
          -[l1:FOLLOWS*]->(m1)
```

Result:

u1	l1	m1
N_3	$[r_4]$	N_1
N_3	$[r_4, r_1]$	N_2
N_3	$[r_4, r_1, r_2]$	N_1
N_3	$[r_4, r_1, r_3]$	N_3

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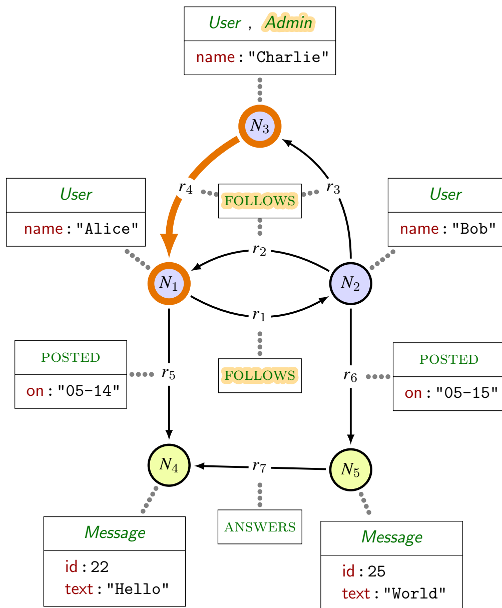

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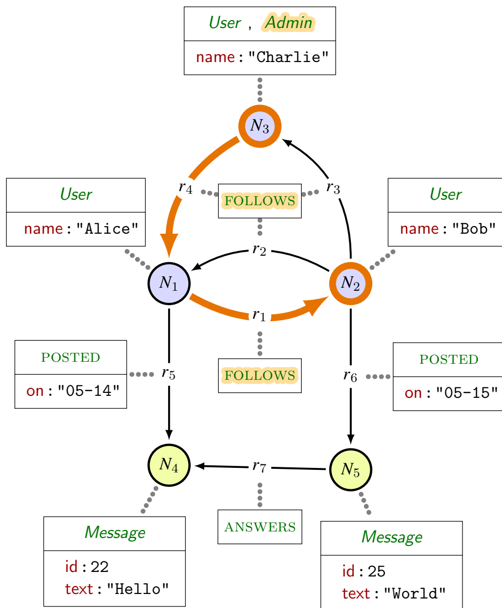

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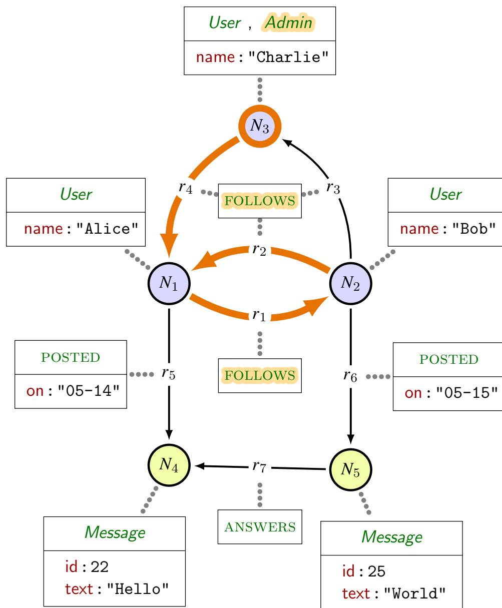
N_3	$[r_4, r_1, r_3]$	N_3

Matching paths (1)



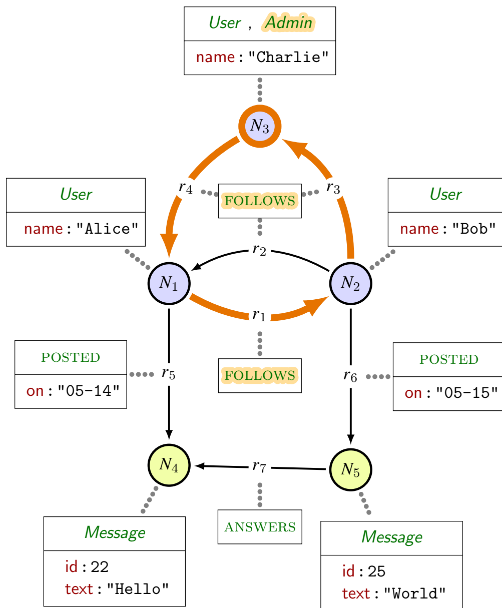
Query:

```
MATCH (u1:Admin)
          -[l1:FOLLOWS*]->(m1)
```

Result:

u1	l1	m1
N_3	$[r_4]$	N_1
N_3	$[r_4, r_1]$	N_2
N_3	$[r_4, r_1, r_2]$	N_1
N_3	$[r_4, r_1, r_3]$	N_3

Matching paths (1)



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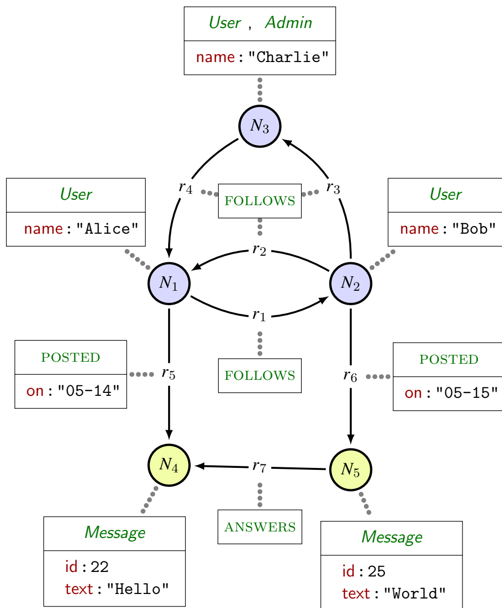
N_3	$[r_4, r_1, r_2]$	N_1
N_3	$[r_4, r_1, r_3]$	N_3



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MATCH (u1:Admin)
        -[l1:FOLLOWS*]->(m1)
```

Result:

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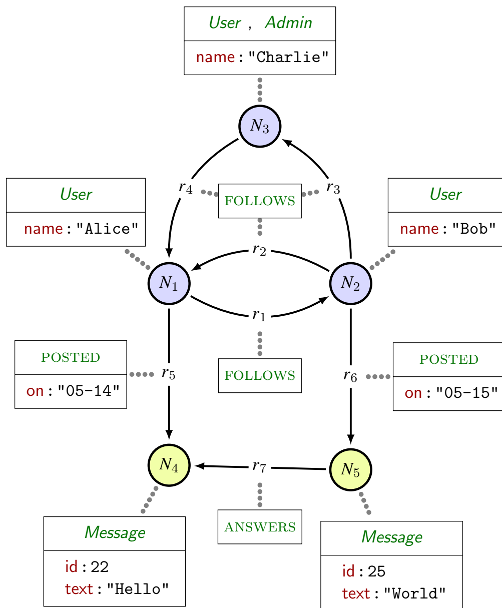
N_3	$[r_4, r_1]$	N_2
N_3	$[r_4, r_1, r_2]$	N_1
N_3	$[r_4, r_1, r_3]$	N_3

Cypher-Morphism

- Each rel. matched ≤ 1 time

$\Rightarrow$ Finitely many results

Matching paths (2)



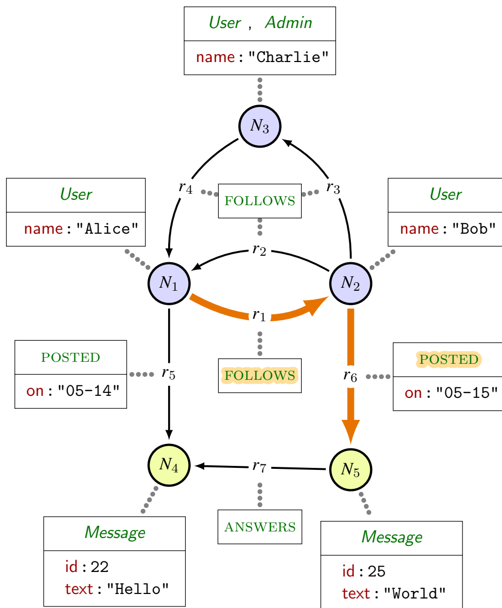
Query:

```
MATCH (u1)-[:FOLLOWS]->()
            -[:POSTED]->(m1)
```

Result:

u1	m1
N_1	N_5
N_2	N_4
N_3	N_5

Matching paths (2)



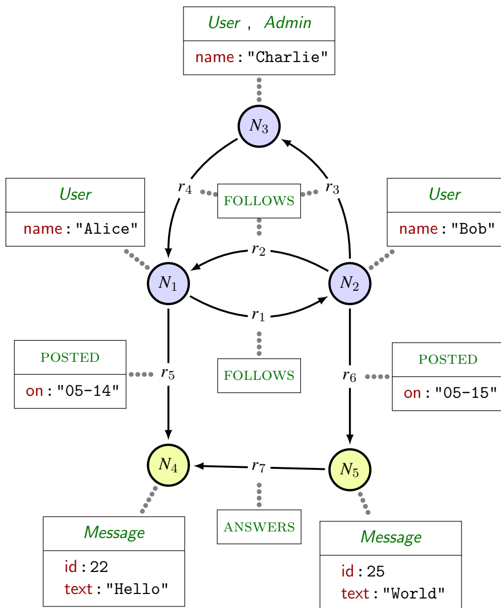
Query:

```
MATCH (u1)-[:FOLLOWS]->()
              -[:POSTED]->(m1)
```

Result:

u1	m1
N_1	N_5
N_2	N_4
N_3	N_5

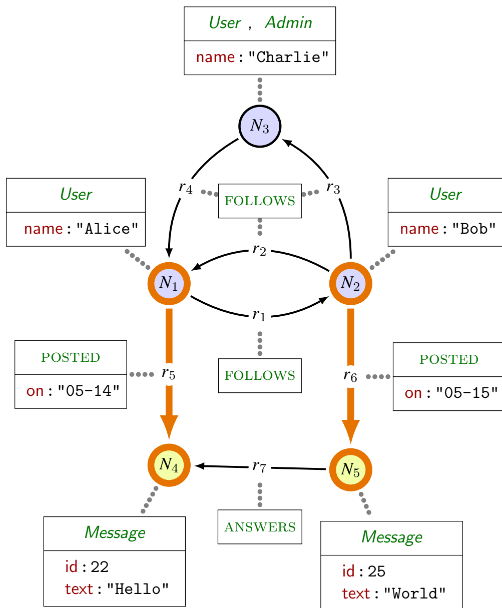
Chaining clauses



Query:

```
MATCH (u1)-[:POSTED]->(m1)
MATCH (u2)<-[:FOLLOWS]-(u1)
               -[:FOLLOWS]->(u3)
```

Chaining clauses



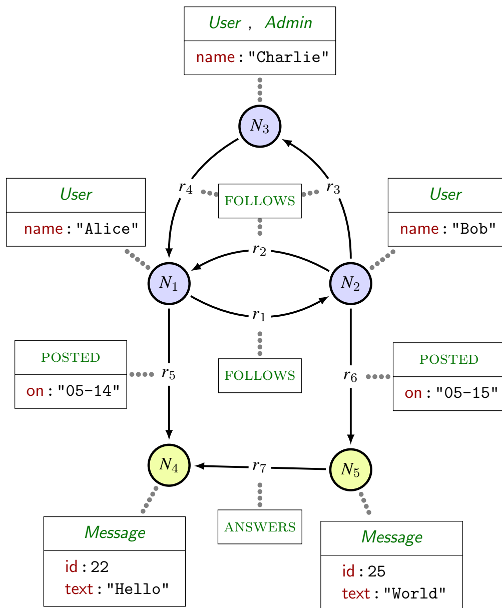
Query:

```
MATCH (u1)-[:POSTED]->(m1)
MATCH (u2)<-[:FOLLOWS]-(u1)
                -[:FOLLOWS]->(u3)
```

Table after first MATCH:

u1	m1
N_1	N_4
N_2	N_5

Chaining clauses



Query:

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MATCH (u1)-[:POSTED]->(m1)
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                  -[:FOLLOWS]->(u3)
```

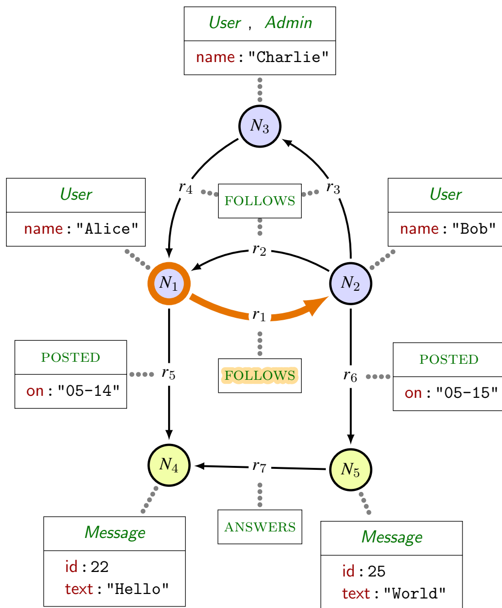
Table after first MATCH:

u1	m1
N_1	N_4
N_2	N_5

Table after second MATCH:

u1	m1	u2	u3
N_1	N_4	·	·
N_2	N_5	·	·

Chaining clauses



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                      -[:FOLLOWS]->(u3)
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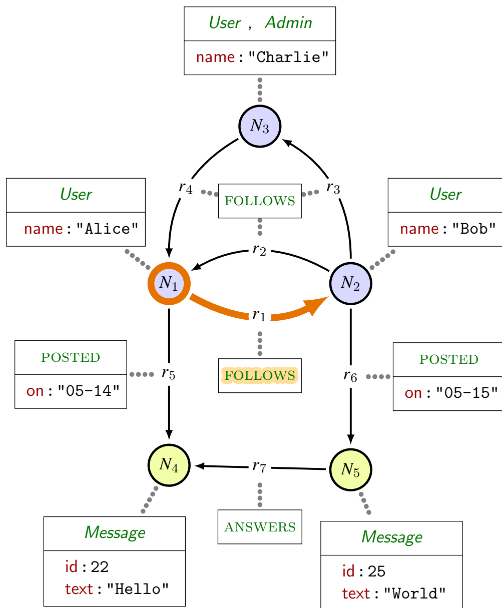
Table after first MATCH:

u1	m1
N_1	N_4
N_2	N_5

Table after second MATCH:

u1	m1	u2	u3
N_1	N_4	·	·
N_2	N_5	·	·

Chaining clauses



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```
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MATCH (u2)<-[:FOLLOWS]-(u1)
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```

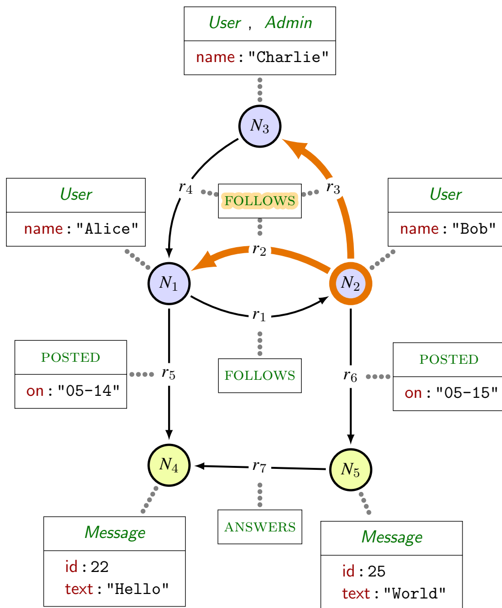
Table after first MATCH:

u1	m1
N_1	N_4
N_2	N_5

Table after second MATCH:

u1	m1	u2	u3
~~N_1~~	~~N_4~~		
N_2	N_5	·	·

Chaining clauses



Query:

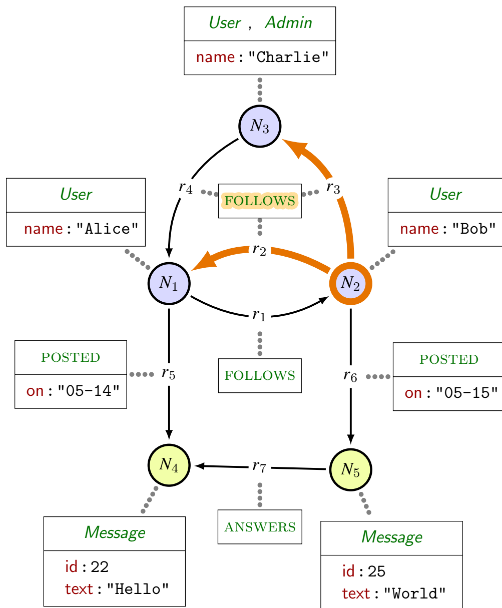
```
MATCH (u1)-[:POSTED]->(m1)
MATCH (u2)<-[:FOLLOWS]-(u1)
                -[:FOLLOWS]->(u3)
```

Table after first MATCH:

u1	m1
N_1	N_4
N_2	N_5

Table after second MATCH:

u1	m1	u2	u3
~~N_1~~	~~N_4~~		
N_2	N_5	·	·



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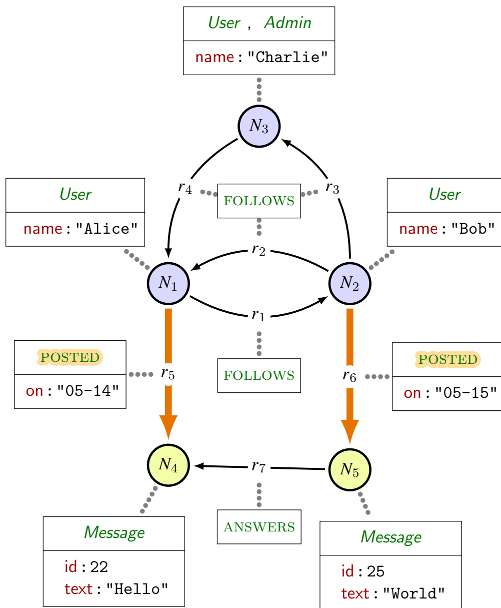
u1	m1
N_1	N_4
N_2	N_5

Table after second MATCH:

u1	m1	u2	u3
N_2	N_5	N_1	N_3
N_2	N_5	N_3	N_1

Column manipulation (WITH clause)



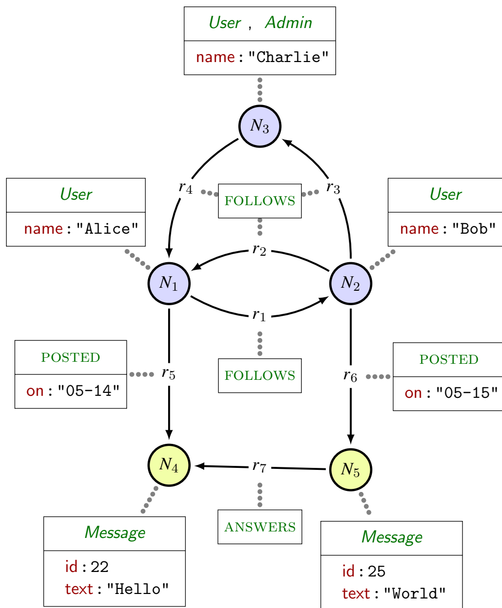
Query:

```
MATCH (u1)-[p1:POSTED]->(m1)
WITH u1, p1, m1.text AS t1
```

After the MATCH clause

u1	p1	m1
N_1	r_5	N_4
N_2	r_6	N_5

Column manipulation (WITH clause)



Query:

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MATCH (u1)-[p1:POSTED]->(m1)
WITH u1, p1, m1.text AS t1
```

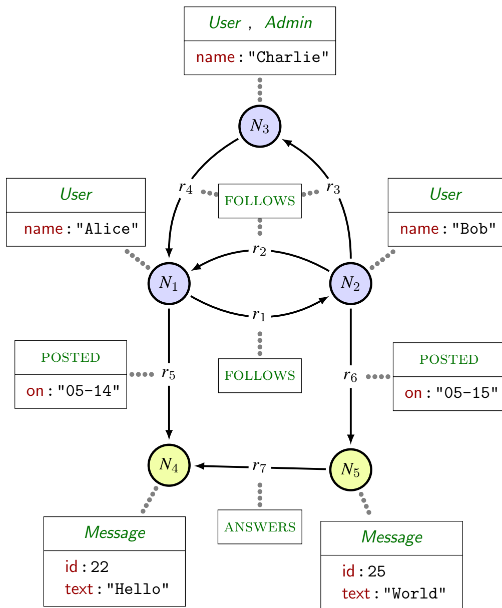
After the MATCH clause

u1	p1	m1
N_1	r_5	N_4
N_2	r_6	N_5

Execution of the WITH clause

u1	p1	t1
N_1	r_5	
N_2	r_6	

Column manipulation (WITH clause)



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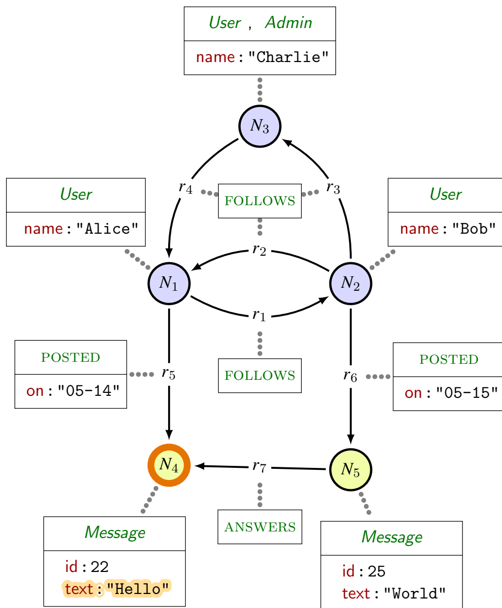

Query:

```
MATCH (u1)-[p1:POSTED]->(m1)
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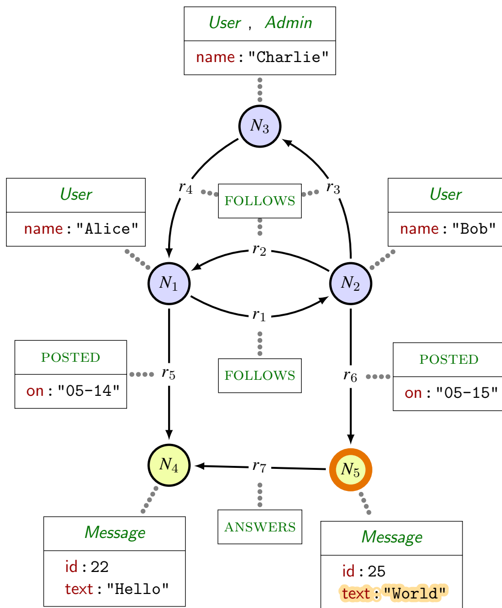
```

After the MATCH clause

u1	p1	m1
N_1	r_5	N_4
N_2	r_6	N_5

Execution of the WITH clause

u1	p1	t1
N_1	r_5	"Hello"
N_2	r_6	



Query:

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MATCH (u1)-[p1:POSTED]->(m1)
WITH u1, p1, m1.text AS t1
```

After the MATCH clause

u1	p1	m1
N_1	r_5	N_4
N_2	r_6	N_5

Execution of the WITH clause

u1	p1	t1
N_1	r_5	"Hello"
N_2	r_6	"World"

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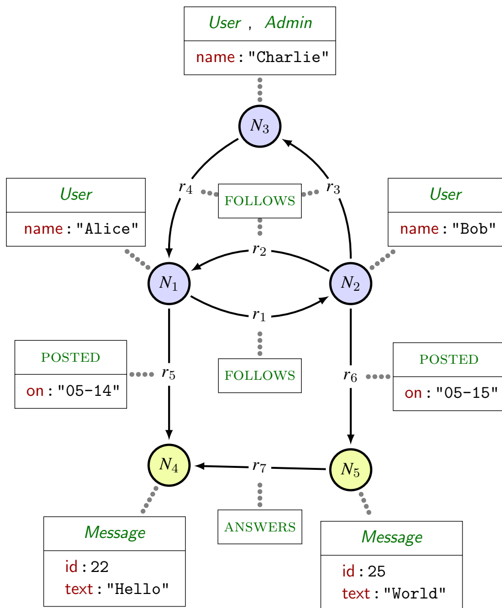


Query:

```
MATCH (u1)-[p1:POSTED]->(m1)
WITH u1, p1, m1.text AS t1
```

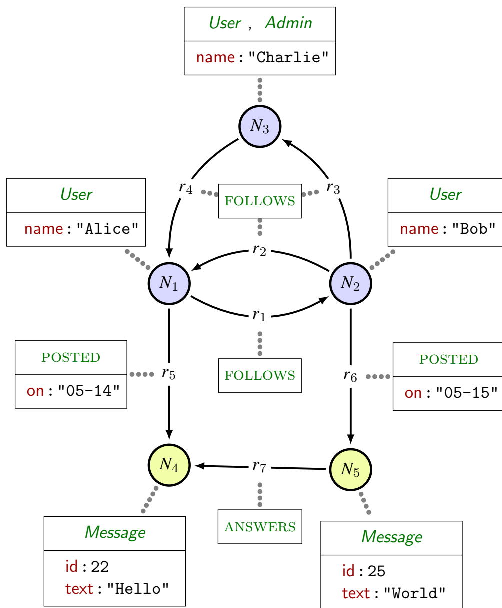
After the MATCH clause

u1	p1	m1
N_1	r_5	N_4
N_2	r_6	N_5

Final result

u1	p1	t1
N_1	r_5	"Hello"
N_2	r_6	"World"

Line filtering (WHERE clause)



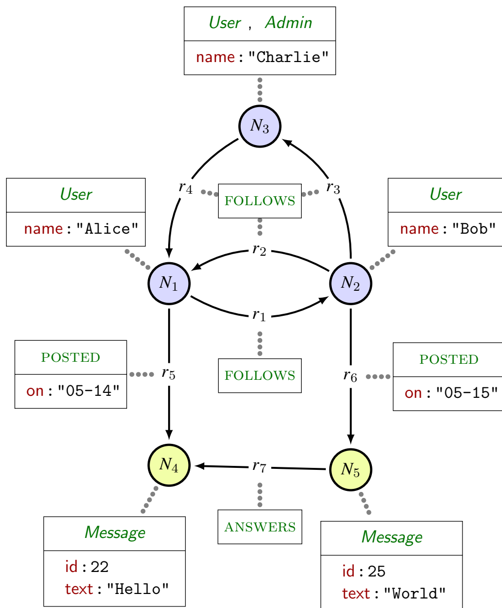
Query:

```
MATCH (u1)-[p1:POSTED]->(m1)
WITH u1, p1, m1.text AS t1
WHERE t1 = "Hello"
```

After the WITH clause

u1	p1	t1
N_1	r_5	"Hello"
N_2	r_6	"World"

Line filtering (WHERE clause)



Query:

```
MATCH (u1)-[p1:POSTED]->(m1)
WITH u1, p1, m1.text AS t1
WHERE t1 = "Hello"
```

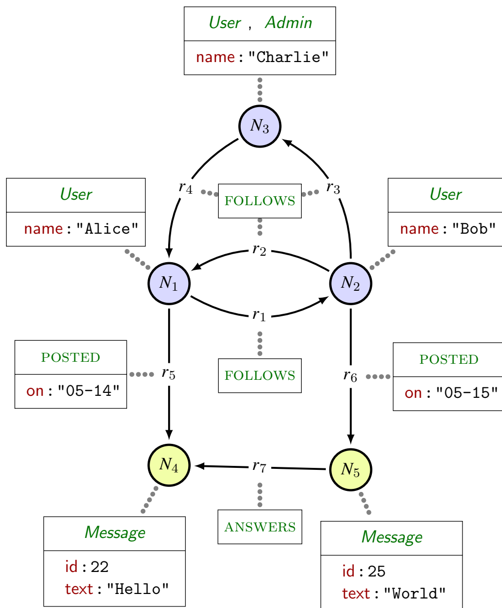
After the WITH clause

u1	p1	t1
N_1	r_5	"Hello"
N_2	r_6	"World"

Execution of the WHERE clause

u1	p1	t1
N_1	r_5	"Hello"
~~N_2~~	~~r_6~~	~~"World"~~

Line filtering (WHERE clause)



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```

After the WITH clause

u1	p1	t1
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Final result

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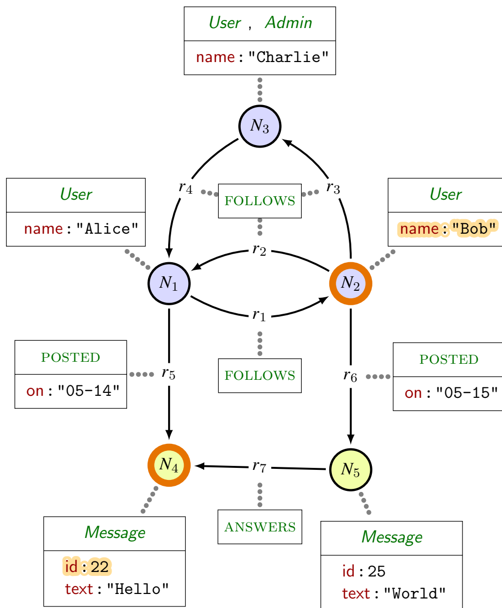
N_1	r_5	"Hello"

A last read-only example



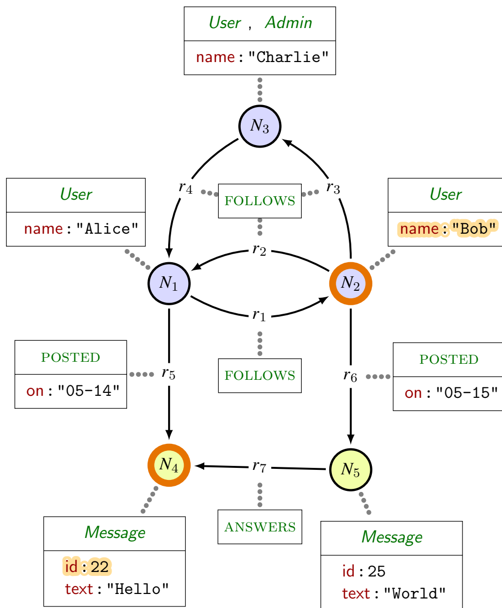
Query:

```
MATCH (a{name:"Bob"})
        -[*]->(b{id:22})
              <-[*]-(a)
```

Question

What does this computes ?

A last read-only example



Query:

```
MATCH (a{name:"Bob"})
        -[*]->(b{id:22})
              <-[*]-(a)
```

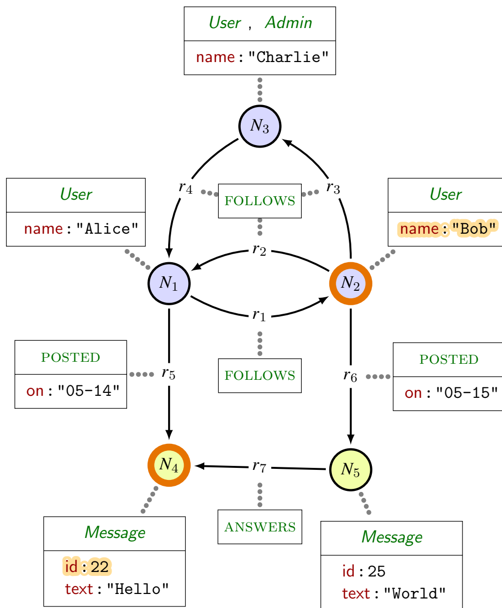
Question

What does this computes ?

Answer

The # of pairs of disjoint paths from N_2 to N_4.

A last read-only example



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              <-[*]-(a)
```

Question

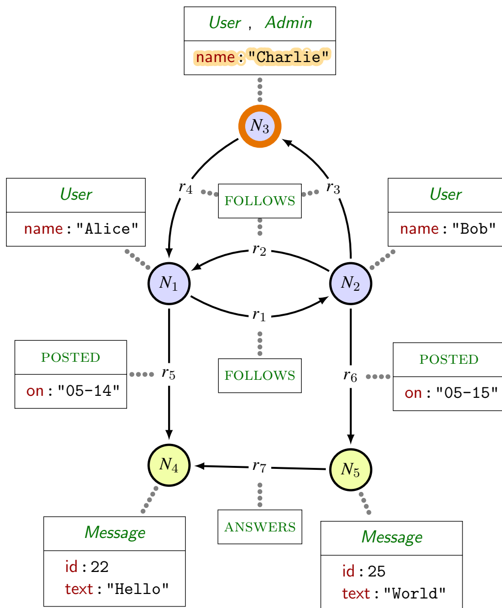
What does this computes ?

Answer

The # of pairs of disjoint paths from N_2 to N_4.

$\Rightarrow$ Evaluation of one constant `MATCH` is NP-HARD

Node and relation creation (CREATE)

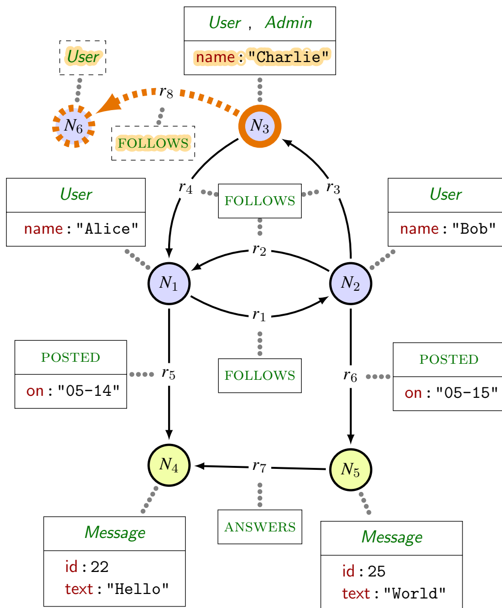


Query:

```
MATCH (a{name:"Charlie"})
CREATE (a)-[:FOLLOWS]->
                  (b:User)
```

Table after MATCH clause:

a
N_3



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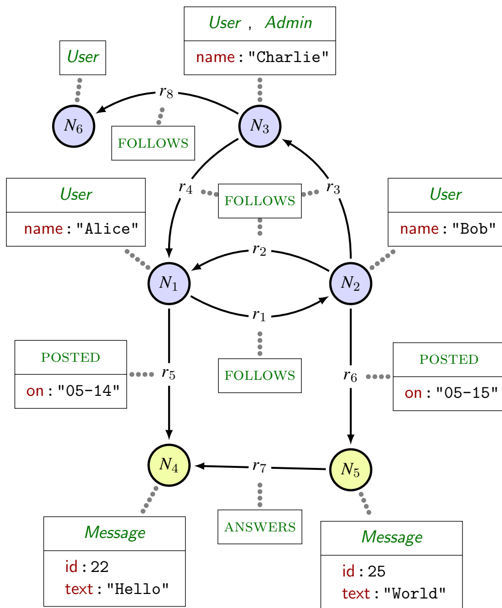
Node and relation creation (CREATE)



Query:

```
MATCH (a{name:"Charlie"})
CREATE (a)-[:FOLLOWS]->
                        (b:User)
```

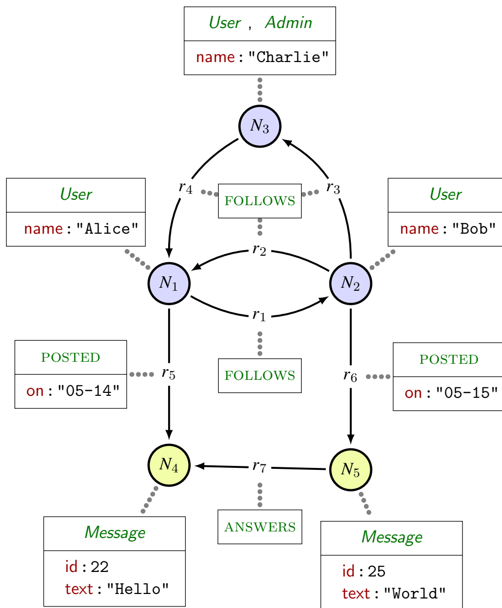
Table after MATCH clause:

a
N_3

Table after CREATE clause:

a	b
N_3	N_6

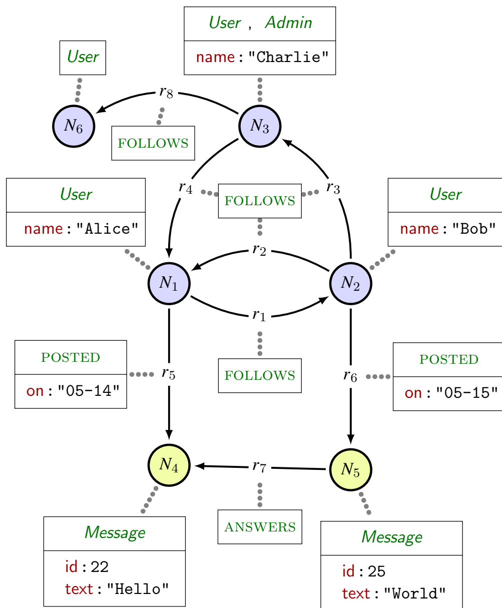
The example graph stored as CREATE clauses



Query:

```
CREATE
  (n1:User{name:"Alice"}),
  (n2:User{name:"Bob"}),
  (n3:User:Admin
            {name:"Charlie"}),
  (n4:Message {id:22,
               text:"Hello"}),
  (n5:Message {id:25,
               text:"World"})
CREATE
  (n1)-[:FOLLOWS]->(n2),
  (n1)-[:POSTED
          {on:"05-04"}]->(n4),
  (n2)-[:FOLLOWS]->(n1),
  (n2)-[:FOLLOWS]->(n3),
  (n2)-[:POSTED
          {on:"05-04"}]->(n5),
  (n3)-[:FOLLOWS]->(n1),
  (n5)-[:ANSWERS]->(n4),
```

Node/Relation modification (SET clause)



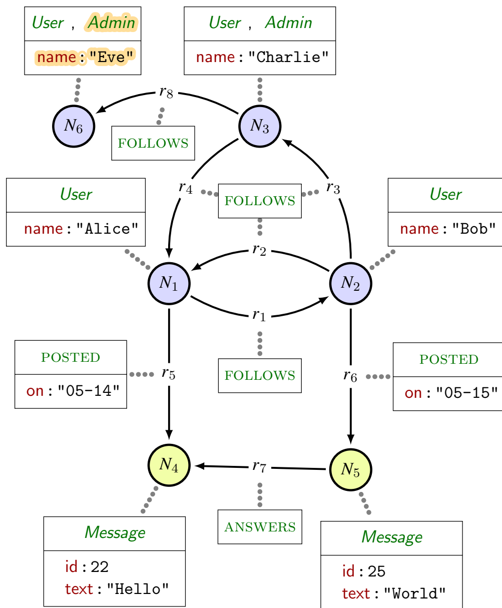
Query:

```
MATCH (a{name:"Charlie"})
CREATE (a)-[:FOLLOWS]->
                    (b:User)
SET b:Admin, b.name="Eve"
```

Table after CREATE clause:

a	b
N_3	N_6

Node/Relation modification (SET clause)



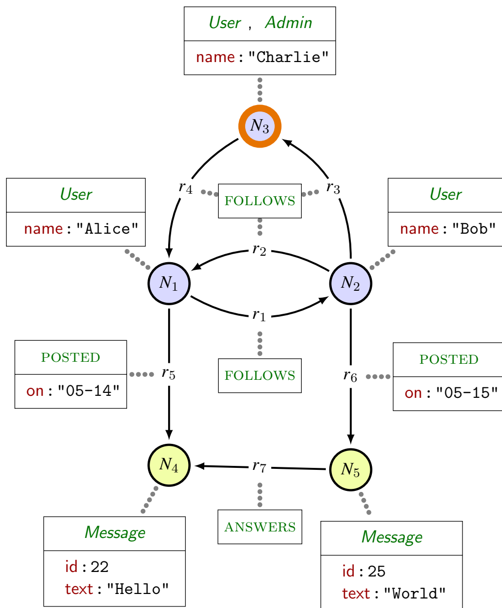
Query:

```
MATCH (a{name:"Charlie"})
CREATE (a)-[:FOLLOWS]->
                    (b:User)
SET b:Admin, b.name="Eve"
```

Table after CREATE clause:

a	b
N_3	N_6

The MERGE clause : MATCH else CREATE



Input table:

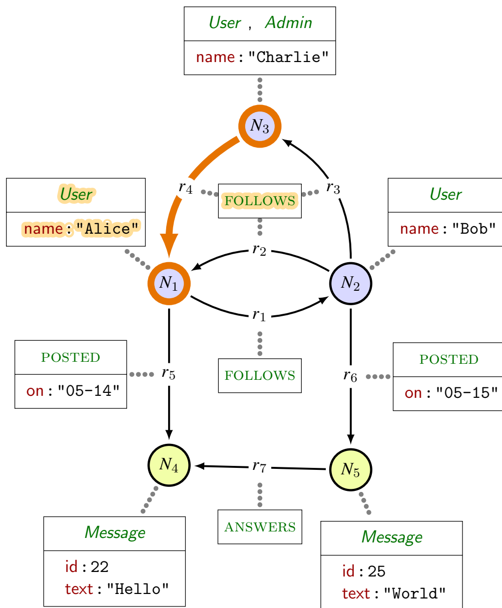
a	n
N_3	"Alice"
N_3	"Eve"

Query:

```
MERGE (a)-[:FOLLOWS]->
          (b:User {name:n})
```

Output table:

a	n	c
N_3	"Alice"	
N_3	"Eve"	



Input table:

a	n
N_3	"Alice"
N_3	"Eve"

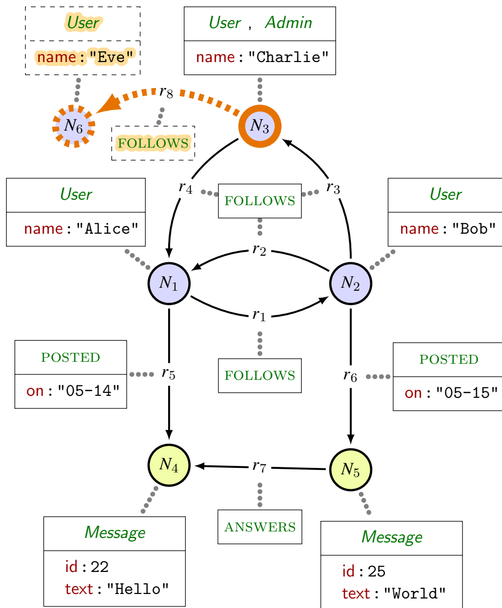
Query:

```
MERGE (a)-[:FOLLOWS]->
            (b:User {name:n})
```

Output table:

a	n	c
N_3	"Alice"	N_1
N_3	"Eve"	

The MERGE clause : MATCH else CREATE



Input table:

a	n
N_3	"Alice"
N_3	"Eve"

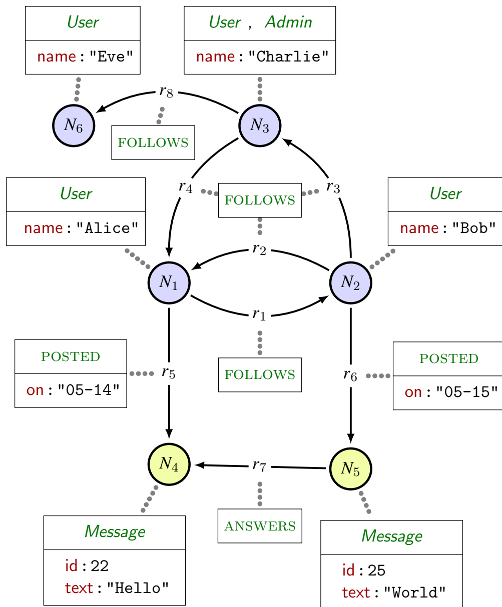
Query:

```
MERGE (a)-[:FOLLOWS]->
          (b:User {name:n})
```

Output table:

a	n	c
N_3	"Alice"	N_1
N_3	"Eve"	N_6

The MERGE clause : MATCH else CREATE



Input table:

a	n
N_3	"Alice"
N_3	"Eve"

Query:

```
MERGE (a)-[:FOLLOWS]->
          (b:User {name:n})
```

Output table:

a	n	c
N_3	"Alice"	N_1
N_3	"Eve"	N_6

Other constructs

- DELETE deletes node and relations.

Ex: `MATCH (a{name:"Eve"}) DELETE a`

- REMOVE removes labels or properties.

Ex: `MATCH (a{name:"Charlie"}) REMOVE a:Admin,a.name`

- WITH allows to perform aggregations.

Ex: `MATCH (a)-[:FOLLOWS]->(b) WITH a, count(b) as c`

- ORDER BY limits size of table.

Ex: `MATCH (a:User) ORDER BY a.name LIMIT 1`

Outline

1 Introduction
2 Property graphs
3 Regular Path Queries
4 Cypher by example
5 Principles of the semantics
6 Towards a standard language for querying property graphs

Record (table row)

A *record* is a partial function from variables to values.

Example: $(x \mapsto \texttt{"Bob"}\ ;\ y \mapsto \texttt{1})$

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Table

A *table* is a multi-set (or bag) of records with the same domain.

Example:

x	y
"Bob"	1
"Alice"	999
"Bob"	1

Modelling Tables

Record (table row)

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Example: $(x \mapsto$ `"Bob"` ; $y \mapsto$ `1`$)$

Table

A *table* is a multi-set (or bag) of records with the same domain.

Example:

x	y
"Bob"	1
"Alice"	999
"Bob"	1

=

y	x
999	"Alice"
1	"Bob"
1	"Bob"

Semantics brackets – Read-Only

G: a graph

Semantics of expressions

$[\![\cdot]\!]_{u,G}$: expression $\longmapsto$ value (where u is a record)

Semantics of clauses

$[\![\cdot]\!]_{G}$: clause $\longmapsto$ (function: Tables $\to$ Tables)

Semantics of queries

$[\![\cdot]\!]_{G}$: query $\longmapsto$ (function: Tables $\to$ Tables)

output : (Graphs $\times$ Queries) $\longmapsto$ Tables

Principle of Read-Only Semantics

G: a graph
Q: a query

To compute the output of Q

- Q is a sequence of clauses $Q = C_1 \, C_2 \, \cdots \, C_n$

Principle of Read-Only Semantics

G: a graph
Q: a query

To compute the output of Q

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- Compute $\left[\!\left[C_1 \right]\!\right]_G, \ \left[\!\left[C_2 \right]\!\right]_G, \ \ldots, \ \left[\!\left[C_n \right]\!\right]_G$

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- Let $[\![Q]\!]_G = [\![C_n]\!]_G \circ \cdots \circ [\![C_2]\!]_G \circ [\![C_1]\!]_G$
- $\text{output}(G, Q) = [\![Q]\!]_G \big(T_{\text{unit}}\big)$

where T_{unit} is the 1-line 0-column table.

Excerpt of Read-Only semantics (2)

- $\left[\!\left[\texttt{WHERE}\ e\right]\!\right]_G(T) = \left\{ u \in T \;\middle|\; \left[\!\left[e\right]\!\right]_{G,u} = \texttt{true} \right\}$

- $\left[\!\left[\texttt{MATCH}\ \bar{\pi}\right]\!\right]_G(T) = \biguplus_{u \in T} \{u \cdot u' \mid u' \in \mathsf{match}(\bar{\pi}, G, u)\}$
- $\left[\!\left[\texttt{MATCH}\ \bar{\pi}\ \texttt{WHERE}\ e\right]\!\right]_G(T) = \left[\!\left[\texttt{WHERE}\ e\right]\!\right]\left(\left[\!\left[\texttt{MATCH}\ \bar{\pi}\right]\!\right]_G(T)\right)$
- $\left[\!\left[\texttt{OPTIONAL MATCH}\ \bar{\pi}\ \texttt{WHERE}\ e\right]\!\right]_G(T)$
  $$= \biguplus_{u \in T} \begin{cases} \left[\!\left[\texttt{MATCH}\ \bar{\pi}\ \texttt{WHERE}\ e\right]\!\right]_G(\{u\}) & \text{if } \left[\!\left[\texttt{MATCH}\ \bar{\pi}\ \texttt{WHERE}\ e\right]\!\right]_G(\{u\}) \neq \emptyset \\ (u, (\mathsf{free}(u, \bar{\pi}) : \texttt{null})) & \text{otherwise} \end{cases}$$
- $\left[\!\left[\texttt{OPTIONAL MATCH}\ \bar{\pi}\right]\!\right]_G(T) = \left[\!\left[\texttt{OPTIONAL MATCH}\ \bar{\pi}\ \texttt{WHERE true}\right]\!\right]_G(T)$

Excerpt of Read-Only semantics (2)

- $\llbracket \texttt{WITH } * \rrbracket_G(T) = T$ if T has at least one column
- $\llbracket \texttt{WITH } *, e_1 \ [\texttt{AS } a_1], \ldots, e_m \ [\texttt{AS } a_m] \rrbracket_G(T) =$
  $$\llbracket \texttt{WITH } b_1 \texttt{ AS } b_1, \ldots, b_q \texttt{ AS } b_q, e_1 \ [\texttt{AS } a_1], \ldots, e_m \ [\texttt{AS } a_m] \rrbracket_G(T)$$
- $\llbracket \texttt{WITH } e_1 \ [\texttt{AS } a_1], \ldots, e_m \ [\texttt{AS } a_m] \rrbracket_G(T) =$
  $$\biguplus_{u \in T} \left\{ (a'_1 : \llbracket e_1 \rrbracket_{G,u}, \ldots, a'_m : \llbracket e_m \rrbracket_{G,u}) \right\}$$

- $\llbracket \texttt{UNWIND } e \texttt{ AS } a \rrbracket_G(T) = \biguplus_{u \in T} \biguplus_{v \in E_u} \{(u, a : v)\}$,
  $$\text{with} \quad E_u = \begin{cases} \biguplus_{0 \le i < m} \{v_i\} & \text{if } \llbracket e \rrbracket_{G,u} = \text{list}(v_0, \ldots, v_{m-1}) \\ \{\} & \text{if } \llbracket e \rrbracket_{G,u} = \text{list}() \\ \left\{ \llbracket e \rrbracket_{G,u} \right\} & \text{otherwise} \end{cases}$$

Semantics of expressions: (Nothing changes)

$[\![\cdot]\!]_{u,G}$: expression $\longmapsto$ value (where u is a record)

Semantics of clauses:

$[\![\cdot]\!]$: clause $\longmapsto$ (function: (Graphs $\times$ Tables) $\rightarrow$ (Graphs $\times$ Tables))

Semantics of queries:

$[\![\cdot]\!]$: query $\longmapsto$ (function: (Graphs $\times$ Tables) $\rightarrow$ (Graphs $\times$ Tables))

output : (Graphs $\times$ Queries) $\longmapsto$ (Graphs $\times$ Tables)

(Computed just like RO $\rightarrow$ composition of clause semantics)

Desirable properties of the semantics at Clause-level

Atomicity:

Each clause is executed as a single unit

Consistency:

Each clause should on valid Graph/Table pair

General scheme of the semantics if a clause is:

1. Ensure that the input Graph/Table is valid w.r.t. clause
2. Compute output Table and all changes to graph
3. Apply all changes to Graph
4. Ensure validity of output Graph/Table

If any fails, semantics is undefined.

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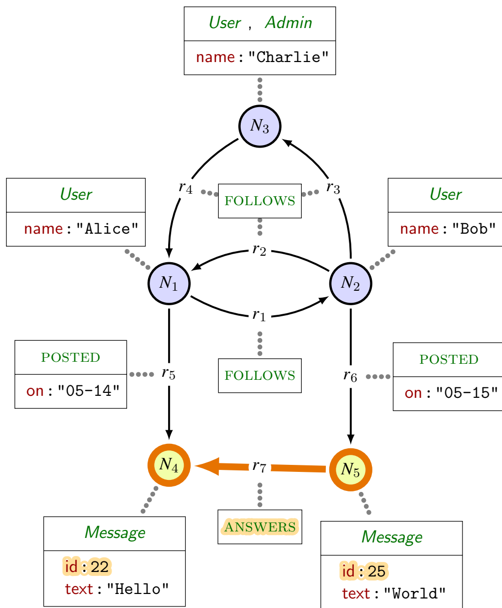
In Neo4j, Atomicity and Consistency are verified at query level only.

In Neo4j, SET violate Atomicity



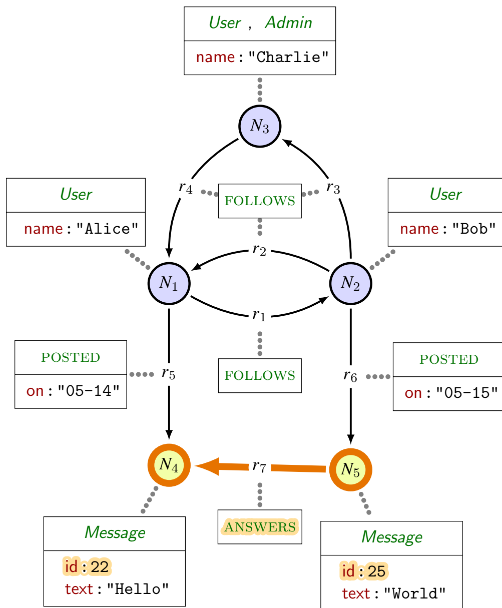
Query:

```
MATCH (m1)-[:ANSWERS]->(m2)
SET m1.id = m2.id,
    m2.id = m1.id
```

Table before SET clause:

m1	m2
N_5	N_4

In Neo4j, SET violate Atomicity



Query:

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MATCH (m1)-[:ANSWERS]->(m2)
SET m1.id = m2.id,
    m2.id = m1.id
```

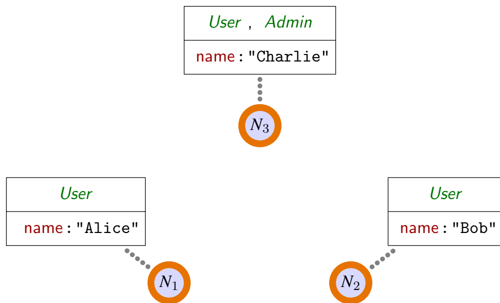
Table before SET clause:

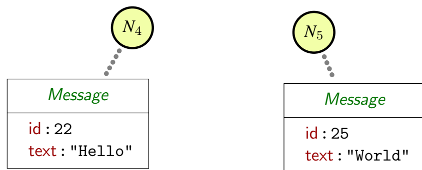
m1	m2
N_5	N_4

Neo4j sets both id to 25.

Semantics exchange id values

In Neo4j, MERGE is highly non-deterministic





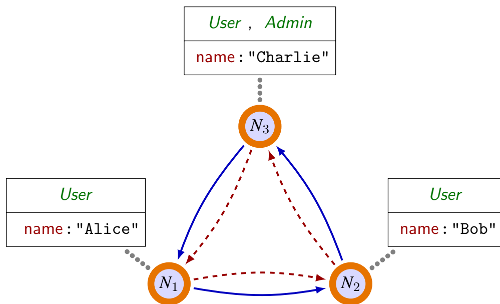
Input Table:

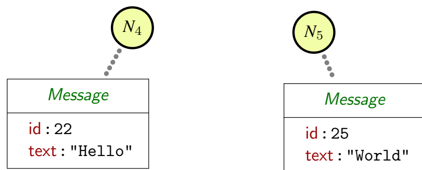
a	b	c
N_1	N_2	N_3
N_2	N_3	N_1
N_3	N_1	N_2

Query:

```
MERGE (a)-[:FOLLOWS]->(b)
         -[:FOLLOWS]->(c)
```

In Neo4j, MERGE is highly non-deterministic





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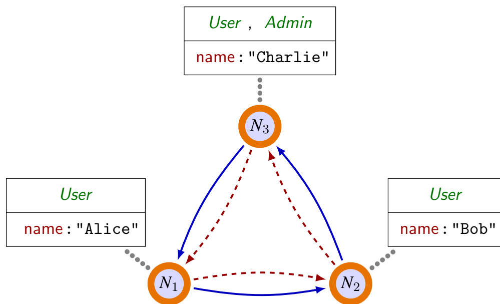
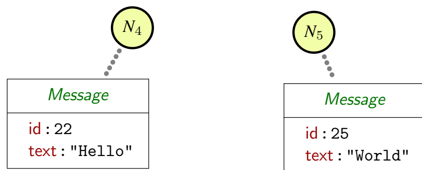
```
MERGE (a)-[:FOLLOWS]->(b)
         -[:FOLLOWS]->(c)
```

Neo4j creates 4 edges

In Neo4j, MERGE is highly non-deterministic





Input Table:

a	b	c
N_1	N_2	N_3
N_2	N_3	N_1
N_3	N_1	N_2

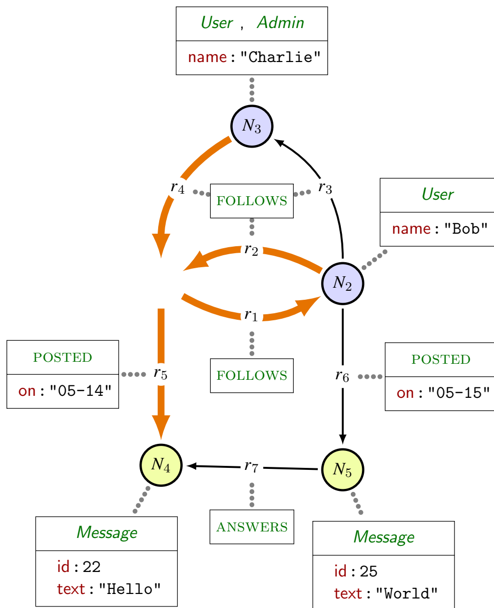
Query:

```
MERGE (a)-[:FOLLOWS]->(b)
         -[:FOLLOWS]->(c)
```

Neo4j creates 4 edges

Semantics propose different semantics creating either 3 or 6.

In Neo4j, DELETE violates clause-level consistency

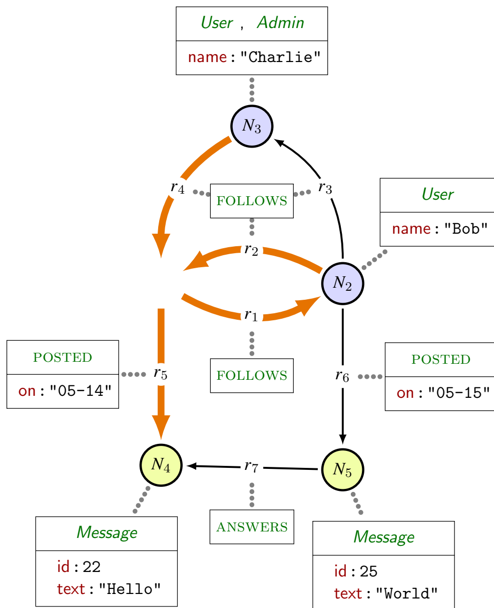


Query:

```
MATCH (a {name:"Alice"})
MATCH (a)-[r]-()
DELETE a
  [...]   // Arbitrary clauses
DELETE r
RETURN a.name AS n
```

- Execution of arbitrary code on invalid graph
- Access to deleted-entity

In Neo4j, DELETE violates clause-level consistency



Query:

```
MATCH (a {name:"Alice"})
MATCH (a)-[r]-()
DELETE a
  [...]    // Arbitrary clauses
DELETE r
RETURN a.name AS n
```

- Execution of arbitrary code on invalid graph
- Access to deleted-entity

Semantics

- is undefined if it should produce invalid graphs
- replaces deleted entities by `null` in the table

Outline

1 Introduction

2 Property graphs

3 Regular Path Queries

4 Cypher by example

5 Principles of the semantics

6 Towards a standard language for querying property graphs

Cypher vs RPQ

Cypher queries vs UCRPQs

- Cypher has bag + cypher-morphism semantics
  (Set+ standard morphism semantics may be emulated...)
- Cypher has relationship/path variables
- (Data model is different)

RPQs not expressible in Cypher

- $(ab)^*$: no concatenation under star in Cypher
- $(a^*)^*$: no nested stars in Cypher
- $(a + b^{-1})^*$: some unions are not allowed under star in Cypher

- $(ab + cd)^3$: nested alternations of concatenations and unions require multi-exponential blow-up of the query
  $\rightarrow (ab)^3 + (ab)^2cd + ab(cd)^2 + (cd)^3$

PGQL (Property Graph Query Language)

- Designed by Oracle Inc.
- Support full UCRPQ
- ASCII-art representation of patterns (similar to Cypher)
- Syntax close to SQL

Example: (from http://pgql-lang.org/)

```
SELECT p1.name
FROM facebook_graph          /* In the Facebook graph,..   */
MATCH (p1:Person)            /* ..find persons such that.. */
WHERE NOT EXISTS (           /* ..there does not exist..   */
  SELECT p2
    FROM twitter_graph       /* ..in the Twitter graph..   */
    MATCH (p2:Person)        /* ..a person..               */
    WHERE p1.name = p2.name  /* ..with the same name.      */
)
```

- Designed by LDBC
- Experimental
- Support full UCRPQ
- ASCII-art representation of patterns (similar to Cypher)
- Support Multiple graphs, graph views, and paths cost
- Multiple semantics

Example:

```
CONSTRUCT (n)-[:To{distance:=c}]->(m)
  MATCH (n) -/SHORTEST p<:KNOWS*> COST c/->(m)
    ON facebook_graph
  MATCH (n),(m) ON upem_graph
```

PGQL

- Read Only
- RPQs
- No Graph Construct/Project;
- Not Composable yet

Oracle PGX

GCORE

Advises

- Create - Read
- RPQs
- Graph Construct/Project;
- Composable

No implementations yet

Cypher

- Create - Read - Update - Delete
- No RPQs
- Graph Construct/Project:
- Composable

- Neo4j DB
- AgensGraph
- Redis Graph
- SAP HANA Graph
- Cypher for Spark/Gremlin
- Memgraph
- inGraph
- Cypher.PL

New, Fused

GQL

- Create - Read - Update - Delete
- RPQs
- Graph Construct/Project;
- Composable

cf. GQL manifesto:
`http://gql.today`

cf. GQL manifesto:
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To be continued...

Detailed outline

1 Introduction
2 Property graphs
3 Regular Path Queries
- RPQ
- CRPQ
- UCRPQ

4 Cypher by example
- Query evaluation
- MATCH
- WITH
- WHERE
- CREATE
- SET
- MERGE
- Others

5 Principles of the semantics
- Modelling tables
- Read-Only
- Read-Write

6 Towards a standard language for querying property graphs
- Cypher vs RPQ
- PGQL
- G-Core

7 The MATCH clause
- Path patterns
- Rigid path patterns
- General case

8 Remainder of the core fragment
- Caveat on the MATCH clause
- OPTIONAL MATCH
- WHERE
- WITH
- UNWIND
- RETURN
- UNION of queries

9 Ongoing additions
- Aggregation
- Line order

Node patterns

⟨node_pattern⟩ ::= (⟨name⟩? ⟨labels⟩? ⟨properties⟩?)

Examples

`()`	Any node
`(a)`	Any node; verify/map id to column a
`(:User)`	Nodes bearing label *User*
`({name:"Alice"})`	Nodes with the property name set to `"Elin"`
`(a:User{name:"Alice"})`	Nodes of type User and with a property name set to `"Elin"`; verify/map id to column a.

Relationship patterns

⟨pattern⟩ ::= **[** ⟨name⟩? ⟨types⟩? ⟨iter⟩? ⟨properties⟩? **]**

Example

`[]`	Any relationship
`[r]`	Any relationship; verify/map relationship id to column r
`[r*]`	Any path; verify/map the list of relationship id to column r
`[*..3]`	Any path of length 1–3
`[:FOLLOWS*3..]`	Any FOLLOWS path of length at least 3
`[*{isNice:true}]`	Path such that every relationship has the property isNice set to true

Path patterns

⟨pattern⟩ ::=
⟨node_pattern⟩ [$<^?$ – ⟨rel_pattern⟩ – $>^?$ ⟨node_pattern⟩ $]^*$

Examples

```
()
()-[*2..3]-(b)
(a)<-[]->()-[{id:220}]->()
(a)<-[*]-(b)-[*]->(a)
({name:a.name})-[*]->(a)
({name:"Alice"})-[:POSTED]->()<-[:ANSWERS*]-()
```

Rigid path-patterns

Definition

A path pattern is *Rigid* if its length is fixed
(*i.e.* all ⟨iter⟩ are absent or derive to $*i..i$ for some $i \in \mathbb{N}$)

Examples

`(a)<-[]->()-[{id:220}]->()`	Rigid
`()-[]->()-[*42..42]->(:User)`	Rigid
`()-[*2..3]->(b)` and `(a)<-[]-(b)-[*]-(a)`	Flexible

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Examples

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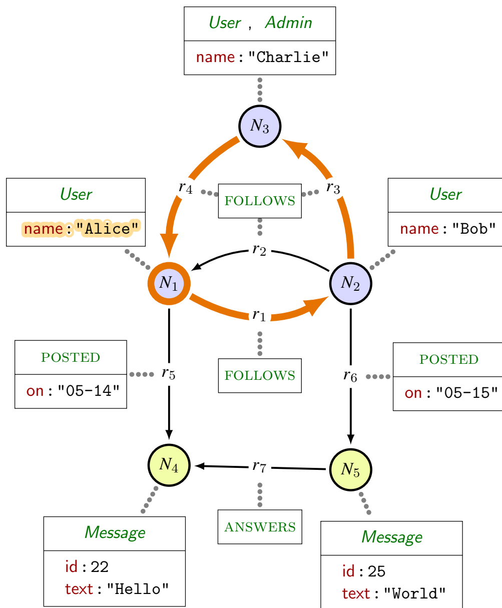
`()-[]->()-[*42..42]->(:User)`	Rigid
`()-[*2..3]->(b)` and `(a)<-[]-(b)-[*]-(a)`	Flexible

Property

A path has only one way to satisfy a rigid path pattern.

Rigid path vs flexible path (1)



Query:

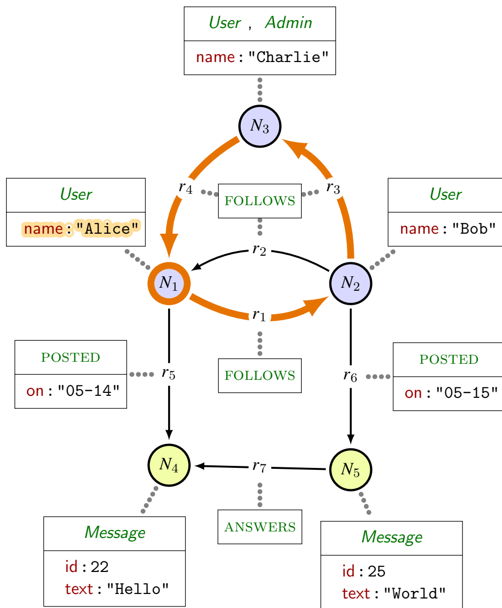
```
MATCH (x {name:Alice})
      -[:FOLLOWS*1..1]->(mid)
      -[:FOLLOWS*2..2]->(x)
```

The path

$$N_1 \xrightarrow{r_1} N_2 \xrightarrow{r_3} N_3 \xrightarrow{r_4} N_1$$

satifies the pattern.

Rigid path vs flexible path (1)



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```

The path

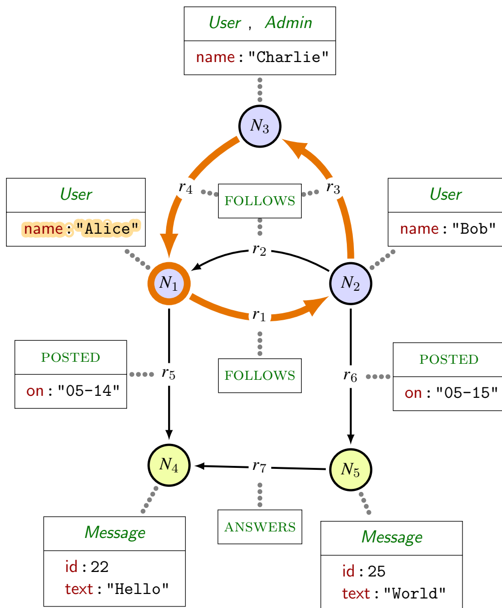
$$N_1 \xrightarrow{r_1} N_2 \xrightarrow{r_3} N_3 \xrightarrow{r_4} N_1$$

satifies the pattern.

Variables are uniquely set

x	mid
N_1	N_2

Rigid path vs flexible path (2)



Query:

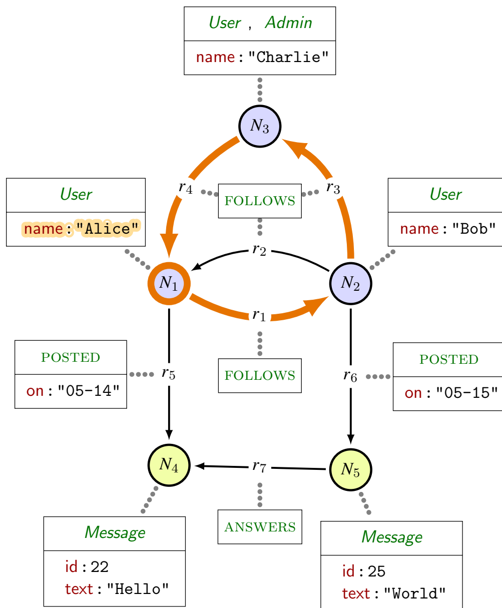
```
MATCH (x {name:"Alice"})
      -[:FOLLOWS*1..2]->(mid)
      -[:FOLLOWS*1..2]->(x)
```

The path

$$N_1 \xrightarrow{r_1} N_2 \xrightarrow{r_3} N_3 \xrightarrow{r_4} N_1$$

satifies this second pattern.

Rigid path vs flexible path (2)



Query:

```
MATCH (x {name:"Alice"})
      -[:FOLLOWS*1..2]->(mid)
      -[:FOLLOWS*1..2]->(x)
```

The path

$$N_1 \xrightarrow{r_1} N_2 \xrightarrow{r_3} N_3 \xrightarrow{r_4} N_1$$

satifies this second pattern.

Two possible assignments

x	mid
N_1	N_2
N_1	N_3

Path satisfying a rigid path-pattern

u: a record

p: a path

π: a rigid path-pattern

n: length of π

Definition

$G, p, u \models \pi$ if

- p is of length n
- $\forall i \leq p$, the i-th node of p satisfies the i-th node pattern of π (variable, labels, properties)
- $\forall i < p$, the i-th relationship of p satisfies the i-th relationship pattern of π (variable, types, properties)

rigid(π) = set of all rigid patterns subsumed by π

Examples

`()-[*]->()` $\mapsto$ { `()-[*1..1]->()` , `()-[*2..2]->()` , $\cdots$ }

Rigid path-pattern subsumed by a flexible path-pattern

rigid(π) = set of all rigid patterns subsumed by π

Examples

`()-[*]->()` $\mapsto$ { `()-[*1..1]->()` , `()-[*2..2]->()` , $\cdots$ }

`()-[*2..3]-(b)` $\mapsto$ { `()-[*2..2]-(b)` , `()-[*3..3]-(b)` }

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Examples

`()-[*]->()` $\mapsto$ {`()-[*1..1]->()` , `()-[*2..2]->()` , ⋯}

`()-[*2..3]-(b)` $\mapsto$ {`()-[*2..2]-(b)` , `()-[*3..3]-(b)`}

`({name:"Alice"})-[*]->()-[:CITES*]->()` $\mapsto$
{`({name:"Alice"})-[*1..1]->()-[:CITES*1..1]->()` ,
`({name:"Alice"})-[*1..1]->()-[:CITES*2..2]->()` ,
`({name:"Alice"})-[*2..2]->()-[:CITES*1..1]->()` , ⋯}

rigid(π) = set of all rigid patterns subsumed by π

Examples

`()-[*]->()` ↦ { `()-[*1..1]->()` , `()-[*2..2]->()` , ⋯ }

`()-[*2..3]-(b)` ↦ { `()-[*2..2]-(b)` , `()-[*3..3]-(b)` }

```
({name:"Alice"})-[*]->()-[:CITES*]->() ↦
{({name:"Alice"})-[*1..1]->()-[:CITES*1..1]->() ,
({name:"Alice"})-[*1..1]->()-[:CITES*2..2]->() ,
({name:"Alice"})-[*2..2]->()-[:CITES*1..1]->() , ⋯}
```

WLOG: rigid(π) contains no pattern longer than the graph size

The MATCH clause

Semantics of MATCH

$$[\![\texttt{MATCH}\ \pi]\!]_G : \quad T \quad \longmapsto \quad \biguplus_{u \in T} \mathsf{expand}_{G,\pi}(u)$$

The MATCH clause

Semantics of MATCH

$$[\![\texttt{MATCH}\ \pi]\!]_G : \quad T \longmapsto \biguplus_{u \in T} \text{expand}_{G,\pi}(u)$$

Table-row expansion

π: a path-pattern

G: a graph

$$\text{expand}_{G,\pi} : \quad \text{Records} \longrightarrow \text{Tables}$$

$$u \longmapsto \biguplus_{\substack{\text{paths } p \text{ in } G \\ \text{patterns } \pi' \text{ in rigid}(\pi)}} \left\{ \text{records } v \;\middle|\; \begin{array}{l} \blacksquare\ \text{dom}(v) = \text{dom}(u) \cup \text{var}(\pi') \\ \blacksquare\ v \text{ extends } u \\ \blacksquare\ G, p, v \models \pi' \end{array} \right\}$$

The core fragment

Queries are essentially sequences of clauses

$$\langle\text{query}\rangle ::= \langle\text{query}\rangle \;\; \texttt{UNION} \; [\texttt{ALL}]^{?} \;\; \langle\text{query}\rangle \;\; \mid \;\; [\langle\text{clause}\rangle]^{*} \;\; \langle\text{return}\rangle$$

A clause is a main statement followed by subclauses

$$\langle\text{clause}\rangle ::= \langle\text{match}\rangle \;\; [\langle\text{where}\rangle]^{?} \;\; \mid \;\; \langle\text{with}\rangle \;\; [\langle\text{where}\rangle]^{?} \;\; \mid \;\; \langle\text{unwind}\rangle$$

Caveat on the MATCH clause (1)

Syntax

⟨match⟩ ::= [OPTIONAL]? MATCH ⟨pattern_tuple⟩

⟨pattern_tuple⟩ ::= ⟨pattern⟩ [, ⟨pattern⟩]*

- MATCH may search for a tuple of path-pattern
  - Cypher-morphism applies to the whole tuple
- MATCH may be "optional" :
  - if MATCH succeeds → no changes
  - if MATCH fails → free variables are set to null instead of deleting the line.

Semantics of MATCH

$\bar{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$: a path-pattern tuple
G: a graph

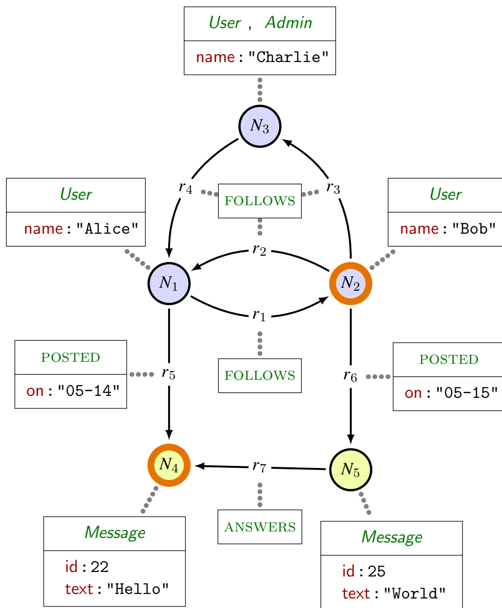
$$\text{rigid}(\bar{\pi}) = \text{rigid}(\pi_1) \times \text{rigid}(\pi_2) \times \cdots \times \text{rigid}(\pi_n)$$

$$\text{expand}_{G,\bar{\pi}} : \quad \text{Records} \longrightarrow \text{Tables}$$

$$u \longmapsto \biguplus_{\substack{\text{paths } \bar{p} \text{ in } G \\ \text{patterns } \bar{\pi}' \text{ in rigid}(\pi)}} \left\{ \text{records } v \;\middle|\; \begin{array}{l} \blacksquare\ \text{dom}(v) = \\ \quad \text{dom}(u) \cup \text{var}(\bar{\pi}') \\ \blacksquare\ v \text{ extends } u \\ \blacksquare\ G, \bar{p}, v \models \bar{\pi}' \end{array} \right\}$$

$$[\![\texttt{MATCH } \bar{\pi}]\!]_G : \quad T \longmapsto \biguplus_{u \in T} expand_{G,\pi}(u)$$

OPTIONAL MATCH – Example



x
N_2
N_4

↓

Query:

```
OPTIONAL MATCH
        (x)-[y:FOLLOWS]->()
```

OPTIONAL MATCH – Example

User, *Admin*
name : "Charlie"

N_3

User
name : "Alice"

r_4 FOLLOWS r_3

User
name : "Bob"

r_2

N_1 N_2

r_1

POSTED
on : "05-14"

r_5 FOLLOWS r_6

POSTED
on : "05-15"

N_4 r_7 N_5

Message
id : 22
text : "Hello"

ANSWERS

Message
id : 25
text : "World"

x
N_2
N_4

Query:

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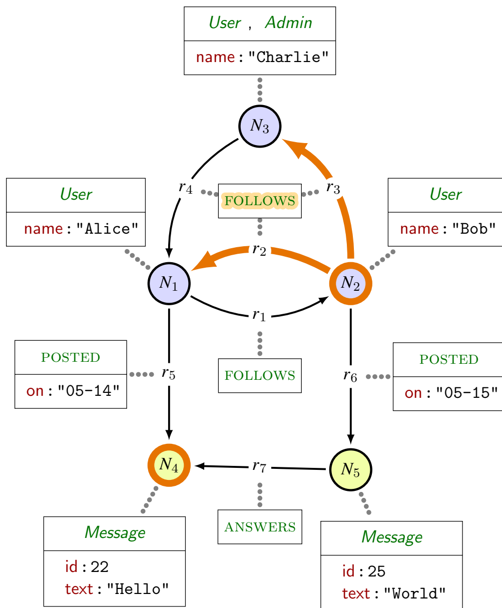
OPTIONAL MATCH – Example



x
N_2
N_4

↓

Query:

```
OPTIONAL MATCH
        (x)-[y:FOLLOWS]->()
```

↓

x	y
N_2	r_2
N_2	r_3
N_4	null

OPTIONAL MATCH – Semantics

u: a record

$\bar{\pi}$: a path-pattern tuple

$\mathsf{compl}(u, \bar{\pi})$ is the record:

- $\mathsf{dom}\Big(\mathsf{cmpl}(u, \bar{\pi})\Big) = \mathsf{dom}(u) \cup \mathsf{var}(\bar{\pi})$
- $a \longmapsto \begin{cases} u(a) & \text{if } a \in \mathsf{dom}(u) \\ \texttt{null} & \text{otherwise} \end{cases}$

$$\Big[\!\!\Big[\texttt{OPTIONAL MATCH } \bar{\pi}\Big]\!\!\Big]_G(T) = \biguplus_{u \in T} \begin{cases} T_u & \text{if } T_u \neq \emptyset \\ \{\mathsf{cmpl}(u)\} & \text{otherwise} \end{cases}$$

where $T_u = \Big[\!\!\Big[\texttt{MATCH } \bar{\pi}\Big]\!\!\Big]_G(\{u\})$

The WHERE subclause (1)

Syntax

$$\langle \text{where} \rangle ::= \quad \texttt{WHERE} \quad \langle \text{expr} \rangle$$

Semantics

$$\left[\!\left[\texttt{WHERE}\ e \right]\!\right] : \quad T \quad \longmapsto \quad \biguplus_{u \in T} \begin{cases} \{u\} & \text{if } \left[\!\left[e \right]\!\right]_{G,u} = \texttt{true} \\ \emptyset & \text{otherwise} \end{cases}$$

Combined semantics

- $[\![\texttt{MATCH}\ \bar{\pi}\ \texttt{WHERE}\ e]\!]_G(T) = [\![\texttt{WHERE}\ e]\!]\big([\![\texttt{MATCH}\ \bar{\pi}]\!]_G(T)\big)$

- $[\![\texttt{WITH}\ \bar{\pi}\ \texttt{WHERE}\ e]\!]_G(T) = [\![\texttt{WHERE}\ e]\!]\big([\![\texttt{WITH}\ \bar{\pi}]\!]_G(T)\big)$

- $[\![\texttt{OPTIONAL MATCH}\ \bar{\pi}\ \texttt{WHERE}\ e]\!]_G(T)$

$$= \biguplus_{u \in T} \begin{cases} T_u & \text{if } T_u \neq \emptyset \\ \{\text{cmpl}(u)\} & \text{otherwise} \end{cases}$$

where $T_u = [\![\texttt{MATCH}\ \bar{\pi}\ \texttt{WHERE}\ e]\!]_G(\{u\})$
(and $\text{cmpl}(u, \bar{\pi})$ is the completion of u w.r.t. variables of $\bar{\pi}$)

$$\langle\text{with}\rangle ::=$$

| WITH ⟨expr⟩ [AS ⟨name⟩]? , · · · , ⟨expr⟩ [AS ⟨name⟩]?

| WITH * , ⟨expr⟩ [AS ⟨name⟩]? , · · · , ⟨expr⟩ [AS ⟨name⟩]?

- Each expression/name pair will be one column in the output table.
- If the name is missing, a default name is provided (implementation dependent).
- * means "every column previously in the table".

The WITH clause (2)

Example

a	b	c
0	1	2
1	1	1
2	1	0

```
WITH  *,  a+b,  a<c AS order
```

a	b	c	‘a+b‘	order
0	1	2	1	true
1	1	1	2	false
2	1	0	3	false

Semantics

- $\left[\!\left[\texttt{WITH}\ * \right]\!\right]_G (T) = T$

- $\left[\!\left[\texttt{WITH}\ e_1\ \texttt{AS}\ a_1, \ldots, e_m\ \texttt{AS}\ a_m \right]\!\right]_G (T)$
$$= \biguplus_{u \in T} \left\{ (a_1 : \left[\!\left[e_1 \right]\!\right]_{G,u}, \ldots, a_m : \left[\!\left[e_m \right]\!\right]_{G,u}) \right\}$$

- $\left[\!\left[\texttt{WITH}\ e_1\ [\texttt{AS}\ a_1]?, \ldots, e_m\ [\texttt{AS}\ a_m]? \right]\!\right]_G (T)$
$$= \left[\!\left[\texttt{WITH}\ e_1\ \texttt{AS}\ a'_1, \ldots, e_m\ \texttt{AS}\ a'_m \right]\!\right]_G (T)$$
where a'_i equals a_i if it is given, or arbitrary otherwise.

- $\left[\!\left[\texttt{WITH}\ *, \ldots \right]\!\right]_G (T) = \left[\!\left[\texttt{WITH}\ b_1\ \texttt{AS}\ b_1, \ldots, b_q\ \texttt{AS}\ b_q, \ldots \right]\!\right]_G (T)$
where $b_1, \ldots, b_q$ are the column names of T

The UNWIND clause (1)

Syntax

$\langle$unwind$\rangle$ ::= `UNWIND` $\langle$expr$\rangle$ `AS` $\langle$name$\rangle$

- Given expression is supposed to be evaluated to lists
- Each line of the input table is expanded as multiple lines in the output, one for each element of the list.

The UNWIND clause (2)

Example

n	*list*
0	["Hello", "World"]
1	["singleton"]
2	"not_a_list"

```
UNWIND list AS x
```

n	*list*	x
0	["Hello","World"]	"Hello"
0	["Hello","World"]	"World"
1	["singleton"]	"singleton"
2	"not_a_list"	"not_a_list"

$$\left[\!\left[\texttt{UNWIND}\ e\ \texttt{AS}\ a \right]\!\right]_G (T) \quad = \quad \biguplus_{u \in T} \biguplus_{v \in E_u} \{(u, a : v)\}\ ,$$

$$\text{where} \quad E_u = \begin{cases} \biguplus_{0 \le i < m} \{v_i\} & \text{if } \left[\!\left[e \right]\!\right]_{G,u} = \text{list}(v_1, \ldots, v_m) \\ \{\} & \text{if } \left[\!\left[e \right]\!\right]_{G,u} = \text{list}() \\ \left\{ \left[\!\left[e \right]\!\right]_{G,u} \right\} & \text{otherwise} \end{cases}$$

The `RETURN` clause

`RETURN` and `WITH` clauses have the exact same semantics.

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RETURN clause is the “end-marker” of a clause sequence:

- it is mandatory;
- it cannot appear earlier.

UNION and UNION ALL

Syntax

$$\langle\text{query}\rangle ::= \langle\text{query}\rangle\ \texttt{UNION}\ [\texttt{ALL}]?\ \langle\text{query}\rangle$$

- The left and right queries are assumed to have the same column-tables.
- `UNION` is the set-union of tables.
- `UNION ALL` is the bag-union of tables.

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Semantics

- $[\![Q_1 \texttt{ UNION ALL } Q_2]\!]_G(T) = [\![Q_1]\!]_G(T) \cup [\![Q_2]\!]_G(T)$
- $[\![Q_1 \texttt{ UNION } Q_2]\!]_G(T) = \text{distinct}\left([\![Q_1]\!]_G(T) \cup [\![Q_2]\!]_G(T) \right)$

Aggregation (1)

Principle

- Group lines according to some criteria
- Some column of the output table contains aggregated values.

Aggregation functions

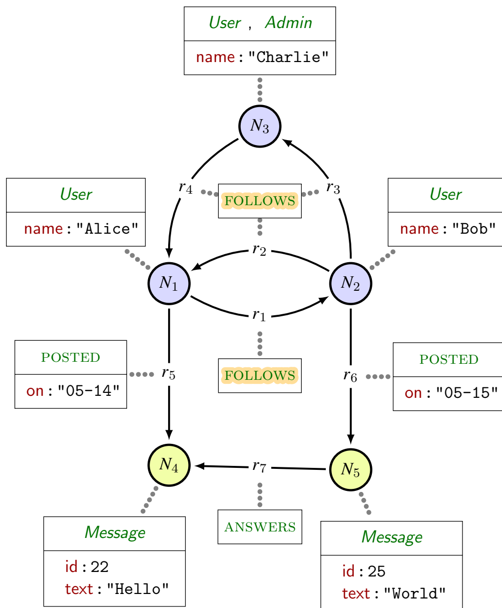
$\mathcal{G}$: a set of functions that map value bags to single values

E.g., `count`, `max` $\in \mathcal{G}$

- In `WITH` clauses, we allow *aggregate expression*:

  $(\mathit{WITH}\ \ \langle\text{aggexpr}\rangle\ [\texttt{AS}\ \ \langle\text{name}\rangle]?)$
- Aggregate expressions are of the form: $g(\langle\text{expr}\rangle)$ where $g \in \mathcal{G}$

Aggregation (2) — Example



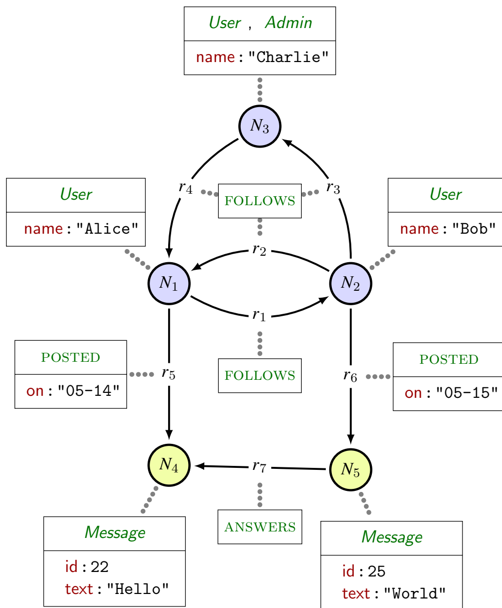
Query:

```
MATCH (a)-[r:FOLLOWS]->()
WITH a.name AS b, count(r) AS c
```

Aggregate

- over expression `a.name`
- and count them

b	c
"Alice"	1
"Bob"	1
"Charlie"	2



Query:

```
MATCH (a)-[r:FOLLOWS]->()
WITH a.name AS b, count(r) AS c
WITH max(c) AS d
```

d
2

Example: WITH e_1 AS a_1, e_2 AS a_2, $max(e_3)$ AS a_3

e_1, e_2 and e_3 are ⟨expr⟩
$max(e_3)$ is an ⟨aggexpr⟩

We group the lines according to the expressions e_1, e_2:

- We partition the input table as subtables $T_1, \ldots, T_k$
- For each $u, v \in T_i$, $\llbracket e_j \rrbracket_{G,u} = \llbracket e_j \rrbracket_{G,v}$ (for $j \in \{1, 2\}$)

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Aggregation (4)

Semantics on an example

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- For $i < k$, let $V_i = \left\{ \llbracket e_3 \rrbracket_{G,v} \middle| v \in T_i \right\}$ and let $u \in T_i$.

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The output table has 3 columns and k lines; the i-th line is:

- For $i < k$, let $V_i = \left\{ [\![e_3]\!]_{G,v} \middle| v \in T_i \right\}$ and let $u \in T_i$.
- The i-th line is $\left(a_1 : [\![e_1]\!]_{G,u},\ a_2 : [\![e_2]\!]_{G,u},\ a_3 : max(V_i)\right)$

Line order (1)

Principle

- `ORDER BY` clause: provides an order
- `LIMIT` subclause: keeps only the first lines of the table
- `SKIP` subclause: removes the first lines of the table

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- In tables, columns and lines are not ordered

- Semantics of queries
- Semantics of clauses
- Semantics of subclauses

} are functions from tables to tables

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} are functions from tables to tables

From now on

- `ORDER BY` and its subclause propagate an order

Line order (2) — Semantics through example

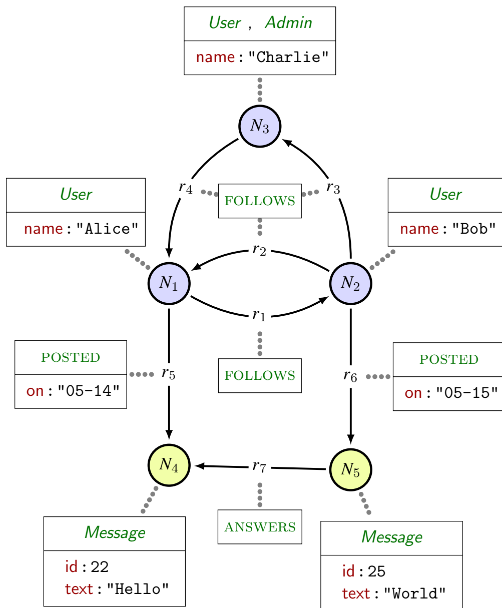
("Alice" < "Bob" < "Charlie")

Query:

```
MATCH (a:User)
ORDER BY a.name
  SKIP 1
  LIMIT 1
RETURN a.name AS b
```

b
"Bob"

Line order (3) — Semantics through example



(`false` < `true`)

Query:

```
MATCH (a:User)
ORDER BY a:Admin
  SKIP 1
  LIMIT 1
RETURN a.name AS b
```

In this case:

- Non-determinism
- 1 column and 1 line
- Cell contains either `"Alice"` or `"Bob"`

Oddities and inconsistencies


```
WHERE []
```

```
WHERE [1] = 1
```

```
MATCH (a)-[r*]->(b)-[r*]->(c)
```

```
MATCH (a)
OPTIONAL MATCH p=(a),(b:LabelAbsentFromTheGraph)
```

```
UNWIND [[1,2],3] AS a
UNWIND a AS b
```

```
UNWIND [1,[2,3]] AS a
UNWIND a AS b
```

```
WITH a + b + count(c) AS x
```

```
WITH a + count(c) + b AS x
```

```
MATCH (n)-[r]-() AS a
DELETE n
```

```
MATCH (n)-[r]-() AS a
DELETE n
  [Arbitrary clauses]
DELETE r
```