Formal semantics of the query-language Cypher

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joint work with

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Outline

Representing data as graphs

Most of database use the relational model

- Relational algebra in theory
- The language SQL in practice

Representing data as graphs

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- The language SQL in practice

Some data have intrinsically the structure of graphs:

- Semantic web
- Social Networks
- Bioinformatic networks

Native representation of data as graphs allows:

- Efficient algorithms on graphs
- Pattern matching
- Optimisations

Graph databases are becoming more widespread

Data model

Property graphs, RDF

Query languages

Cypher, Gremlin, PGQL, SparQL

Engines

JanusGraph, Jena, Neo4j, Virtuoso

Domain

Fraud detection, Investigative journalism

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Cypher

- Language for querying and updating *property graphs*

- Invented by Neo Technology
- Originally, part of engine Neo4j $\rightarrow$ commercial success
- Now, in multiple datagraph engines (e.g., SAP HANA Graph, Redis Graph, Agens Graph)

Cypher

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The openCypher project

- Since 2015
- Seeks to make of Cypher a standard (SQL for data graphs?)
  - Community-led evolution
  - Complete specification

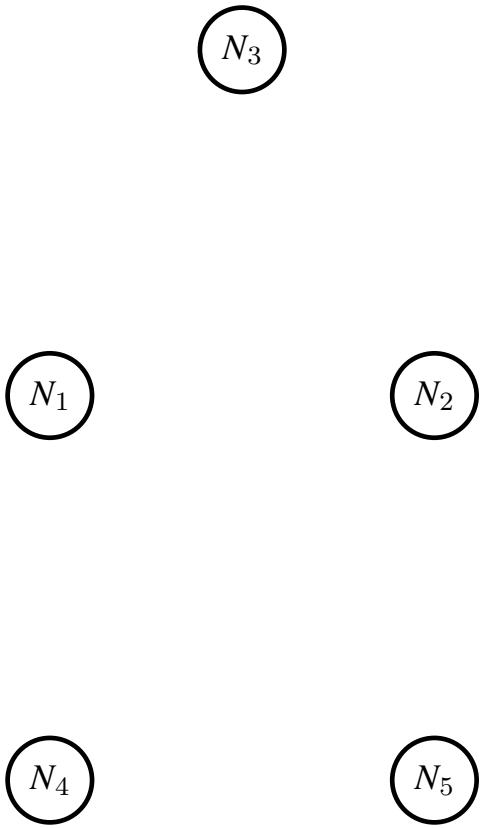
Our goal

Full denotational semantics for the language Cypher.

- Industrial partnership Neo Technology/University of Edinburgh
- Reverse engineering and formalisation from Neo4j
- What I present here: Semantics of the "core fragment" of Cypher

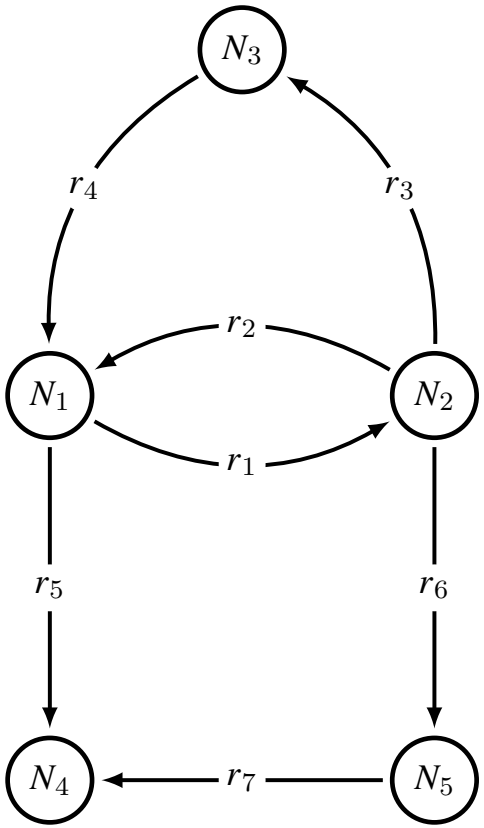
Outline

Property graph



- Nodes : $N_1, N_2, \cdots, N_5$

Property graph



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- Relationships :

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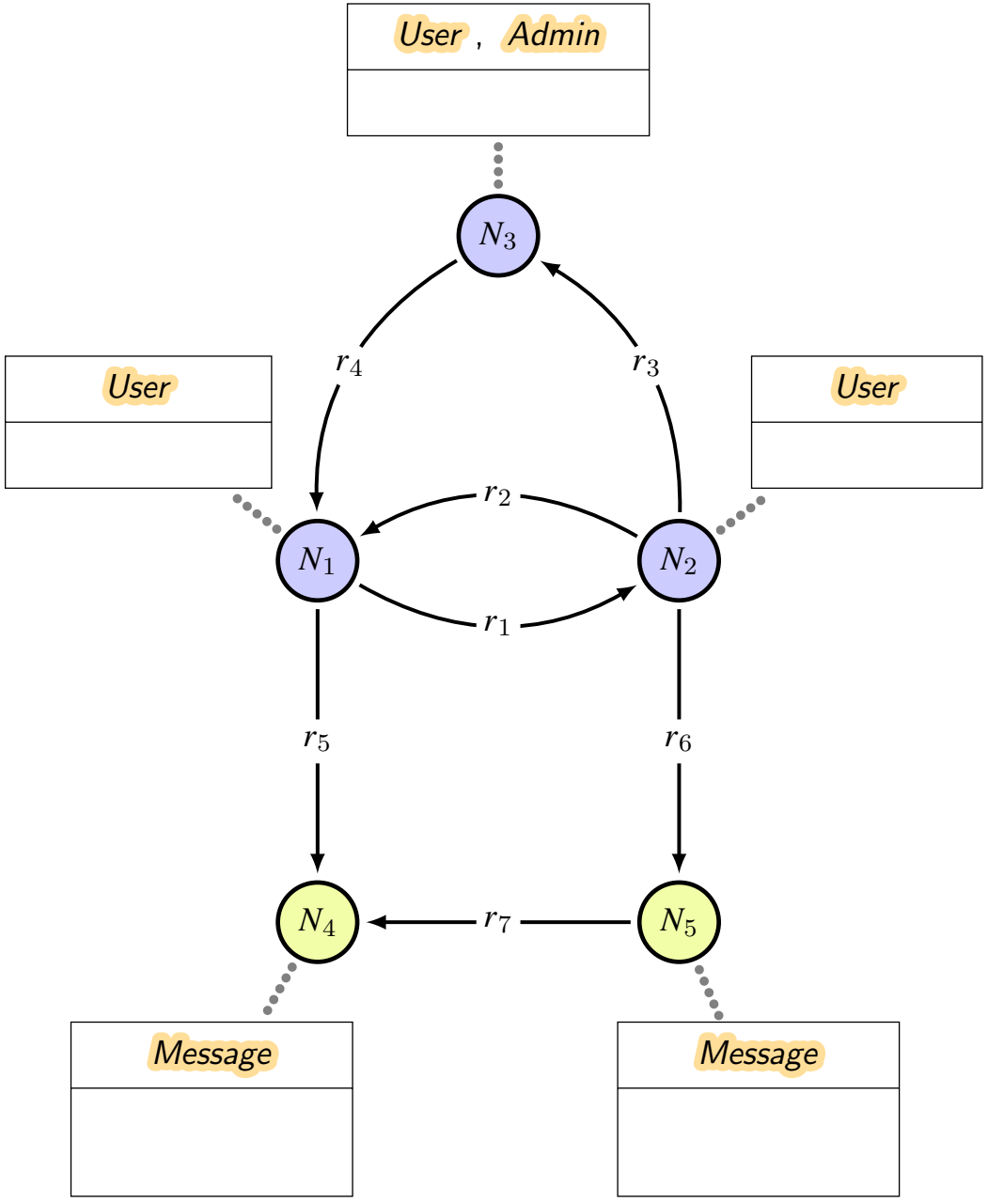
Property graph



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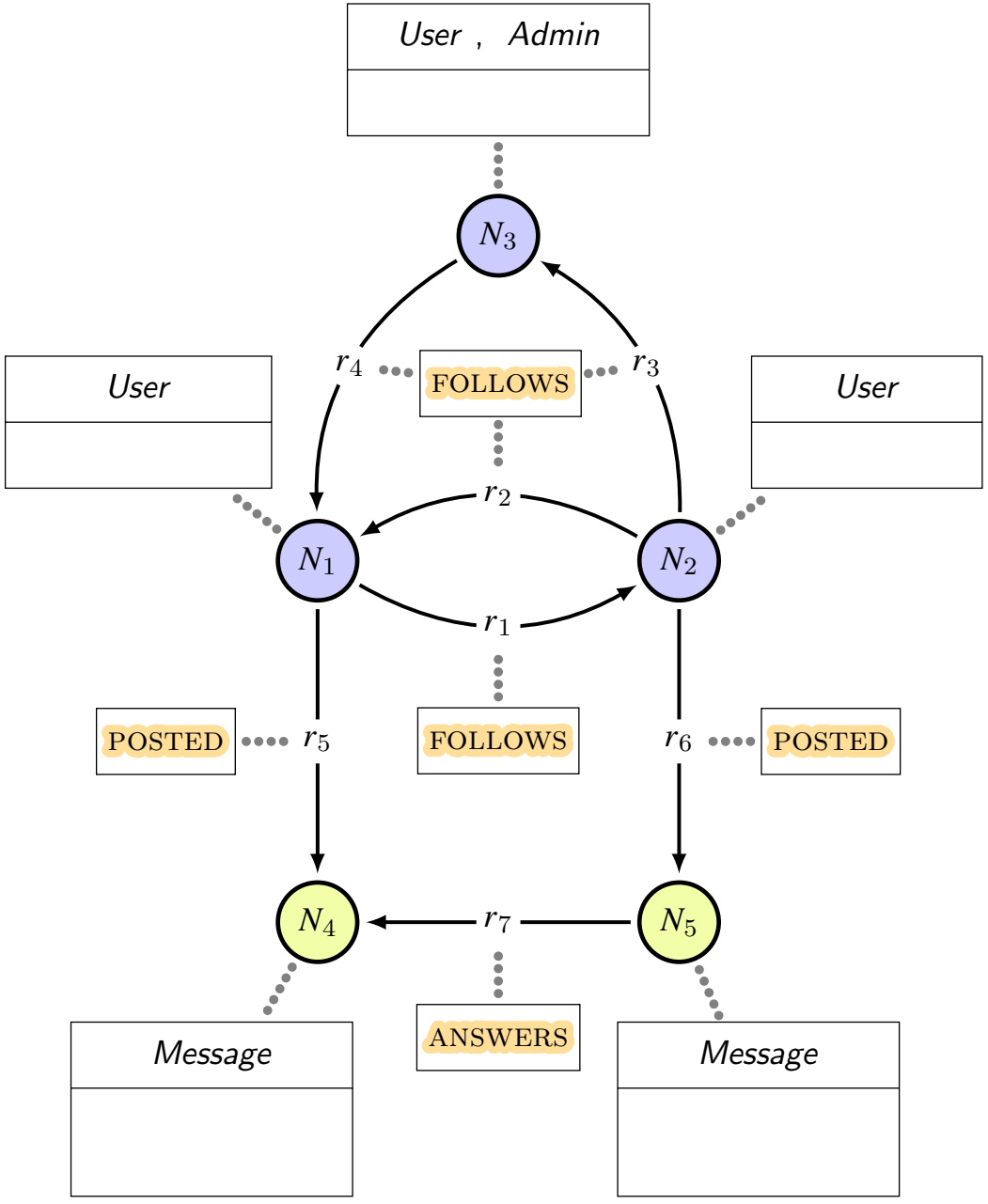
- Labels (de nœuds) :
  - *User*
  - *Message*

Property graph



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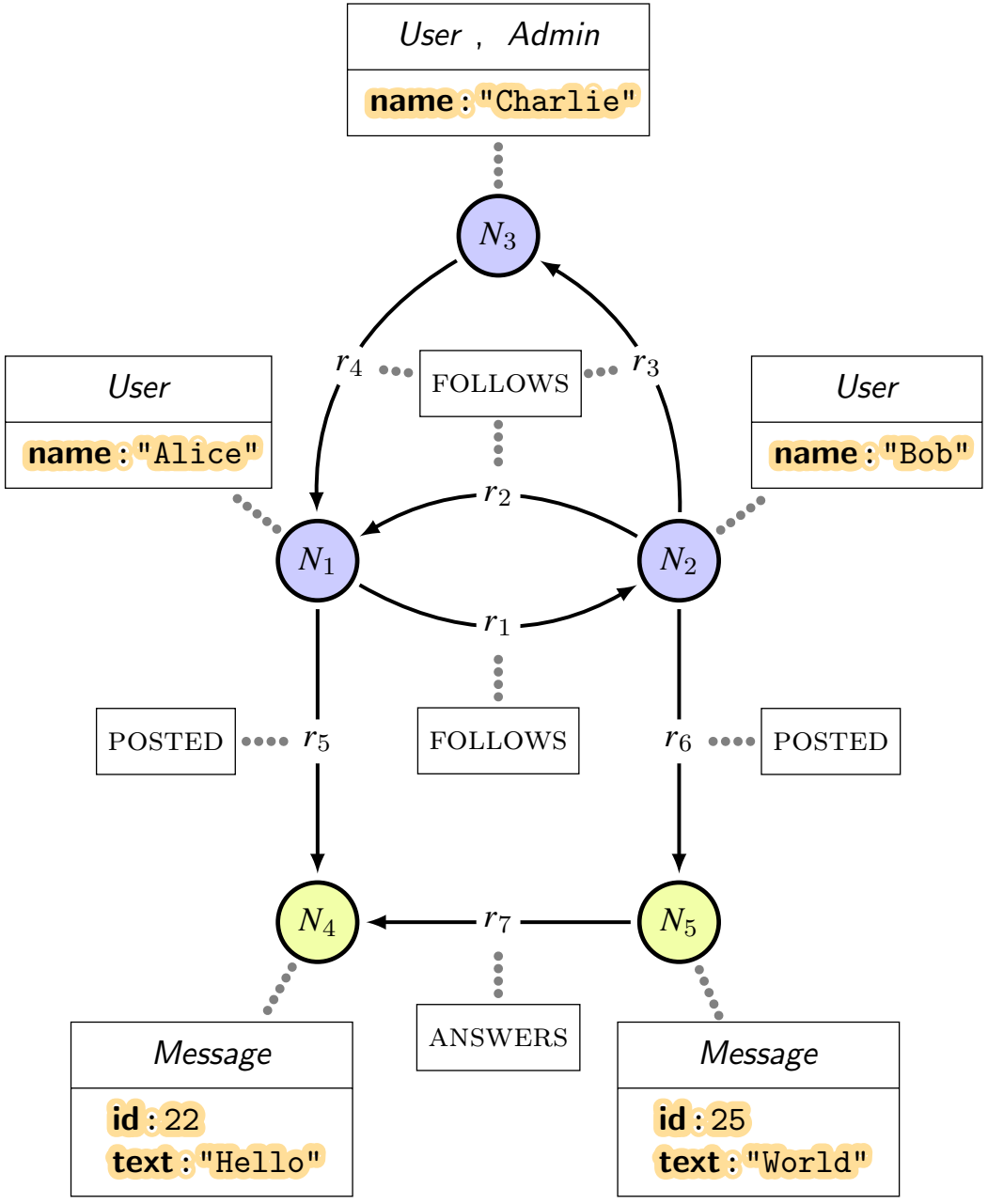

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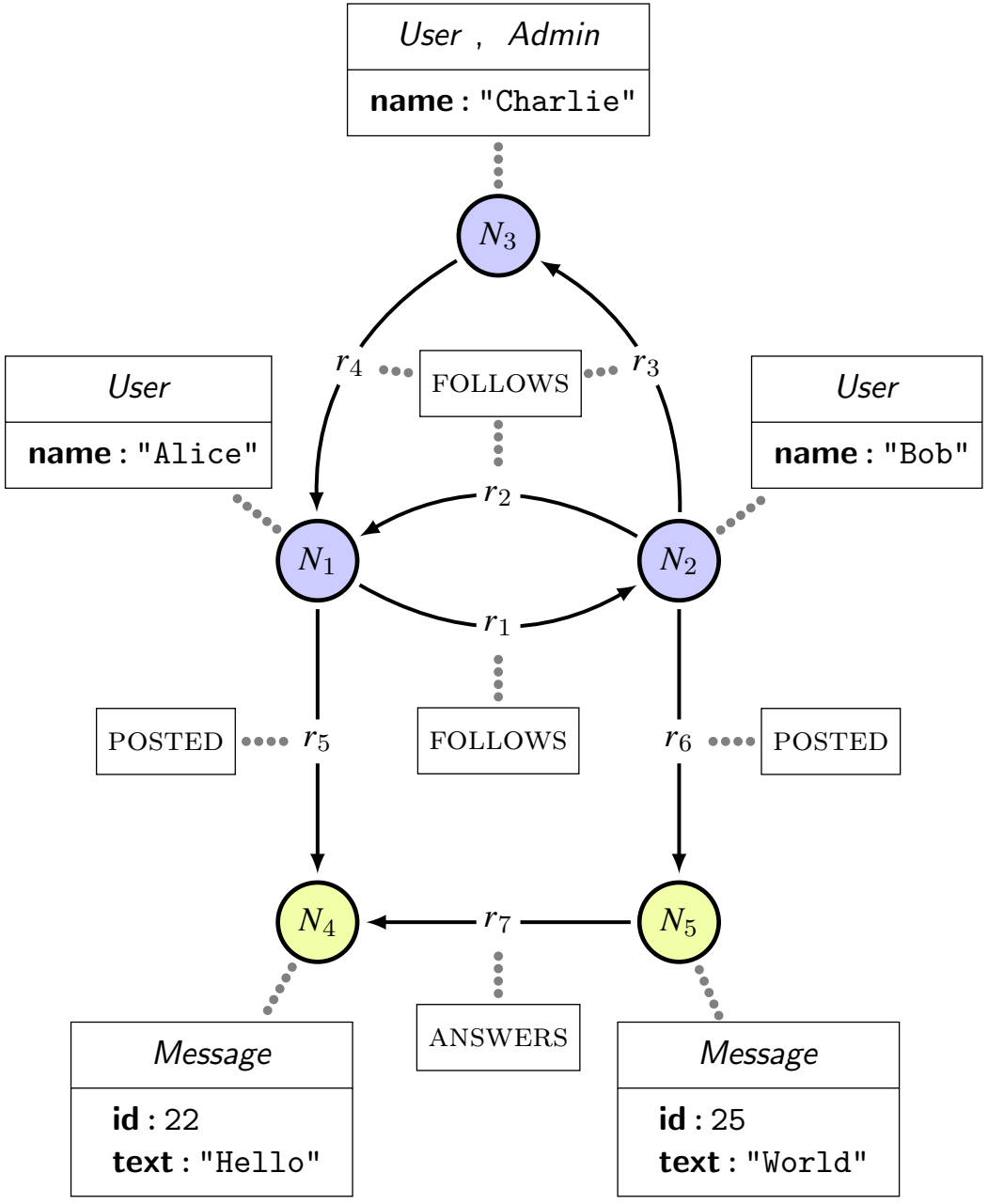
- Properties (*i.e.* Key/Value pairs) :
  - **name** : "Alice"
  - **id** : 22
  - **text** : "Hello"

Property graph



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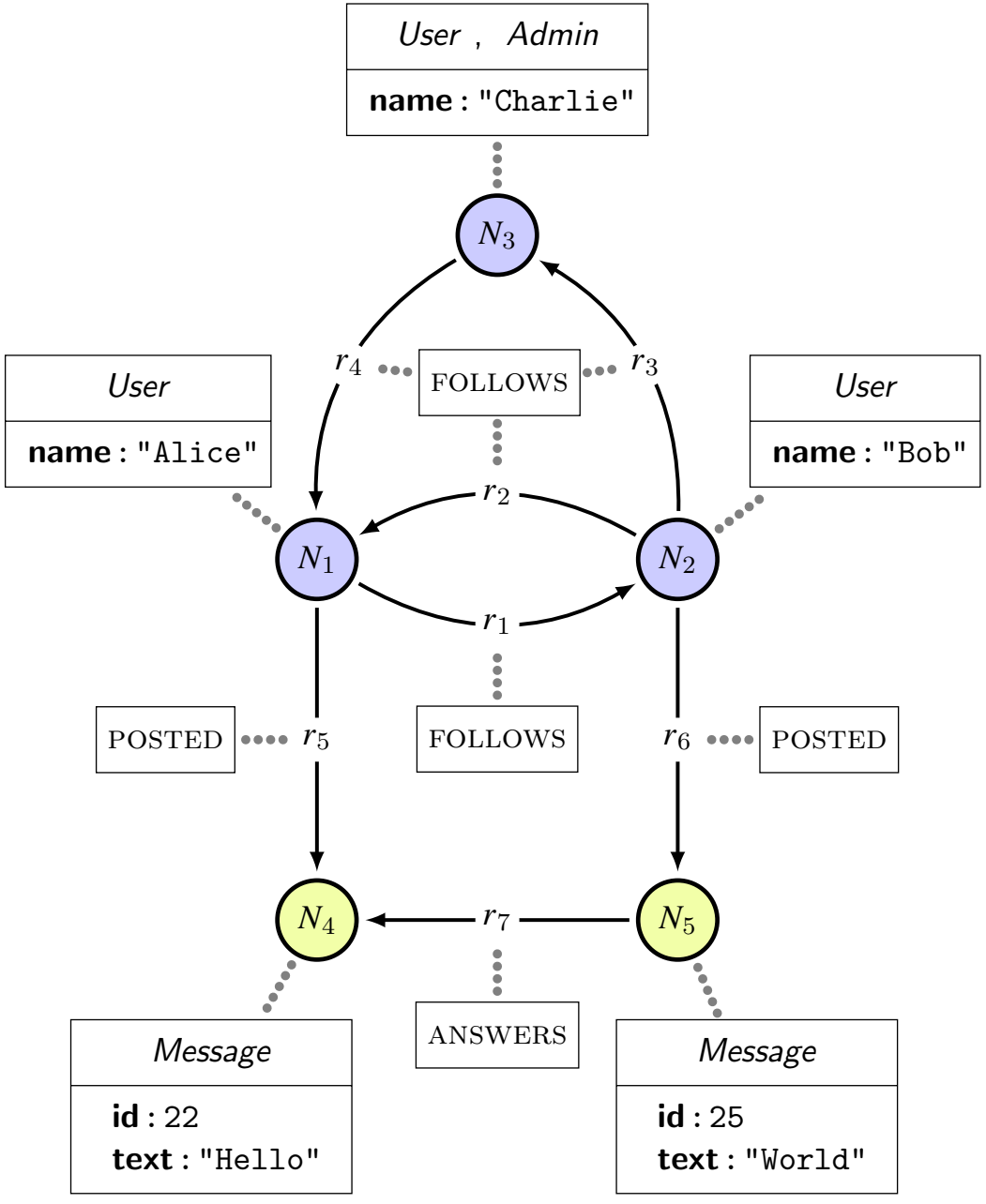
Computing a Cypher query



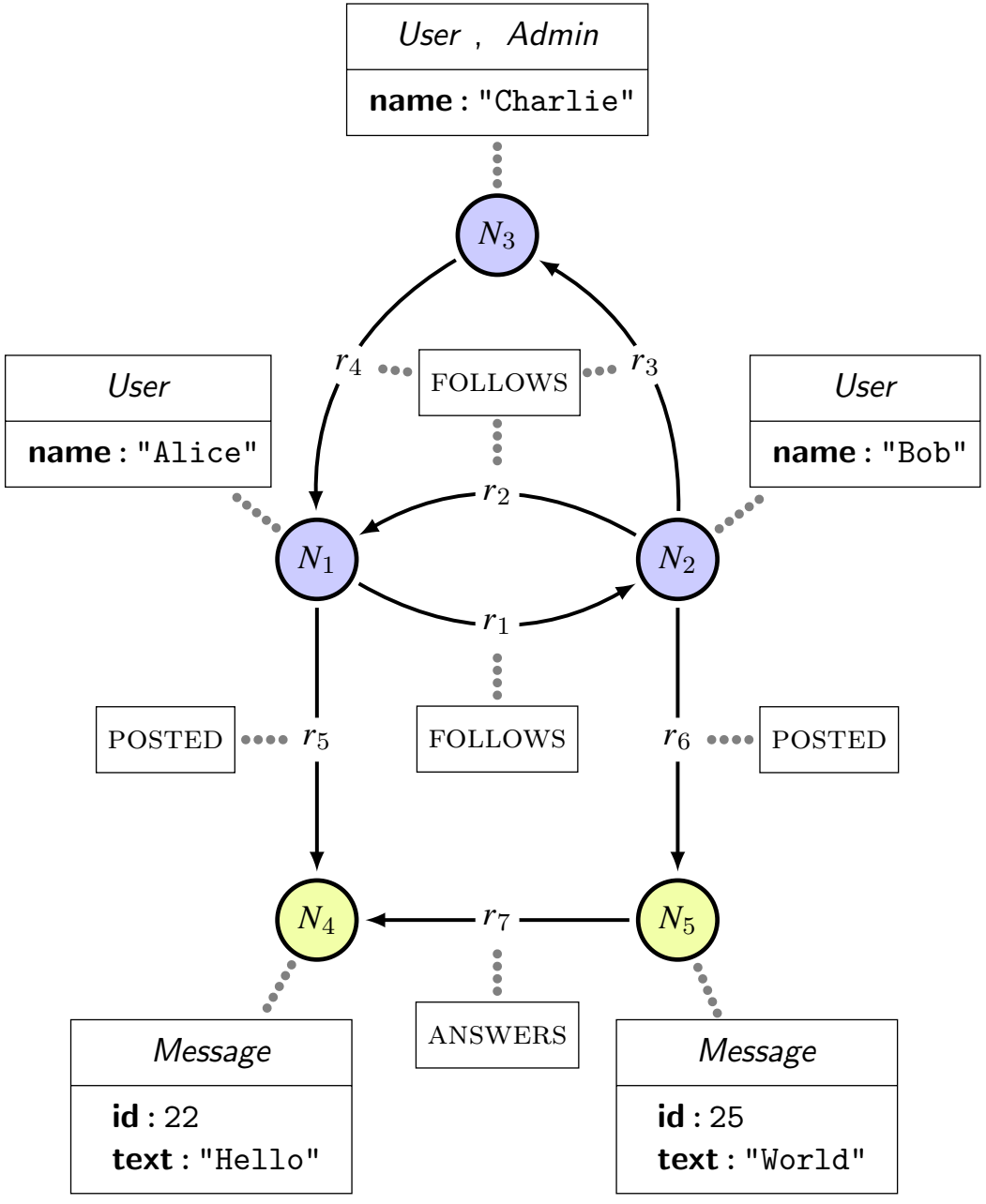
Example of a Cypher query :

```
MATCH (u1)-[p1:POSTE]->(m1)
WITH u1, p1, m1.text AS t1
WHERE t1 = "Hello"
```

A Cypher statement :

- is a sequence of *clauses*

Computing a Cypher query



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```
MATCH (u1)-[p1:POSTE]->(m1)
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WHERE t1 = "Hello"
```
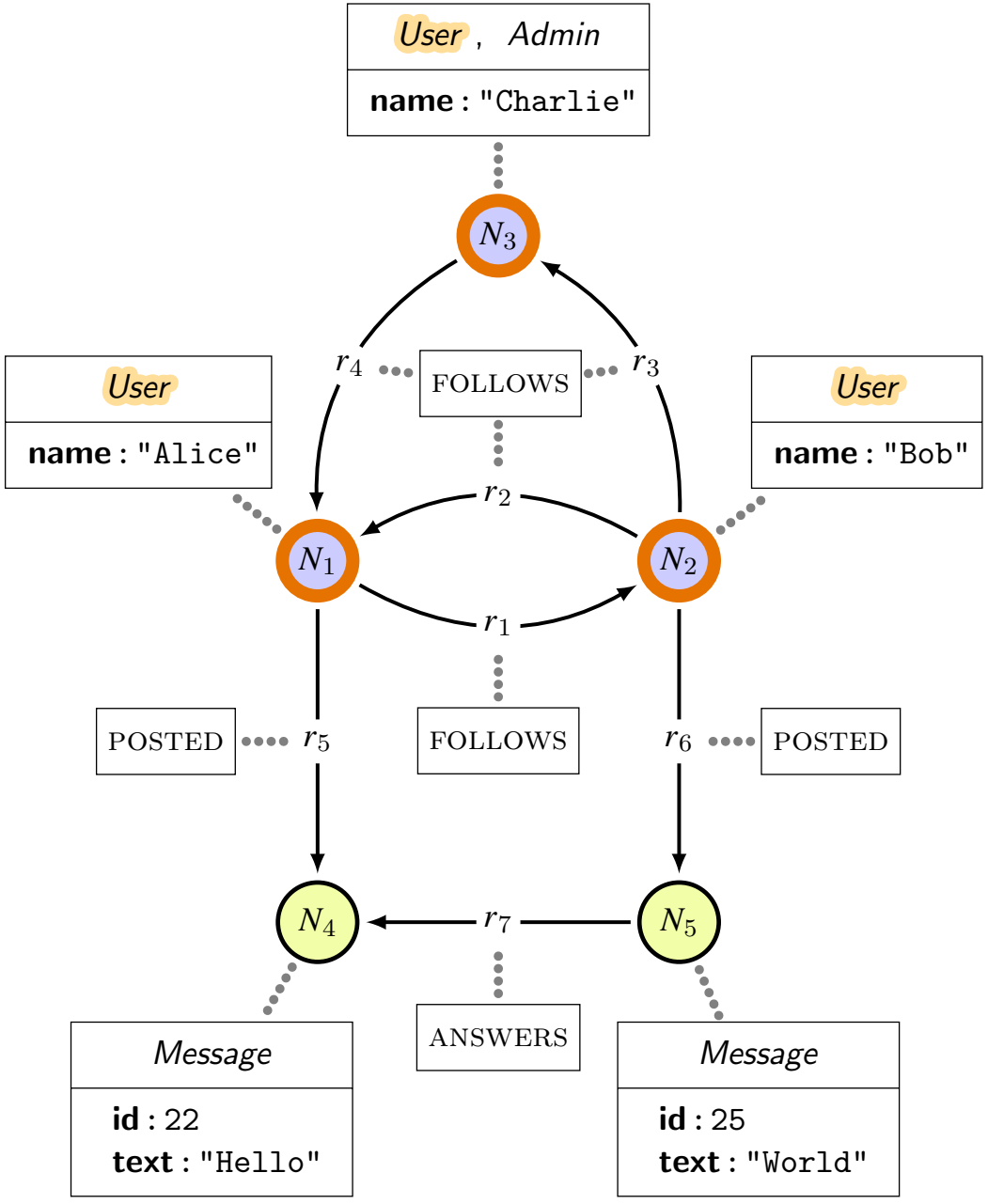

A Cypher statement :

- is a sequence of *clauses*
- queries a graph
- returns a table

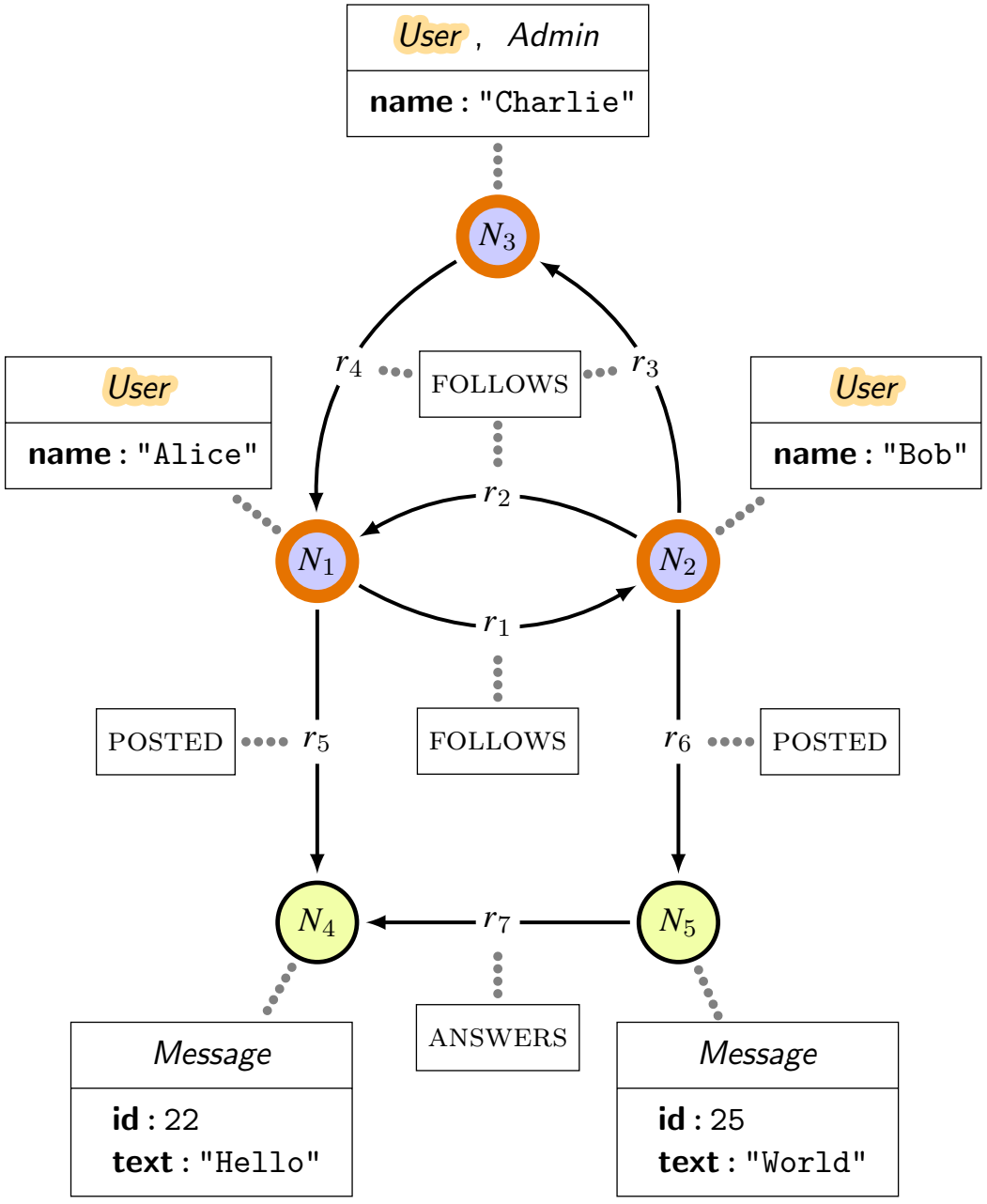
Matching nodes (1)



Query :

```
MATCH (u1:User)
```

Matching nodes (1)



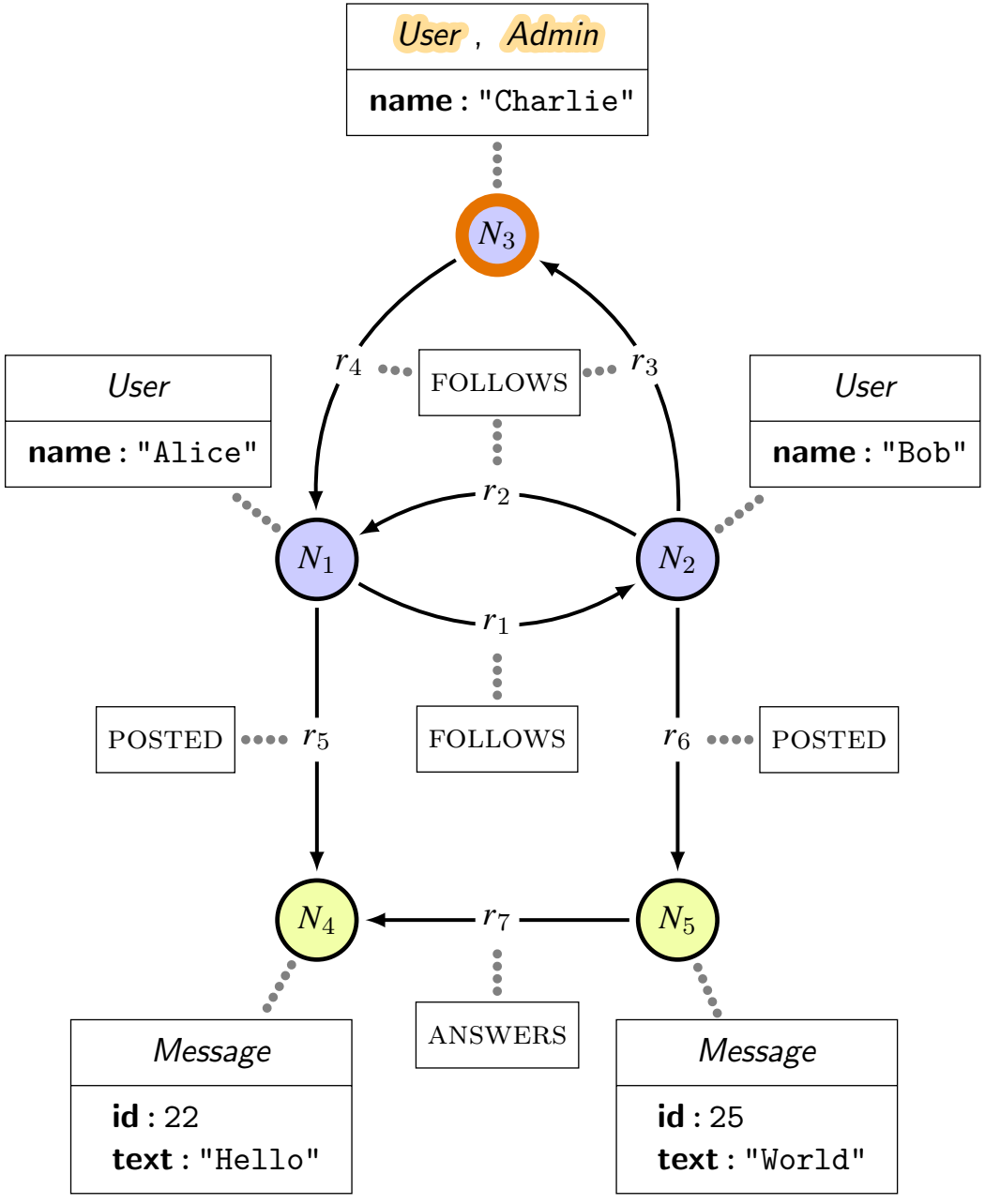
Query :

```
MATCH (u1:User)
```

Result :

u1
N_1
N_2
N_3

Matching nodes (2)



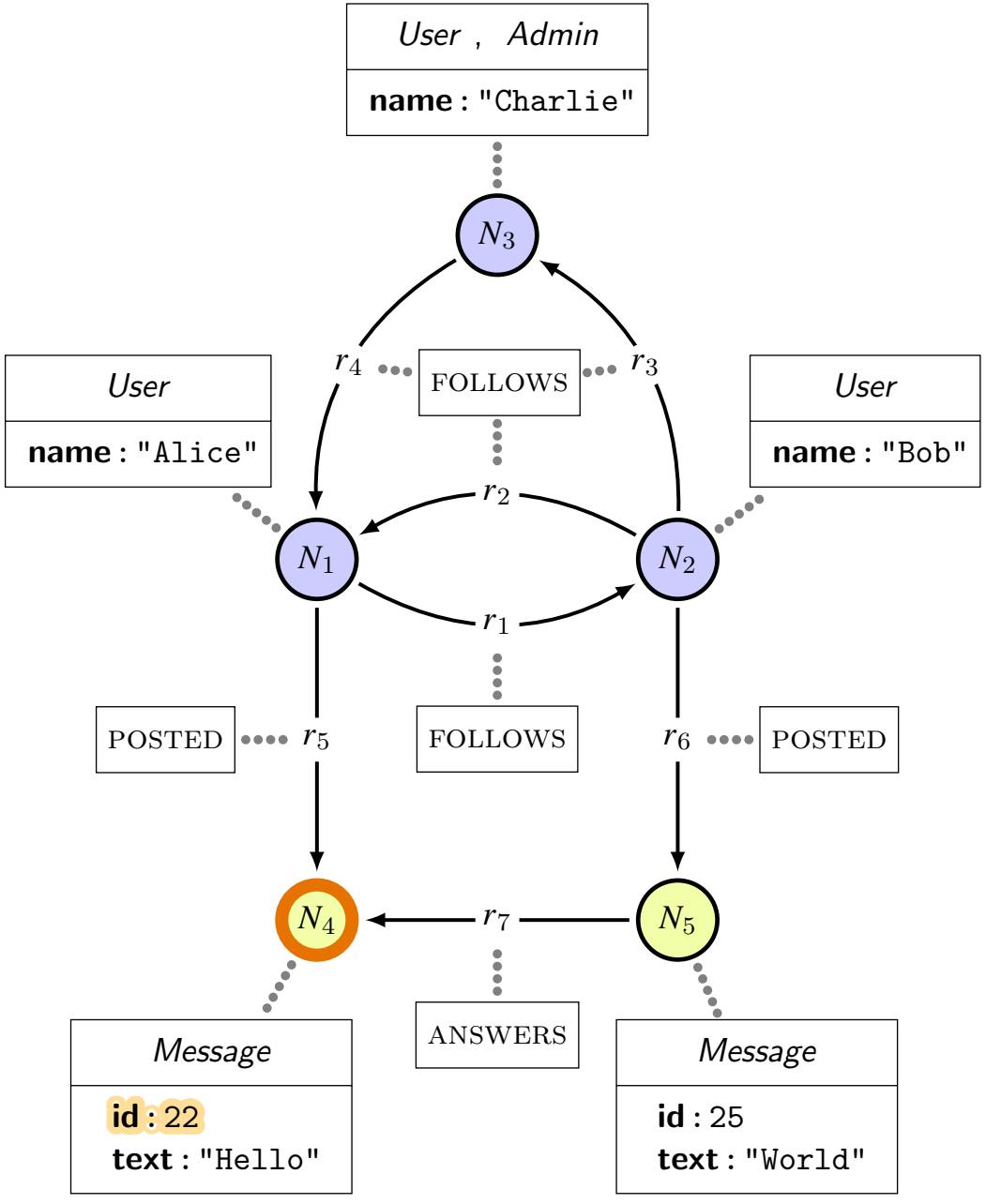
Query :

```
MATCH (u1:User:Admin)
```

Result :

u1
N_3

Matching nodes (3)



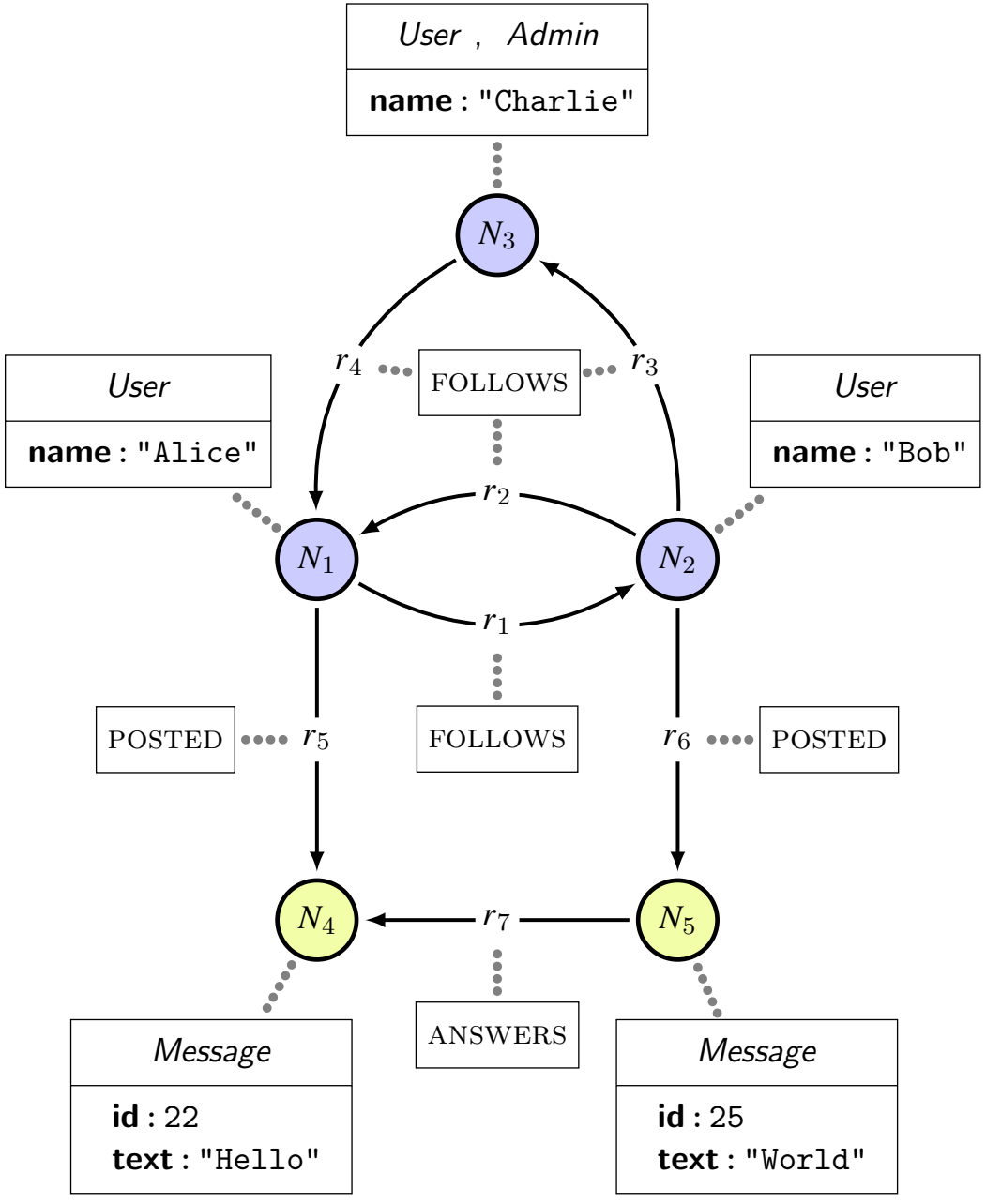
Query :

```
MATCH (u1{id:22})
```

Result :

u1
N_4

Matching relationships (1)



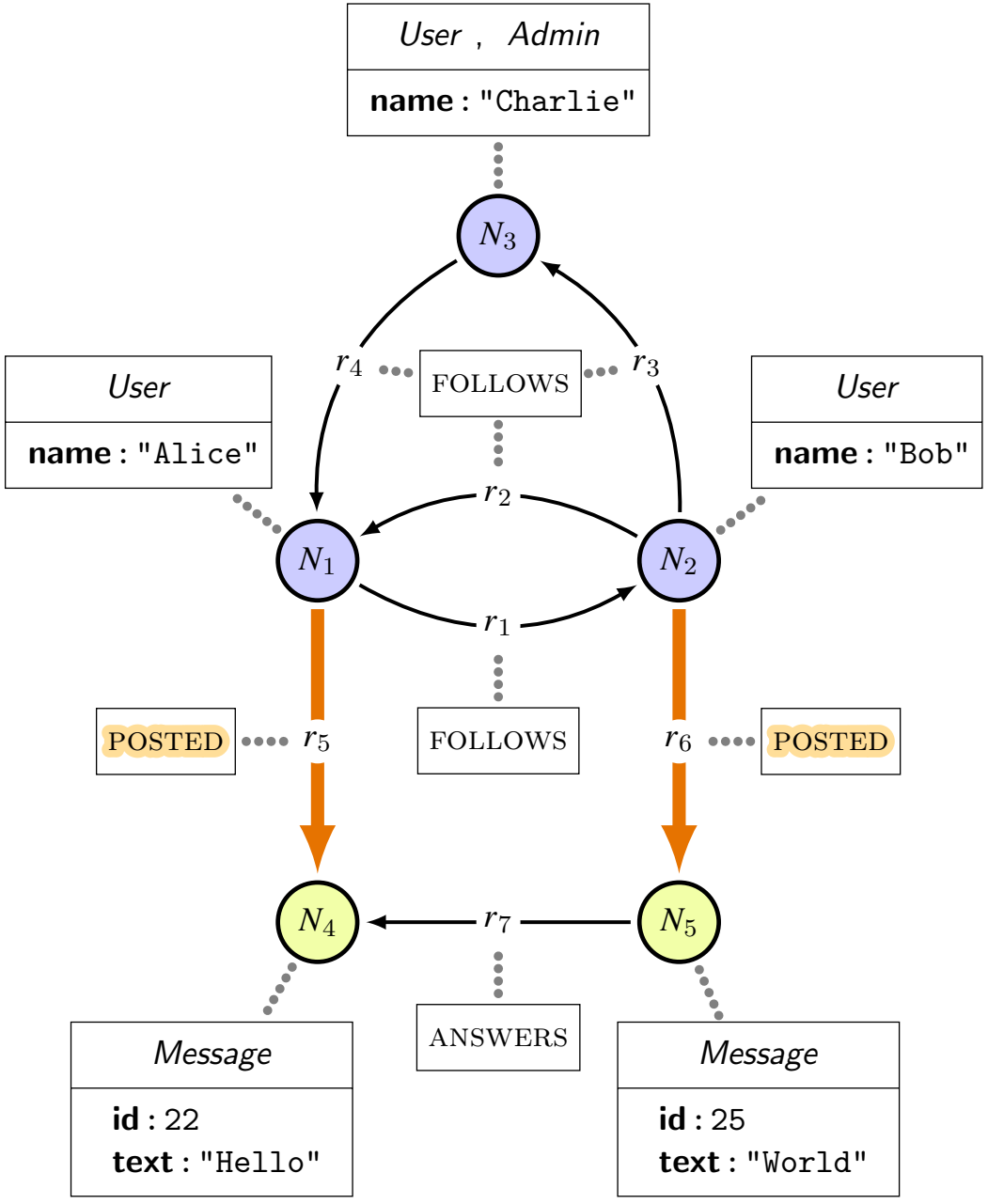
Query :

```
MATCH ()-[p1]->()
```

Result :

p1
r_1
r_2
r_3
r_4
r_5
r_6
r_7

Matching relationships (2)



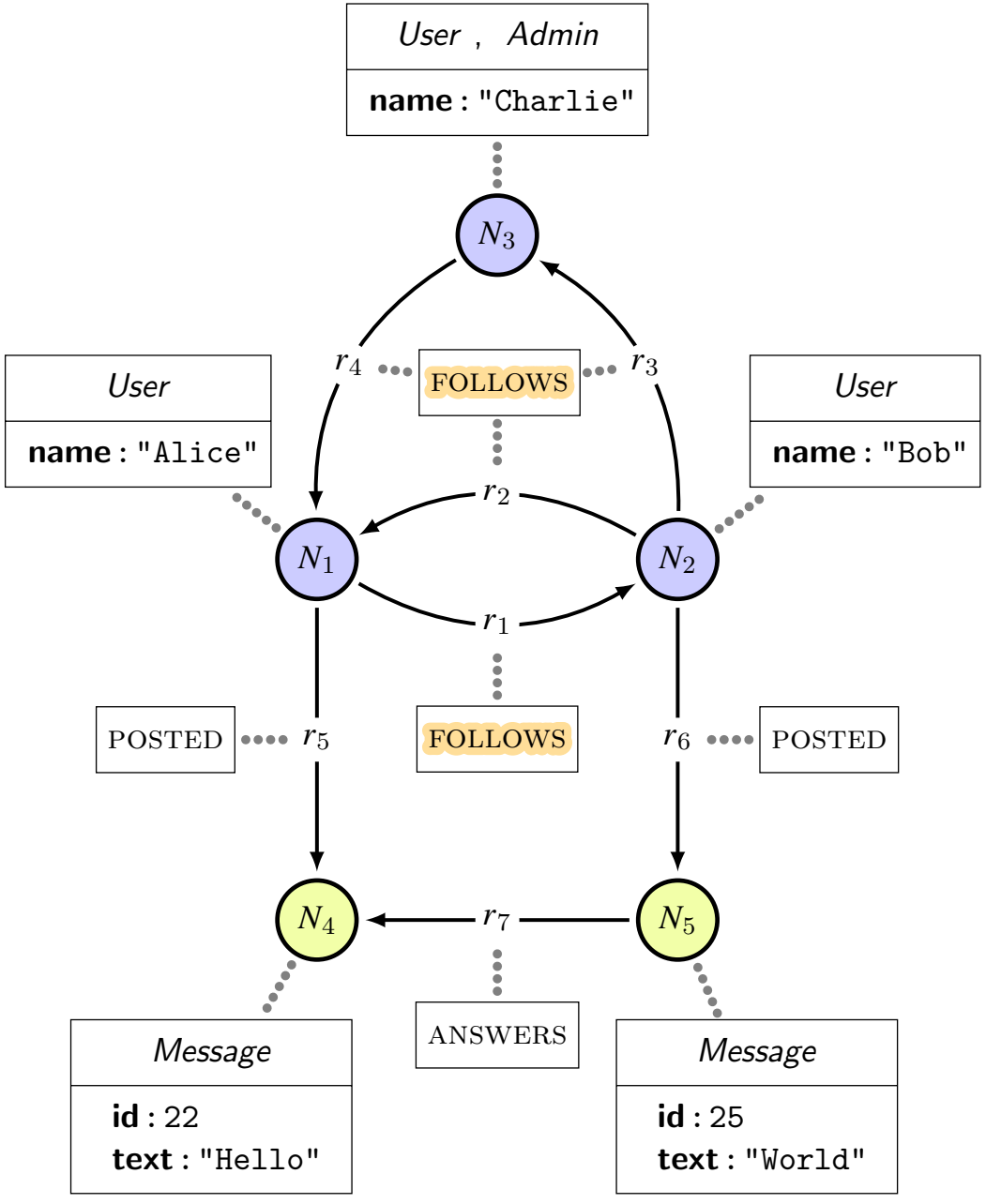
Query :

```
MATCH (u1)-[p1:POSTED]->(m1)
```

Result :

u1	p1	m1
N_1	r_5	N_4
N_2	r_6	N_5

Matching relationships (3)



Query :

```
MATCH (u1)-[:FOLLOWS]->()
```

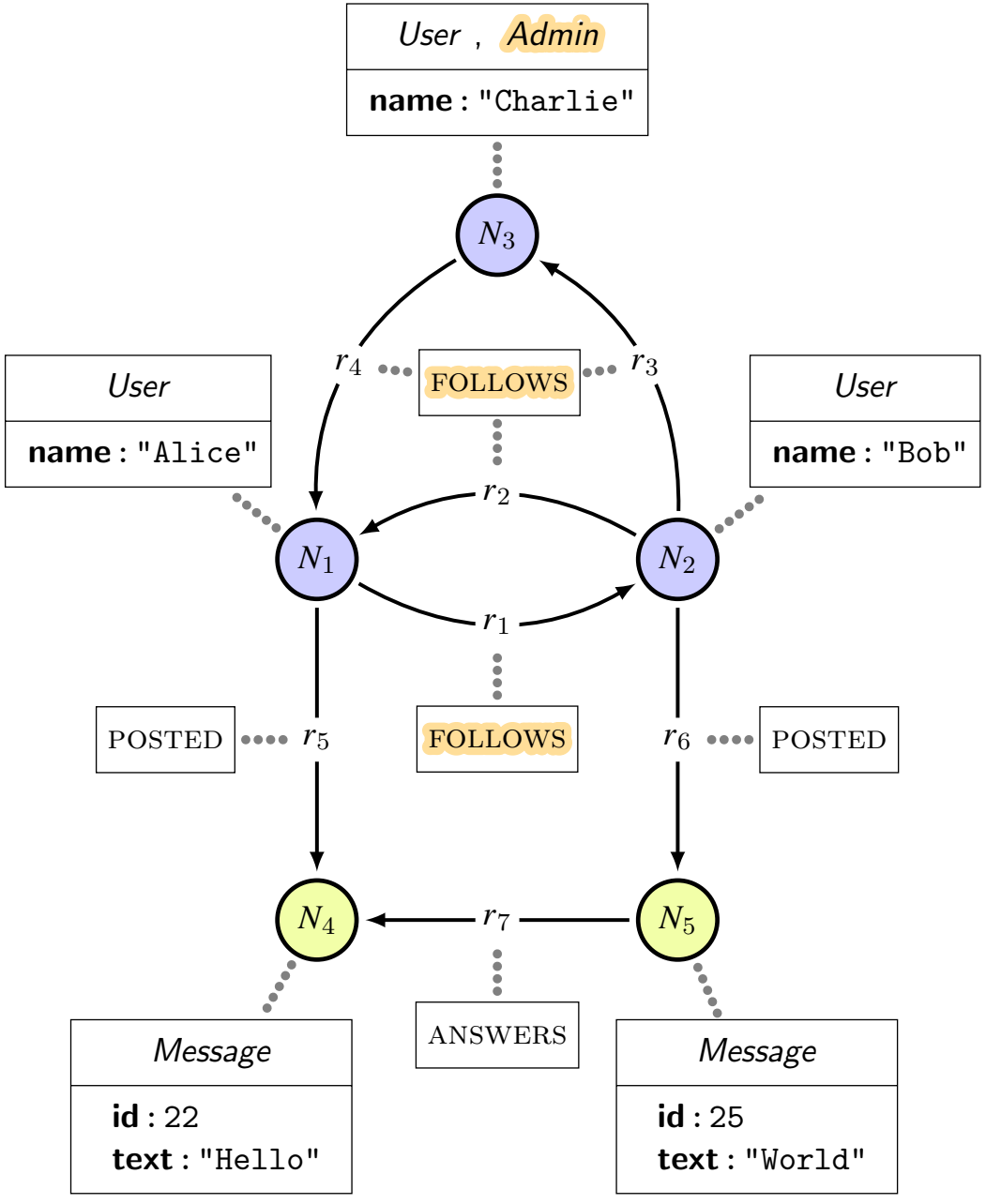
Result :

u1
N_1
N_2
N_2
N_3

Bag semantics :

N_2 has two outgoing FOLLOWS rel. $\Rightarrow$ two lines N_2

Matching relationships (4)



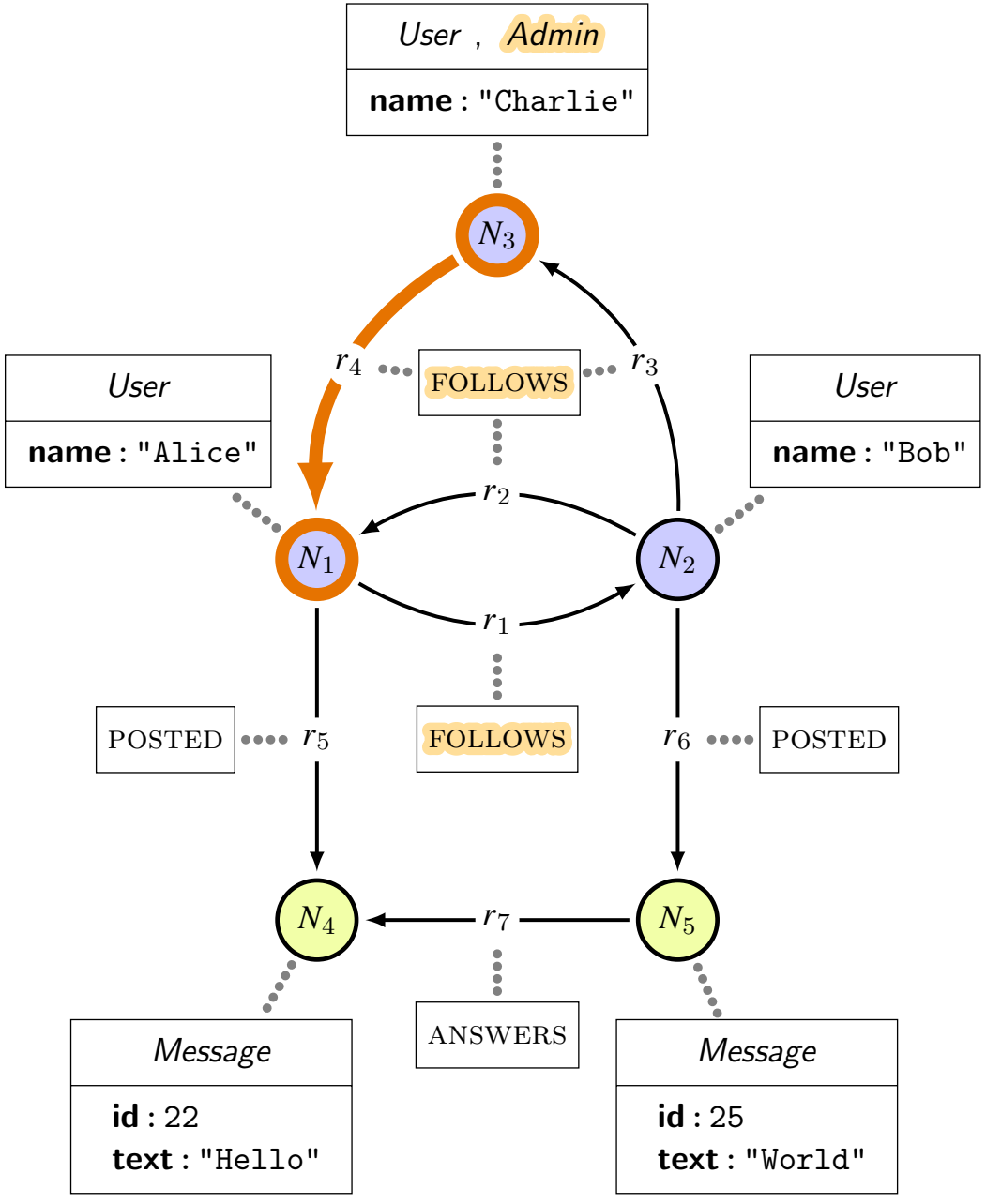
Query :

```
MATCH (u1:Admin)
          -[l1:FOLLOWS*]->(m1)
```

Result :

u1	l1	m1
N_3	$[r_4]$	N_1
N_3	$[r_4, r_1]$	N_2
N_3	$[r_4, r_1, r_2]$	N_1
N_3	$[r_4, r_1, r_3]$	N_3

Matching relationships (4)



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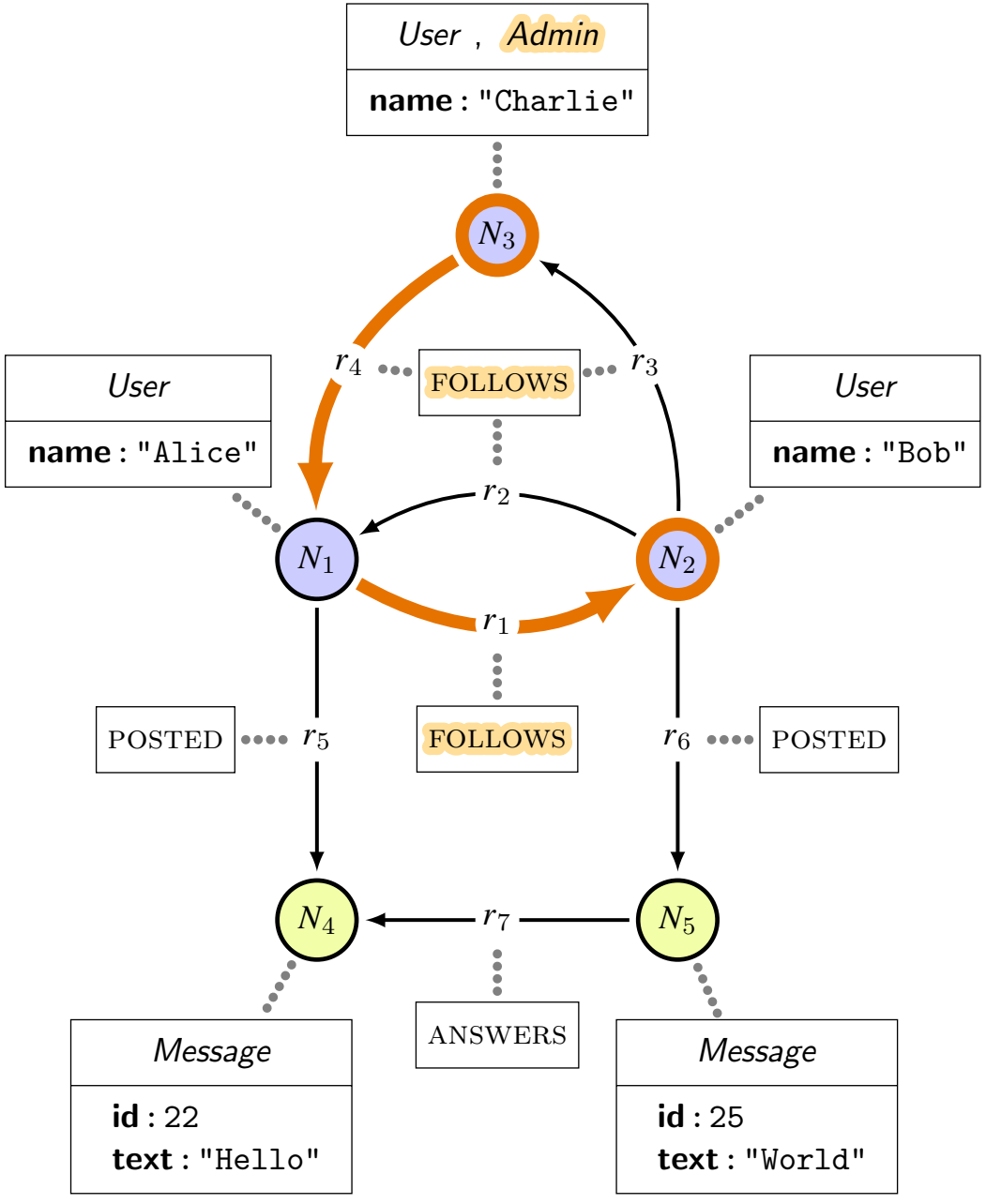

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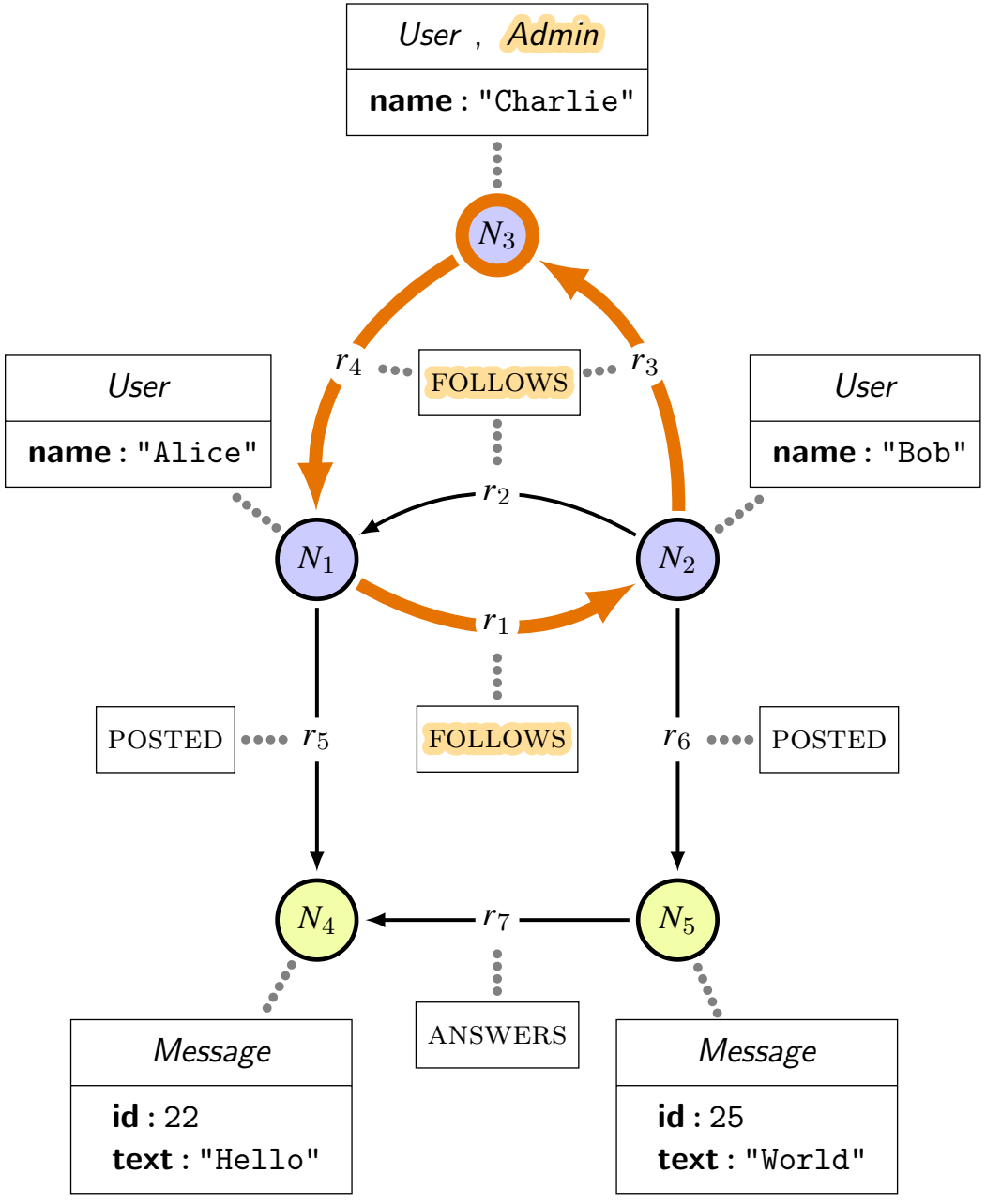

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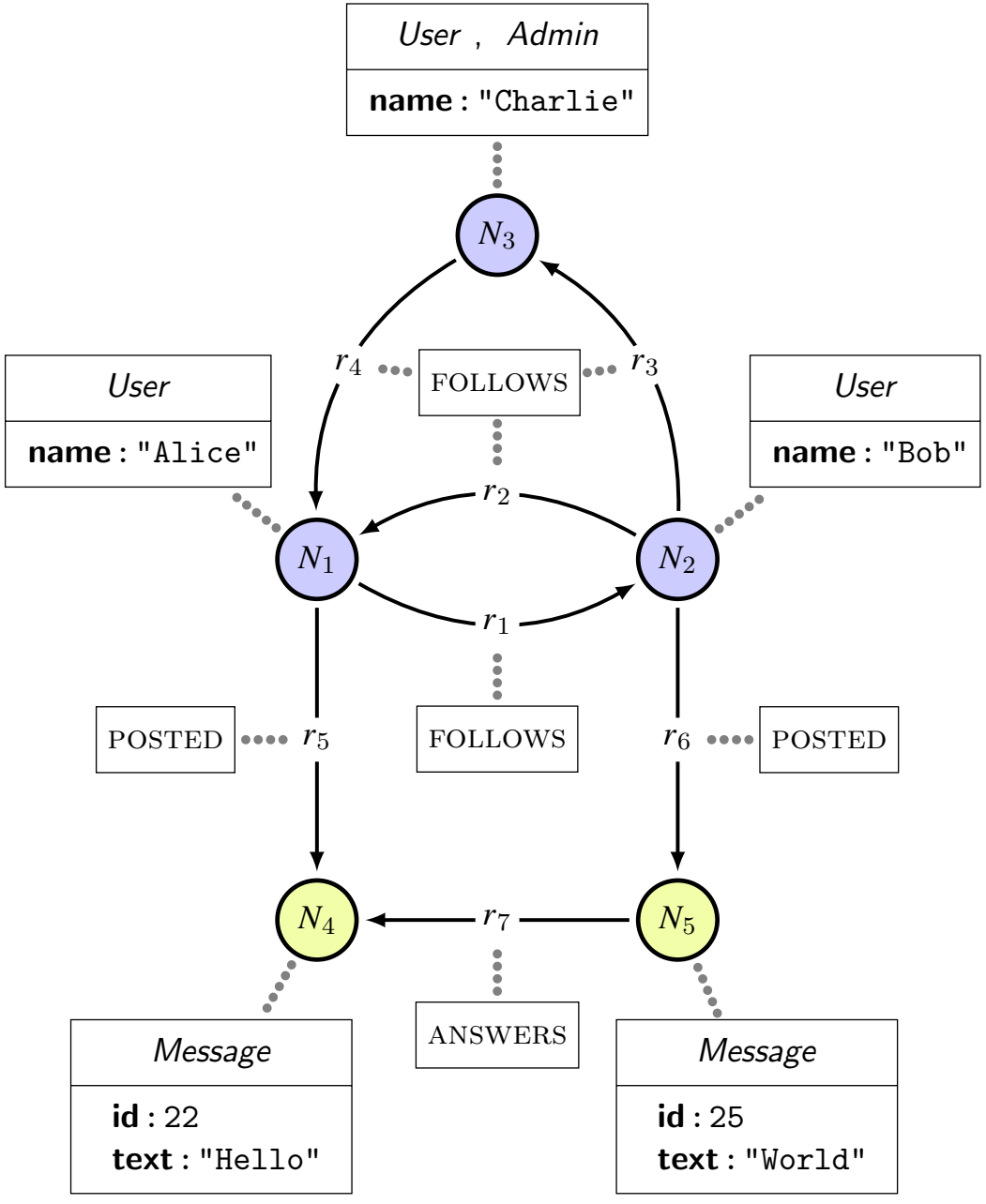

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Result :

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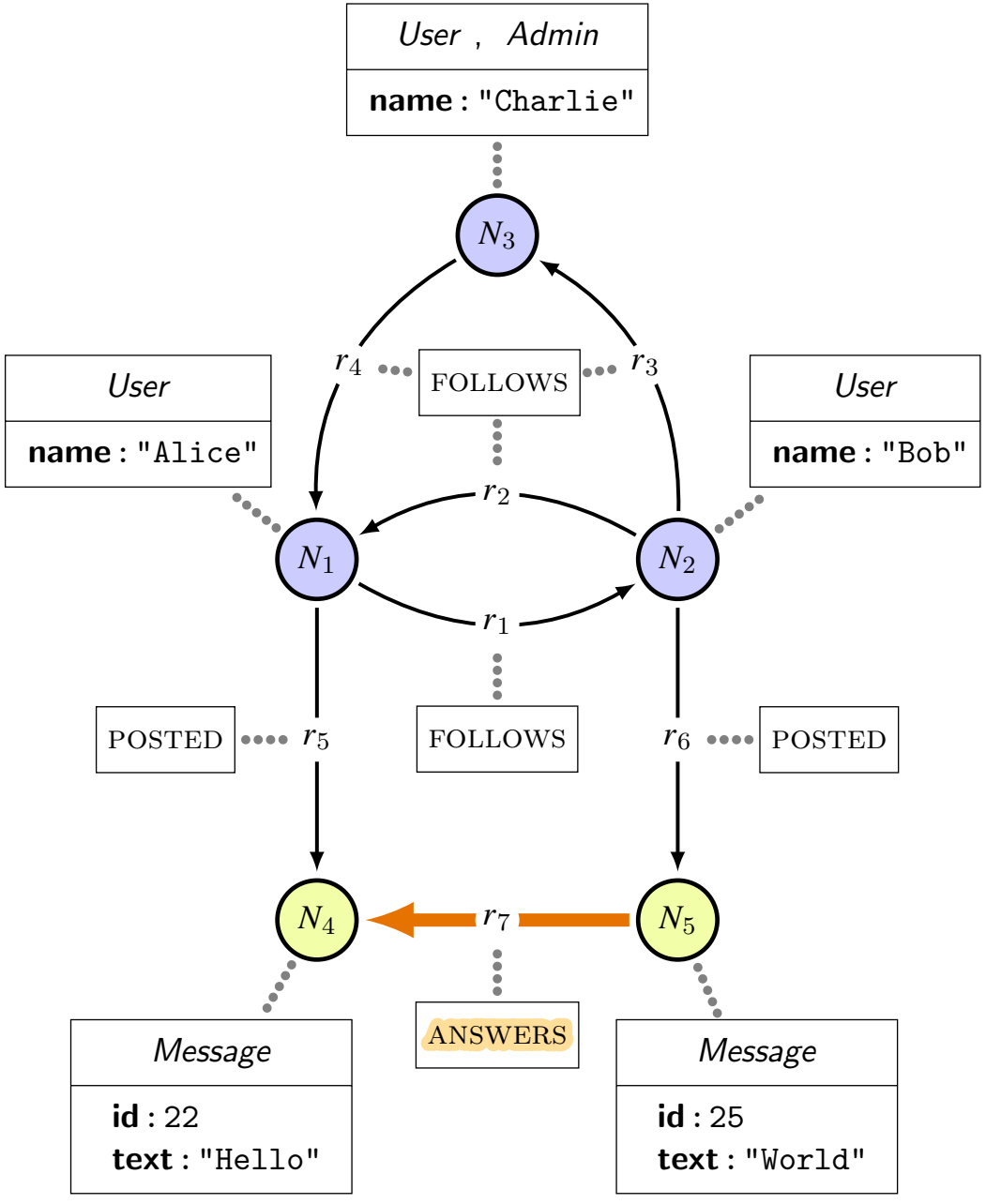
N_3	$[r_4, r_1, r_3]$	N_3

Cypher-Morphism :

- Each rel. matched ≤ 1 time

$\Rightarrow$ Finitely many results

Matching relationships (5)



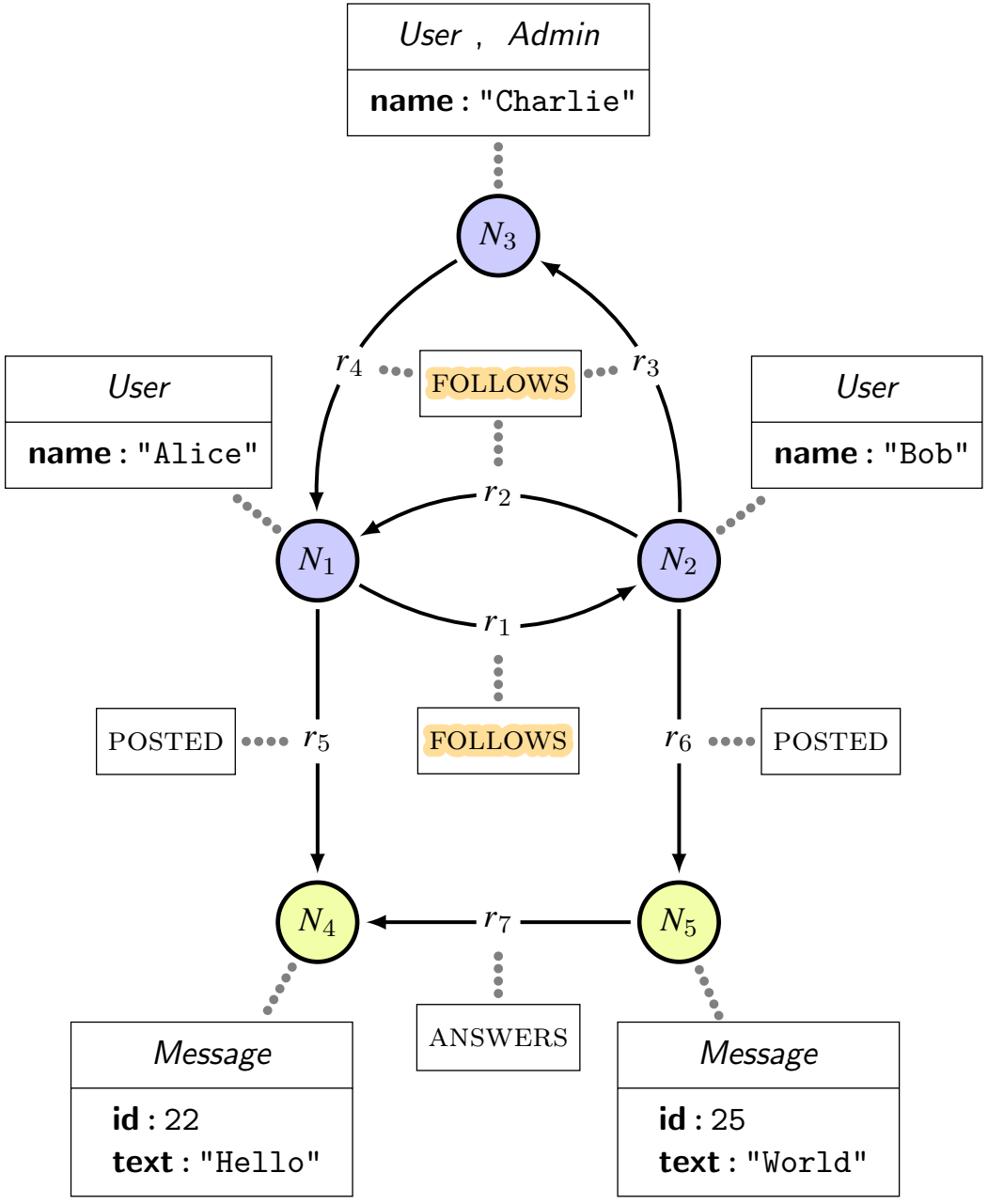
Query :

```
MATCH (m1)-[:ANSWERS]-(m2)
```

Result :

m1	m2
N_4	N_5
N_5	N_4

Matching relationships (6)



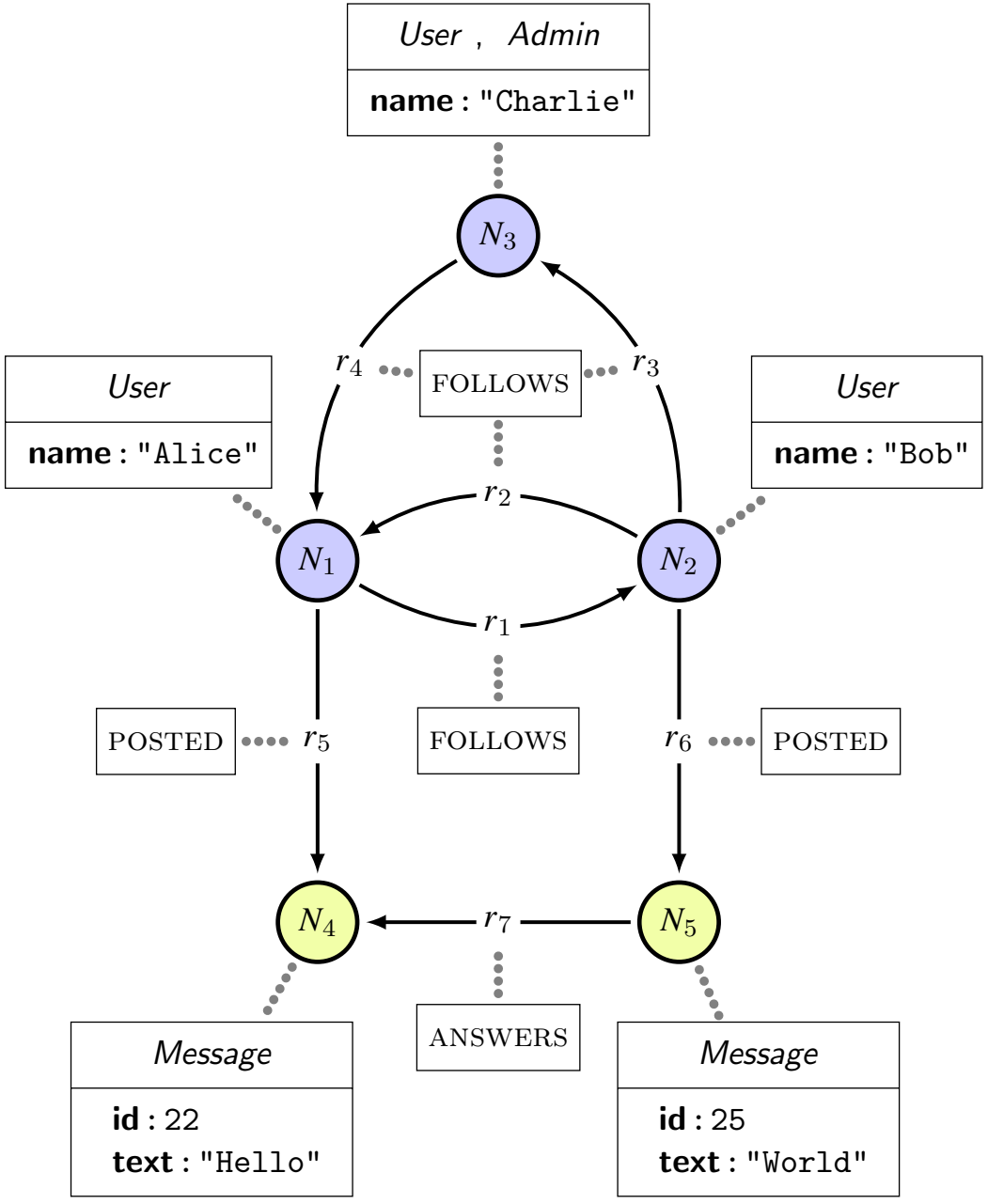
Query :

```
MATCH (u1)-[:FOLLOWS*]->(u1)
```

Result :

u1
N_1
N_1
N_2
N_2
N_3

Matching paths (1)



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MATCH (u1)-[:FOLLOWS]->()
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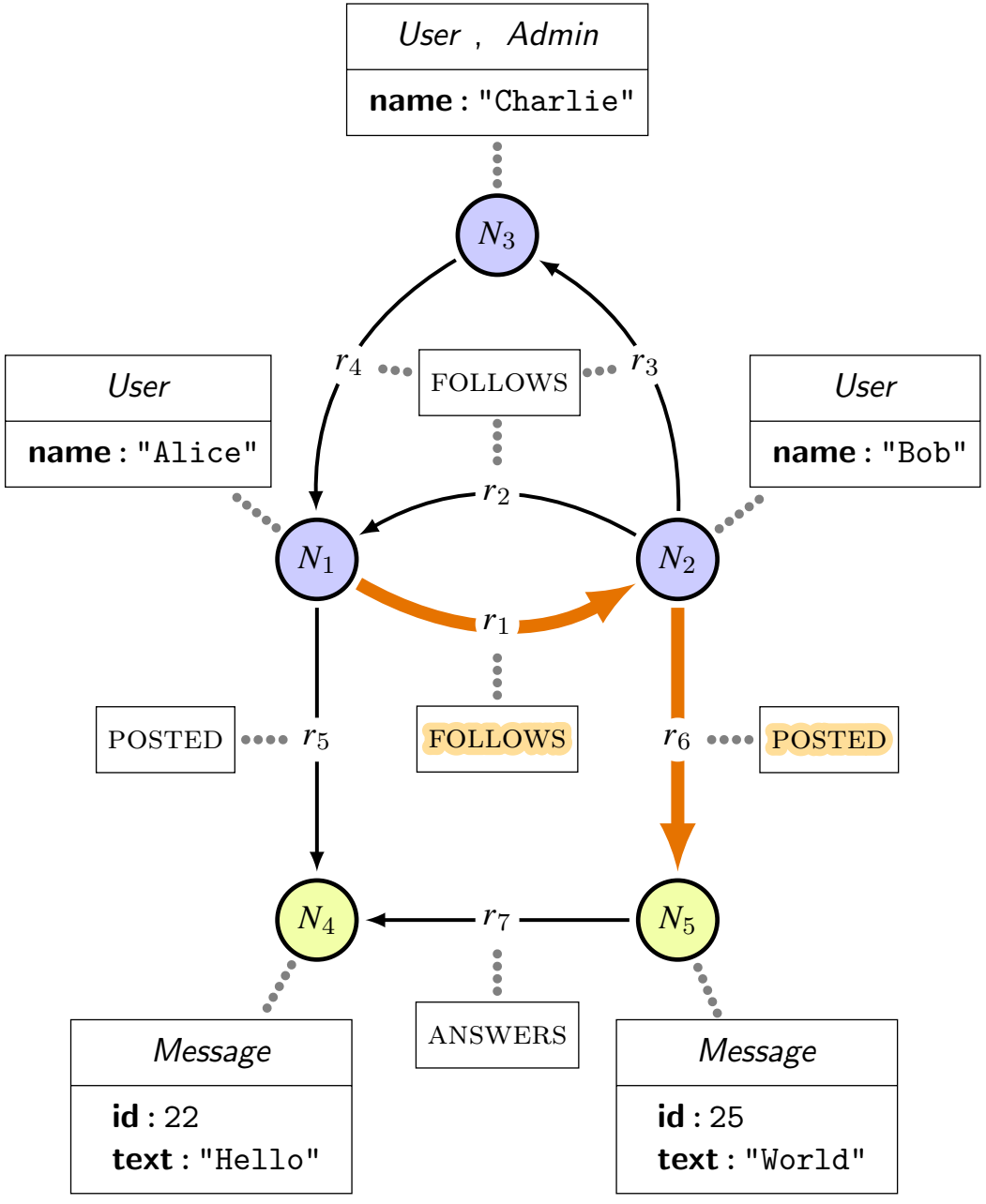

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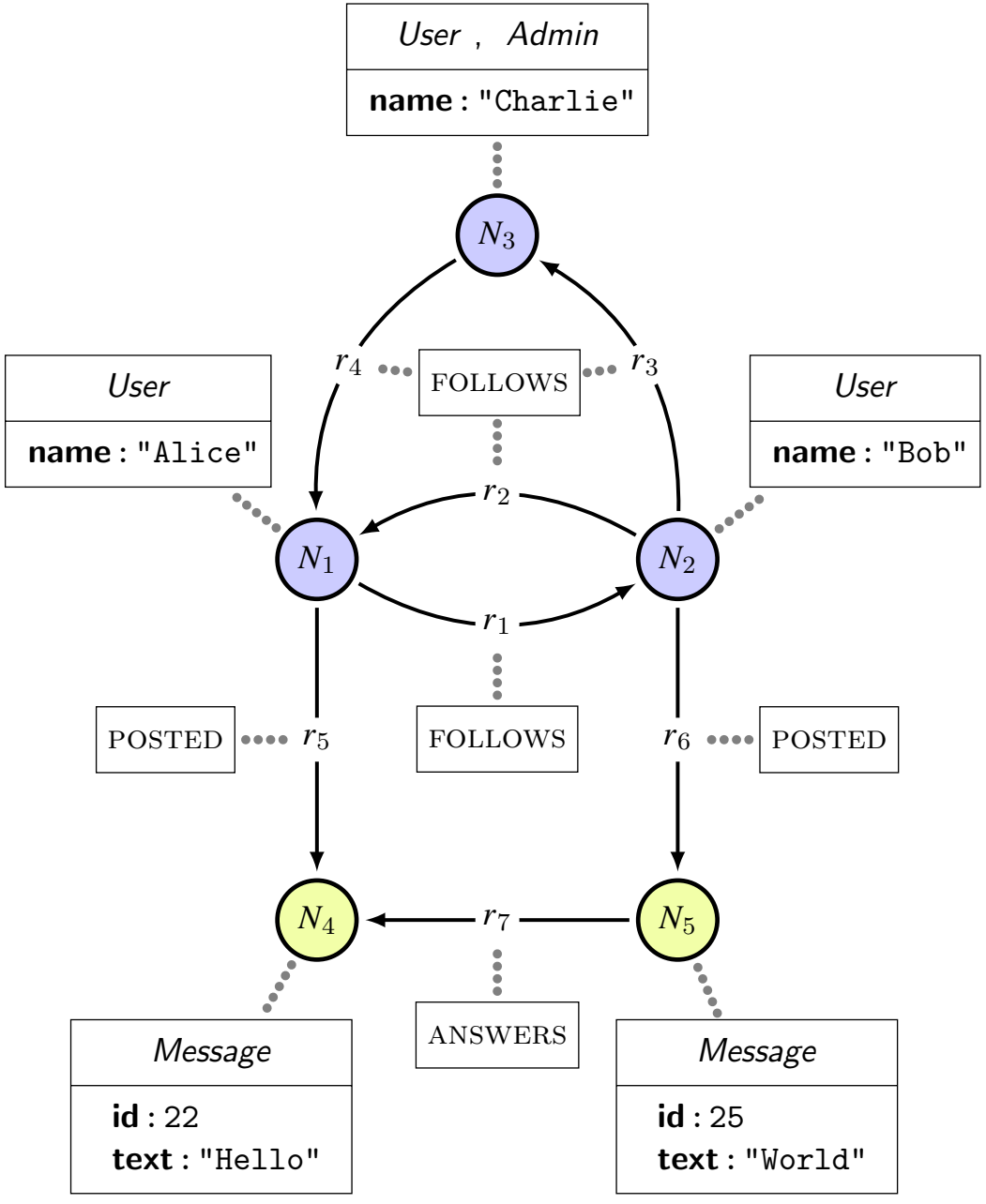
Matching paths (2)



Query :

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MATCH (u1)-[:FOLLOWS]->()
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      -[:ANSWERS]->({id:22})
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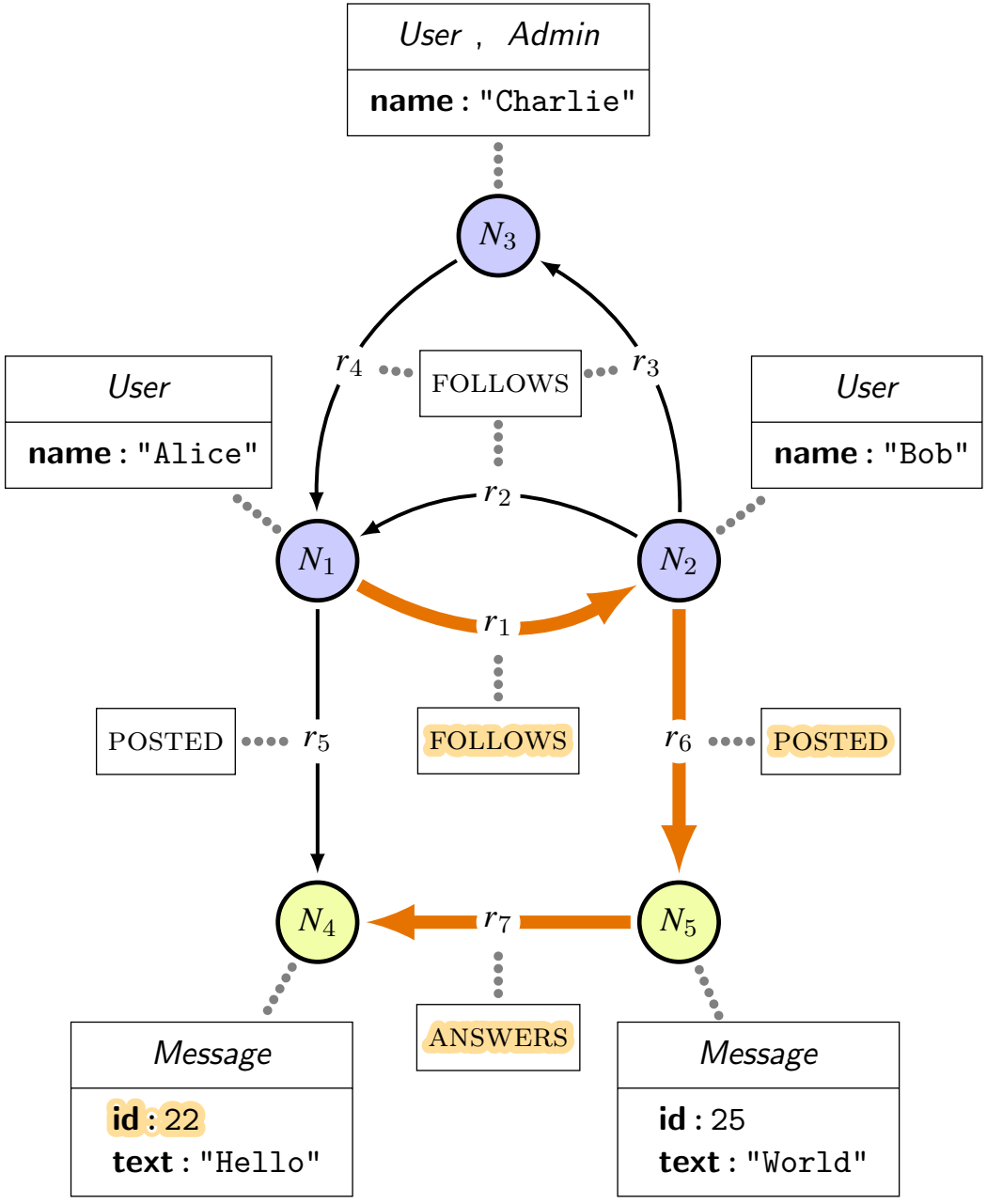
N_1	N_5

Matching paths (2)



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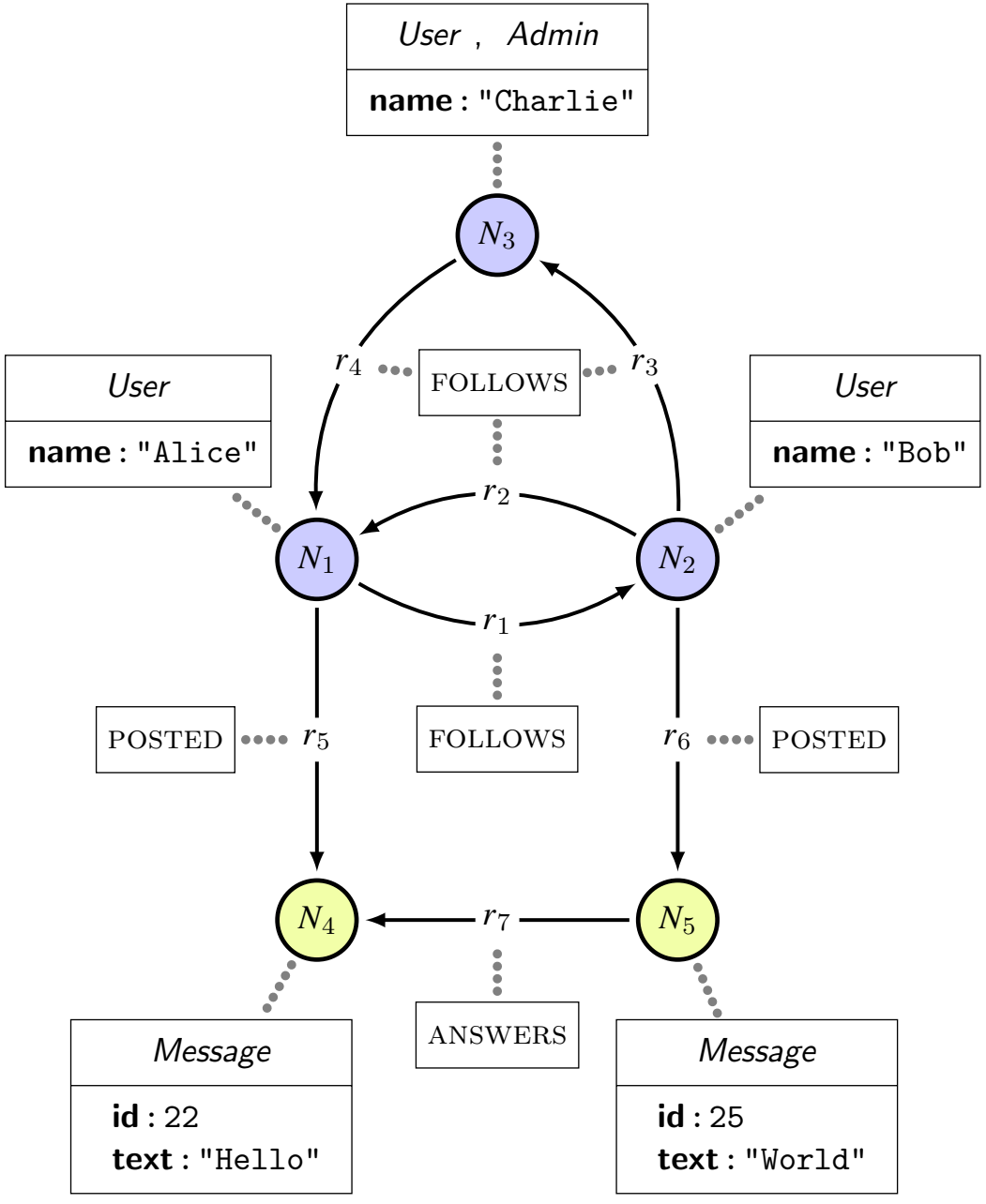
Chaining clauses



Query :

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MATCH (u2)<-[:FOLLOWS]-(u1)
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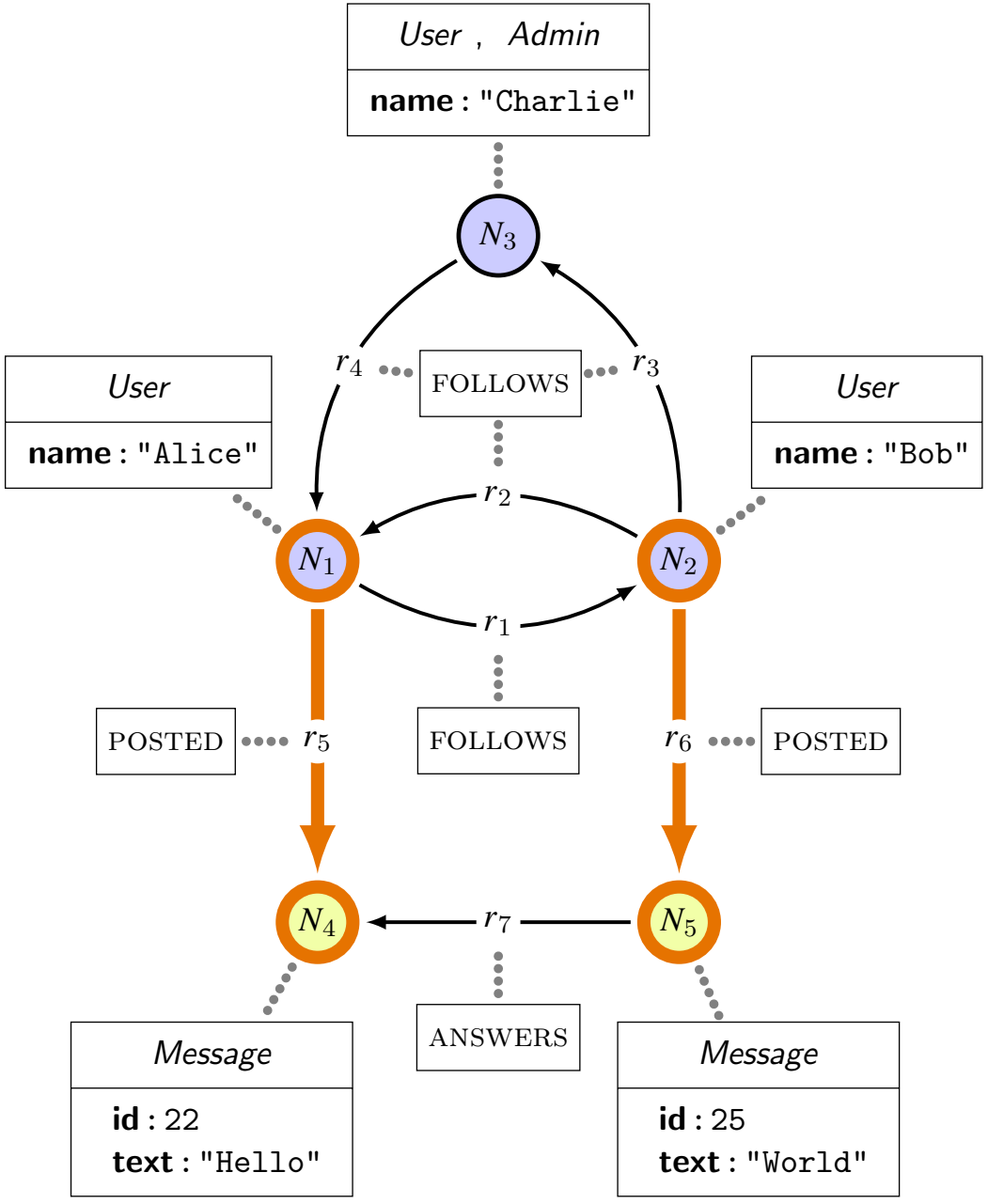

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After first `MATCH` :

u1	m1
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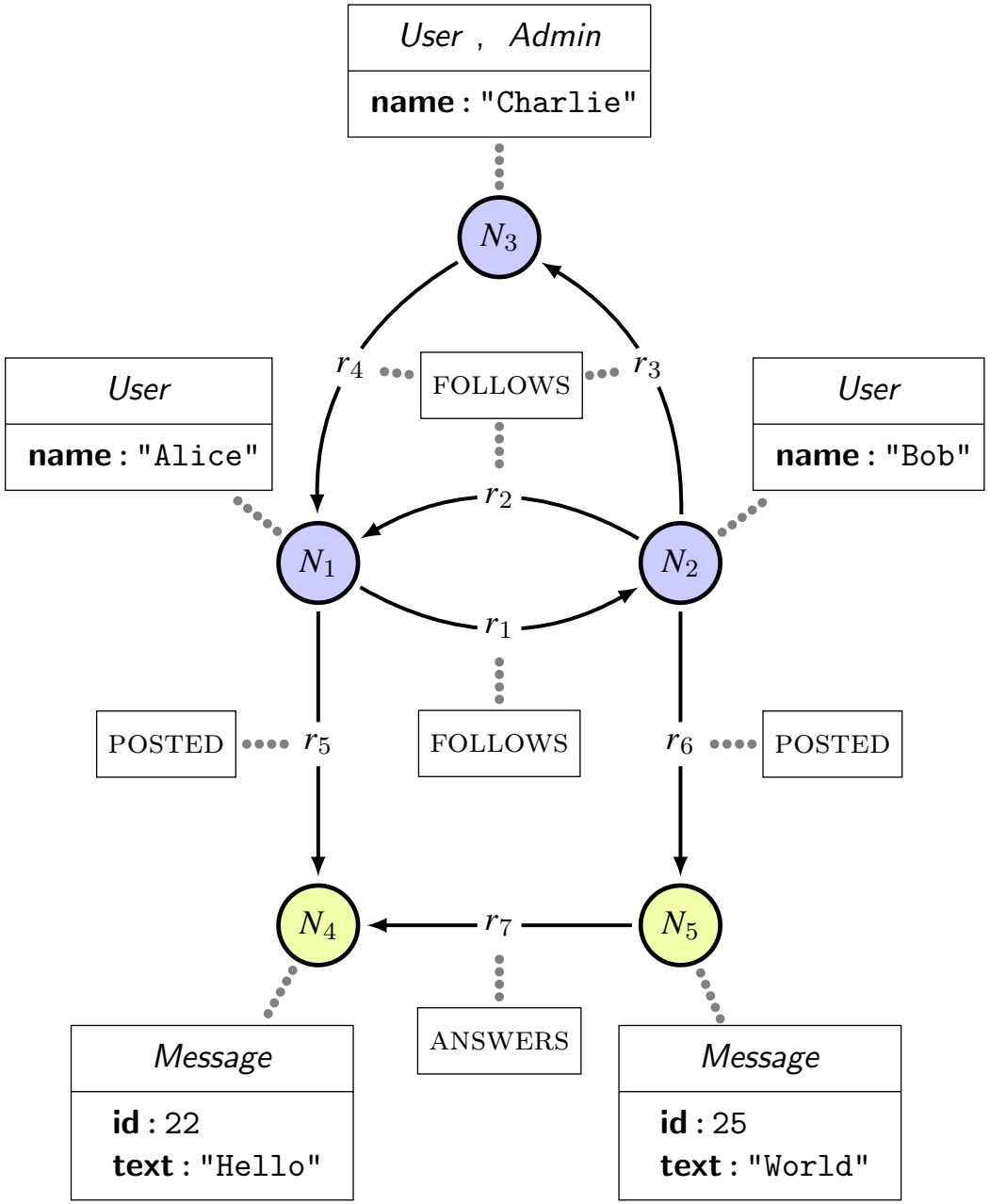
Chaining clauses



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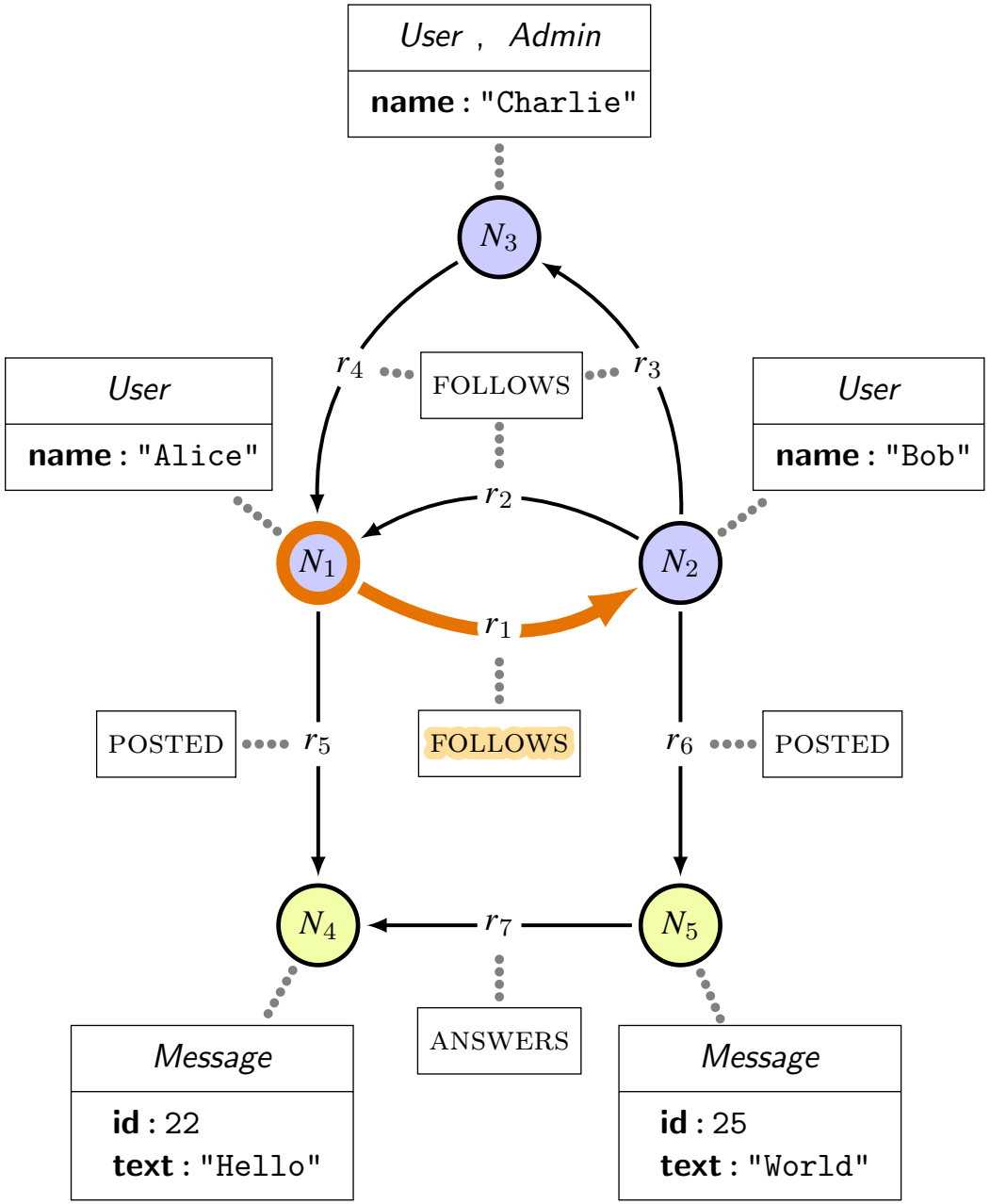
u1	m1
N_1	N_4
N_2	N_5

After second `MATCH` :

u1	m1	u2	u3
N_1	N_4	·	·
N_2	N_5	·	·

Chaining clauses



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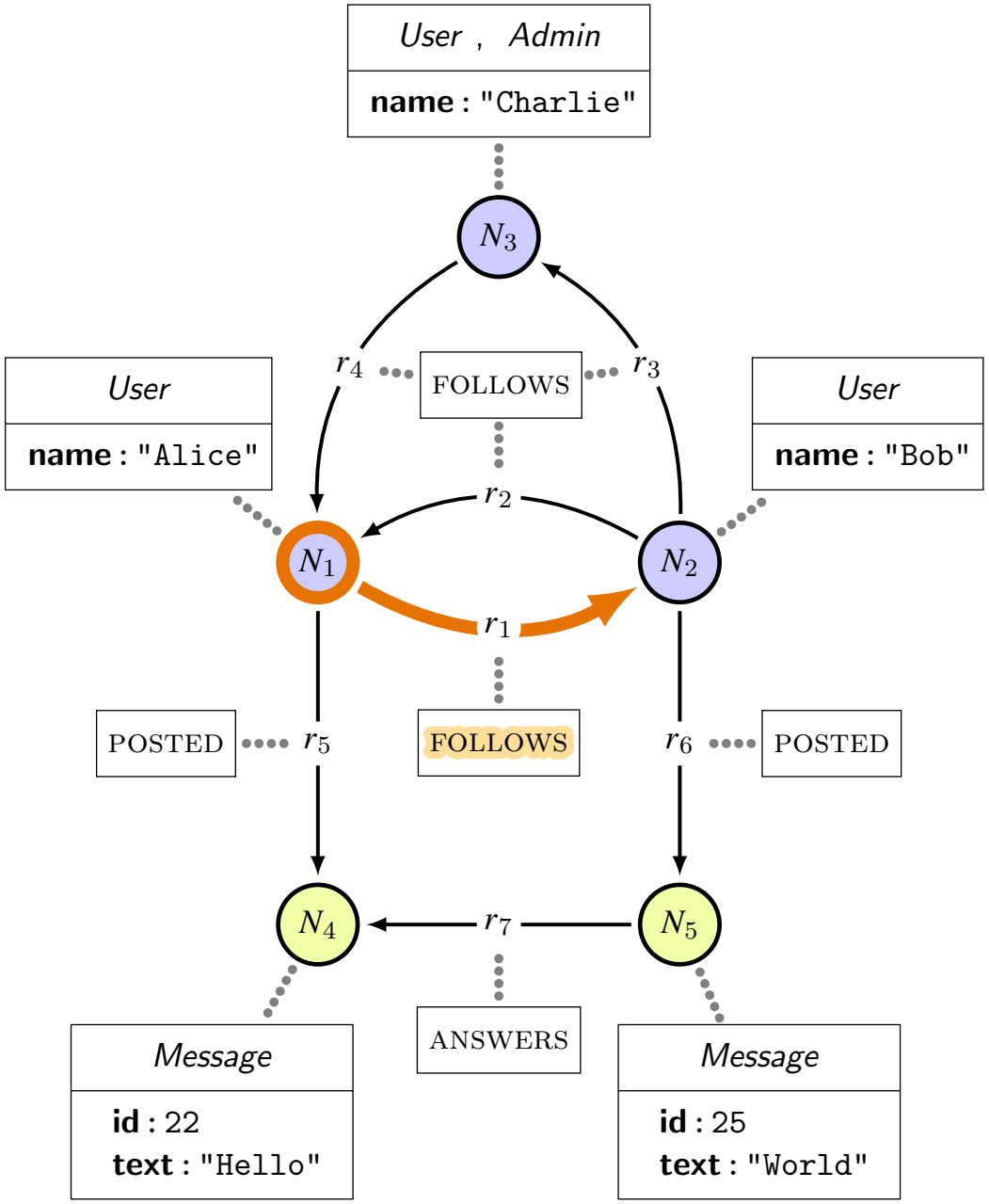
u1	m1	u2	u3
N_1	N_4	·	·
N_2	N_5	·	·

Chaining clauses



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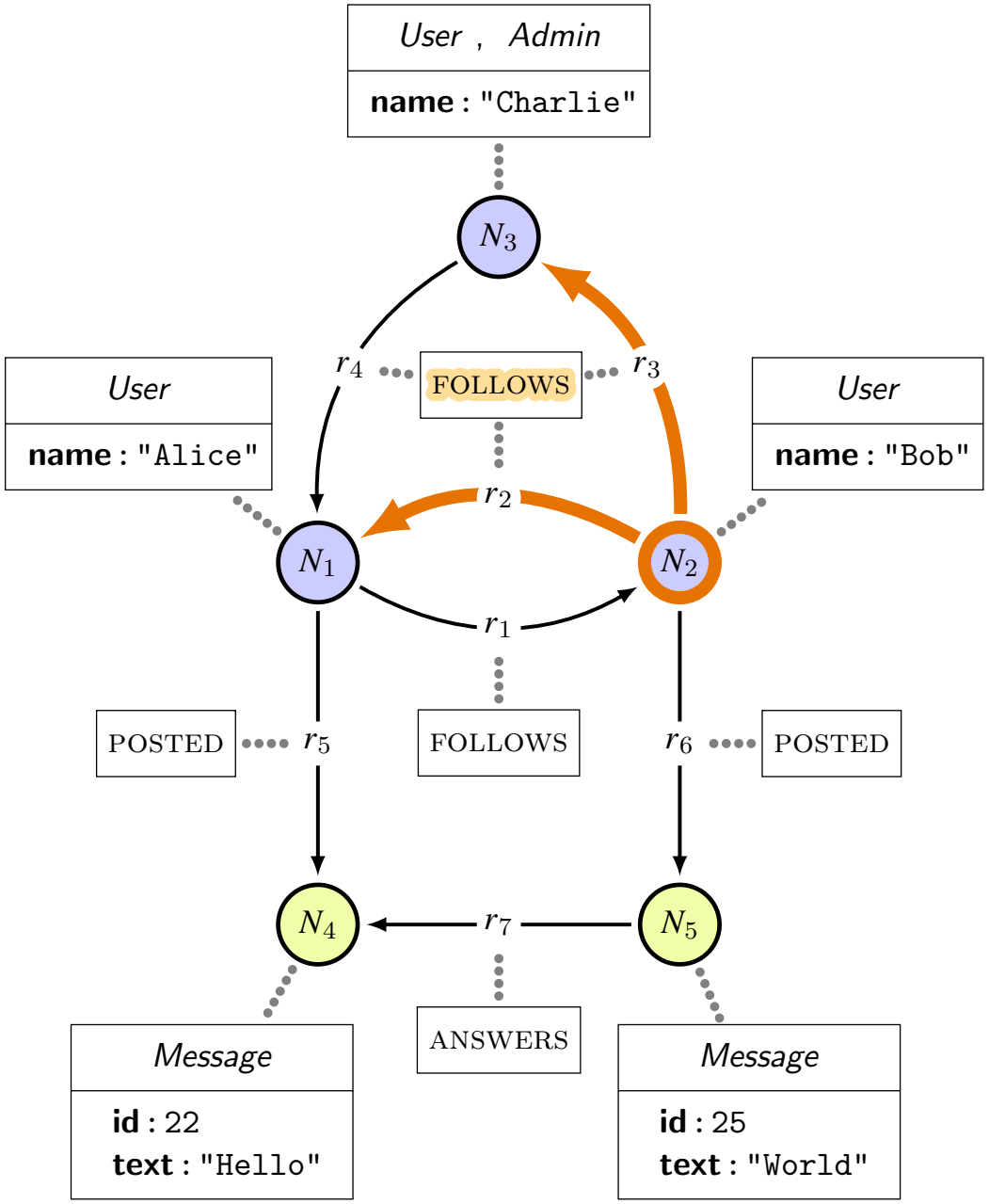
u1	m1
N_1	N_4
N_2	N_5

After second `MATCH` :

u1	m1	u2	u3
~~N_1~~	~~N_4~~	~~.~~	~~.~~
N_2	N_5	.	.

Chaining clauses



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N_1	N_4
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After second `MATCH` :

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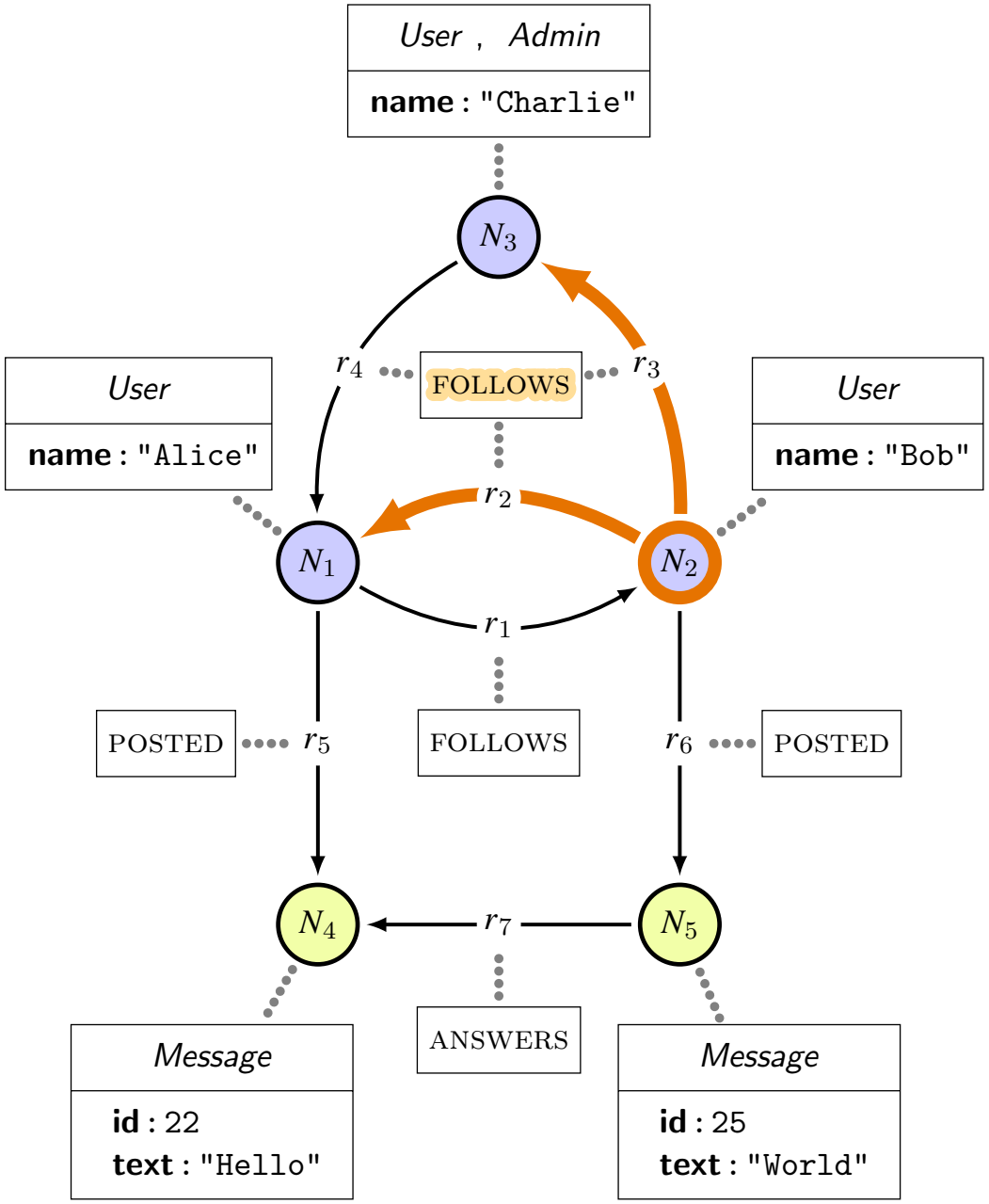
~~N_1~~	~~N_4~~	·	·
N_2	N_5	·	·

Chaining clauses



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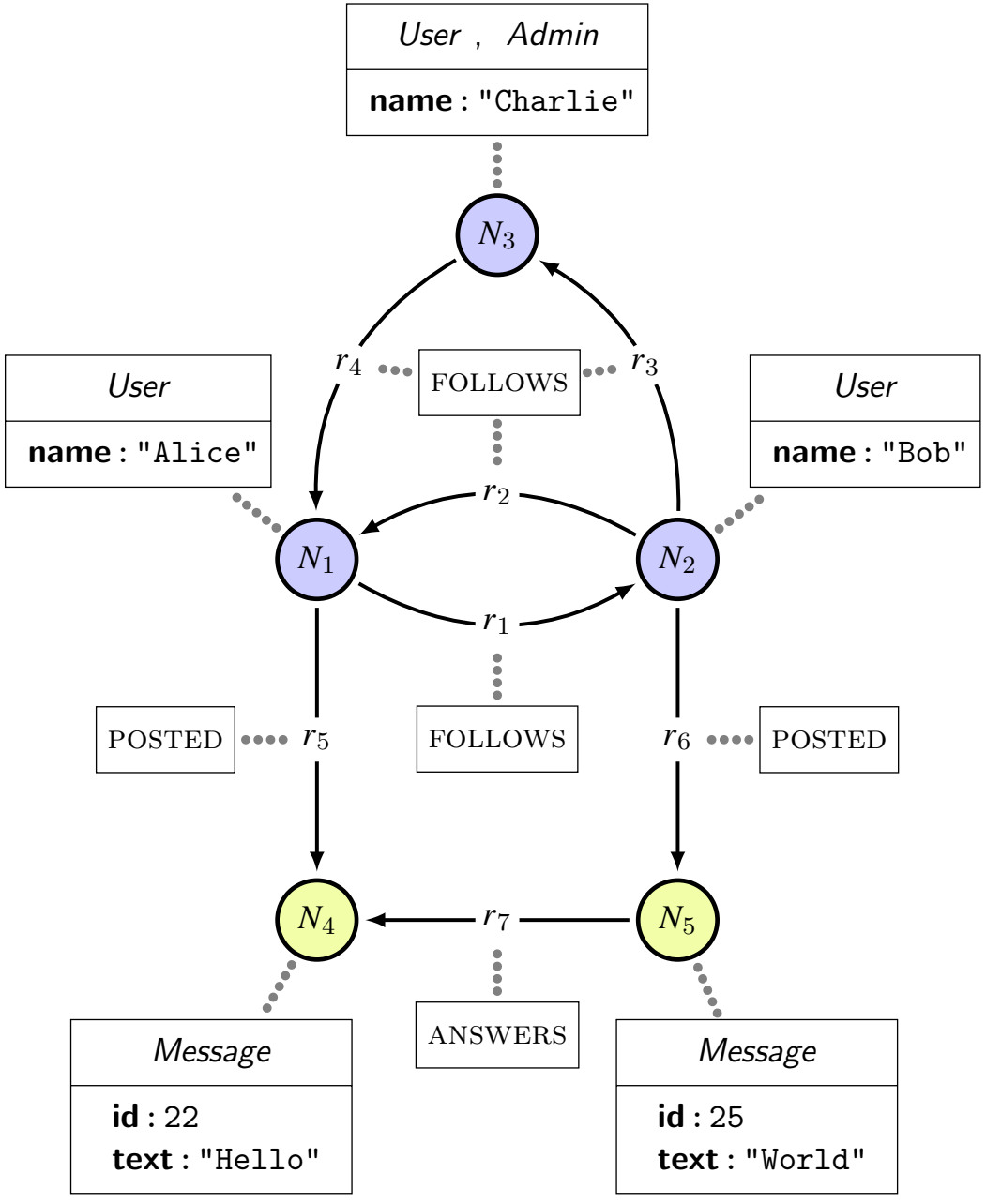
After second `MATCH` :

u1	m1	u2	u3
N_2	N_5	N_1	N_3
N_2	N_5	N_3	N_1

Column manipulation (WITH clause)



Query :

```
MATCH (u1)-[p1:POSTED]->(m1)
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```

After the MATCH clause :

u1	p1	m1
N_1	r_5	N_4
N_2	r_6	N_5

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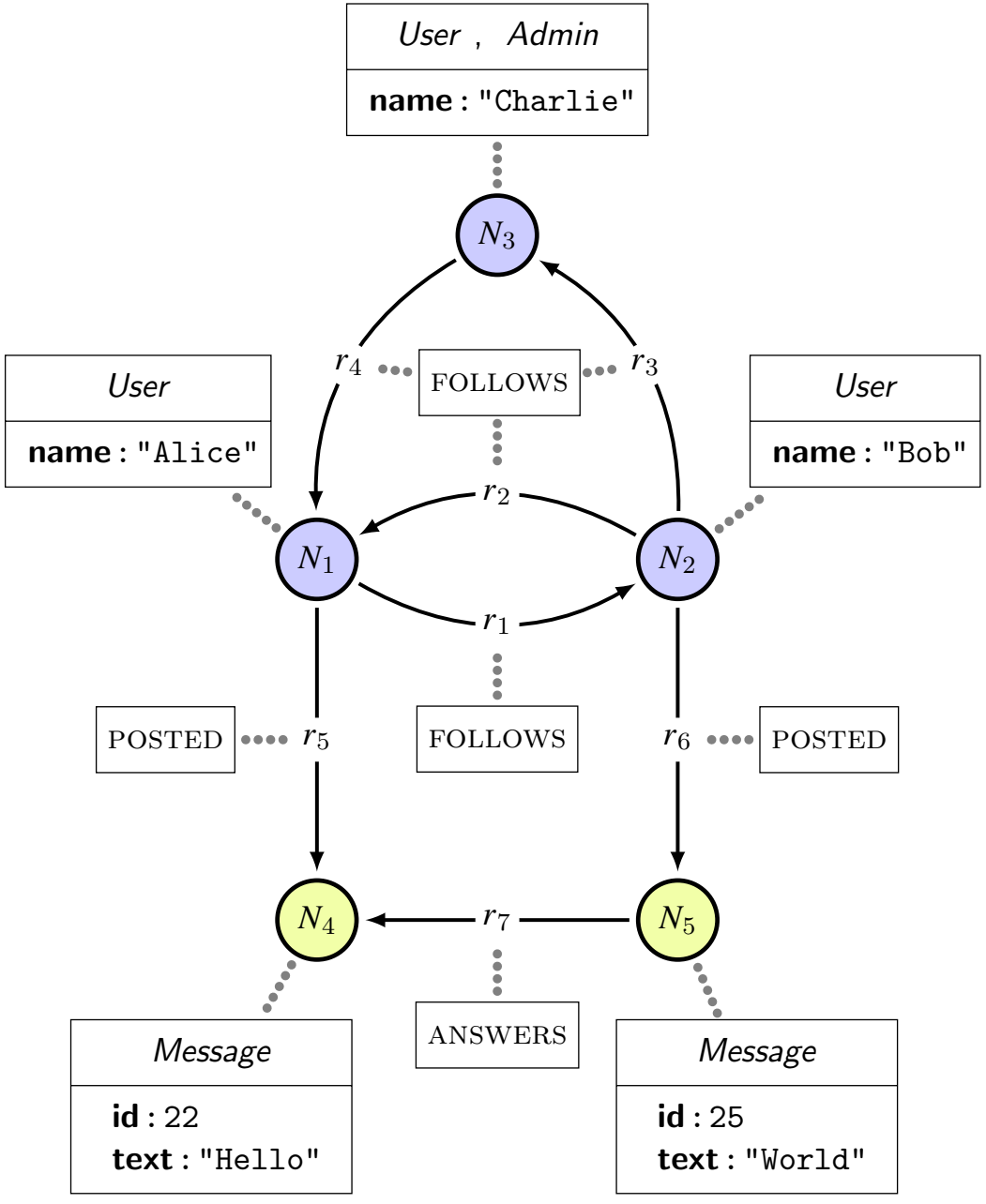

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Execution of the WITH clause :

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N_1	r_5	
N_2	r_6	

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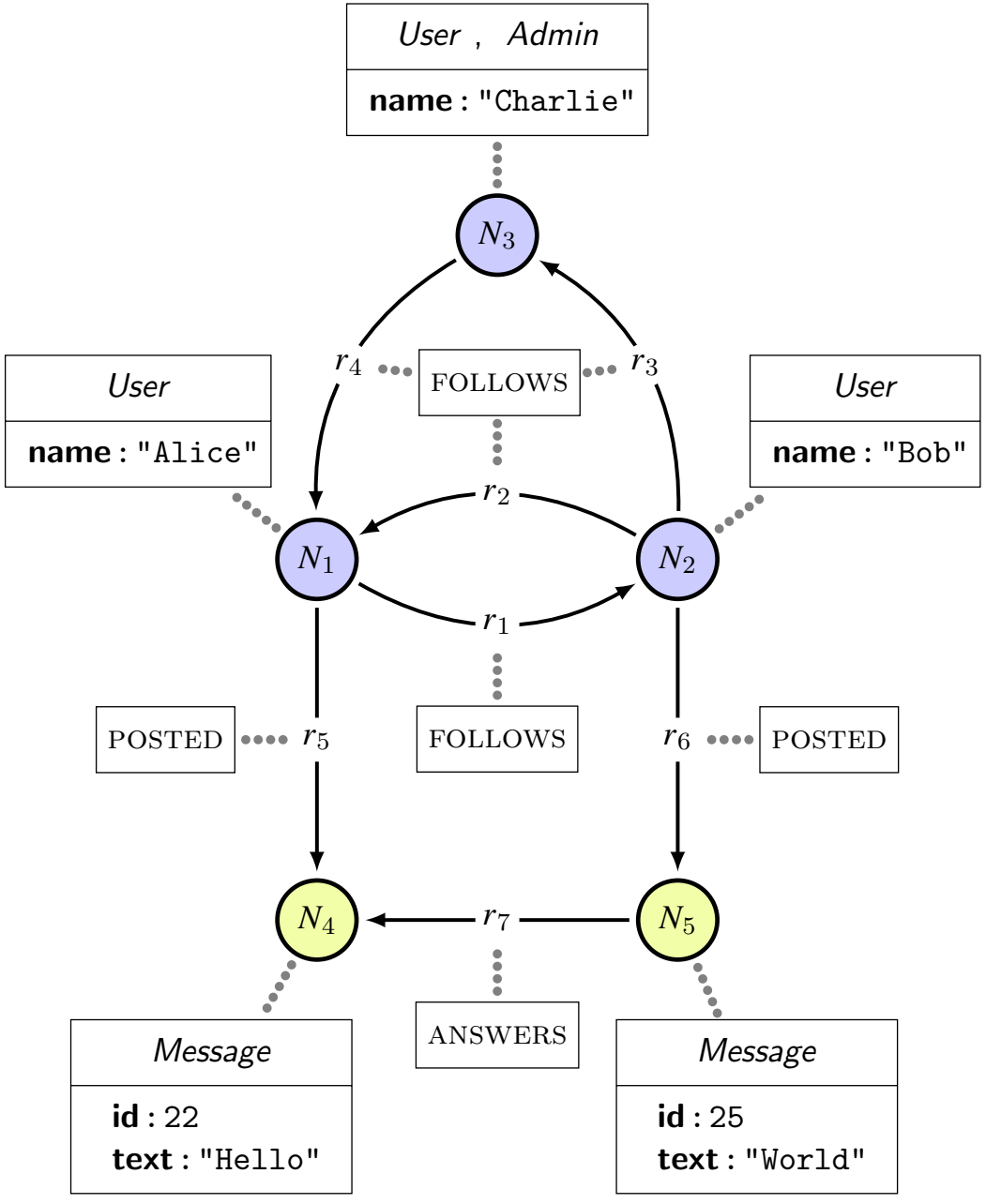

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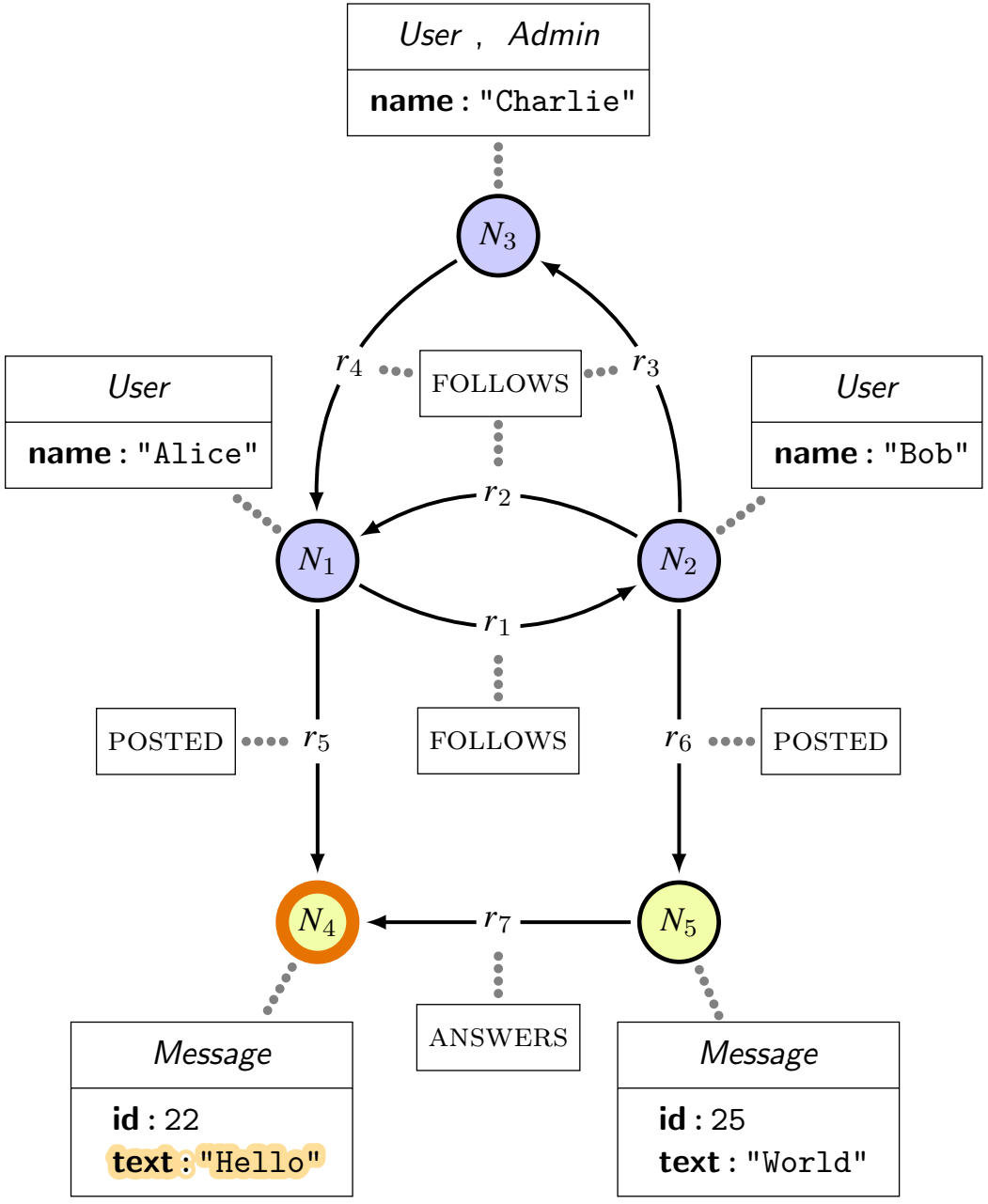

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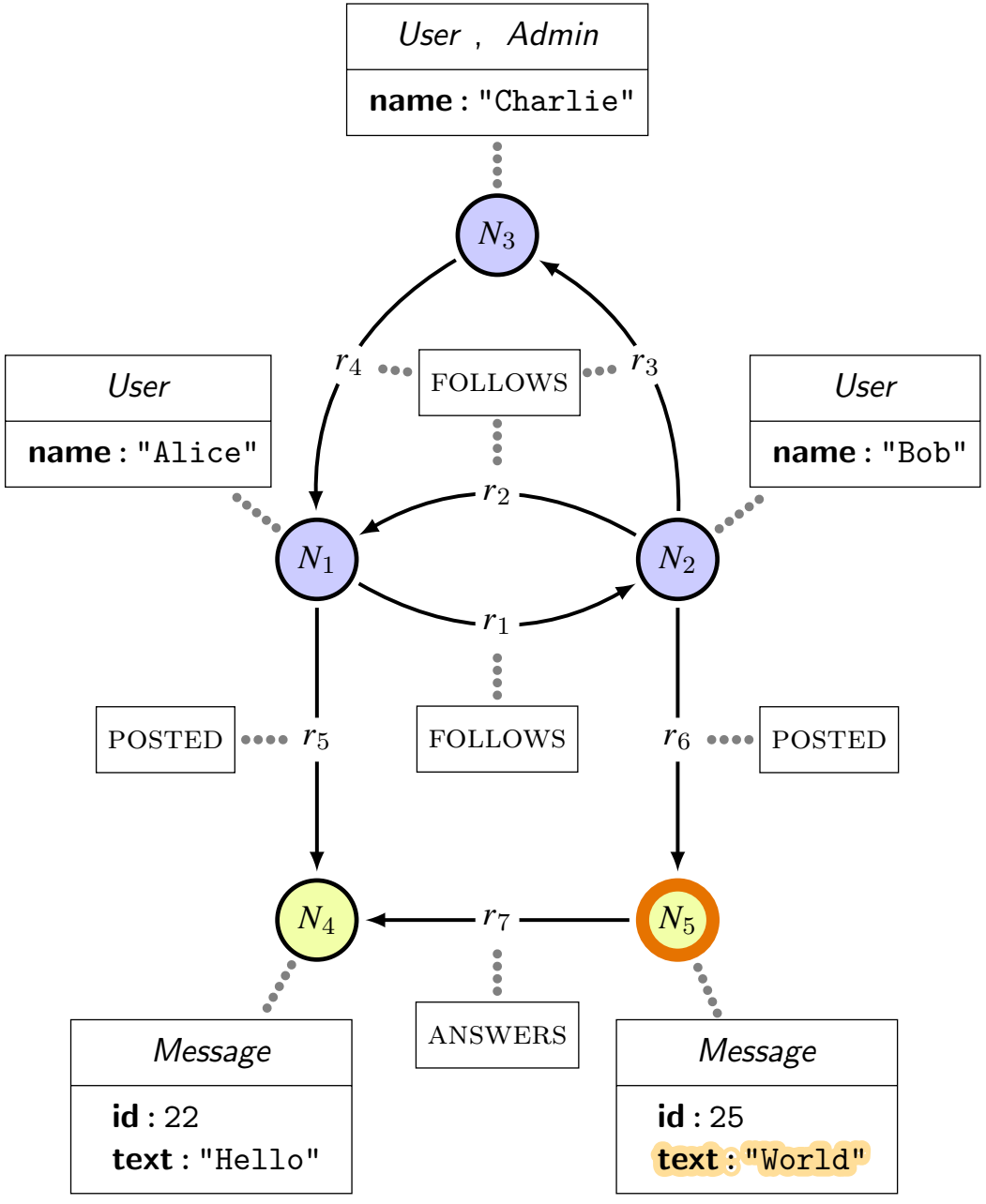

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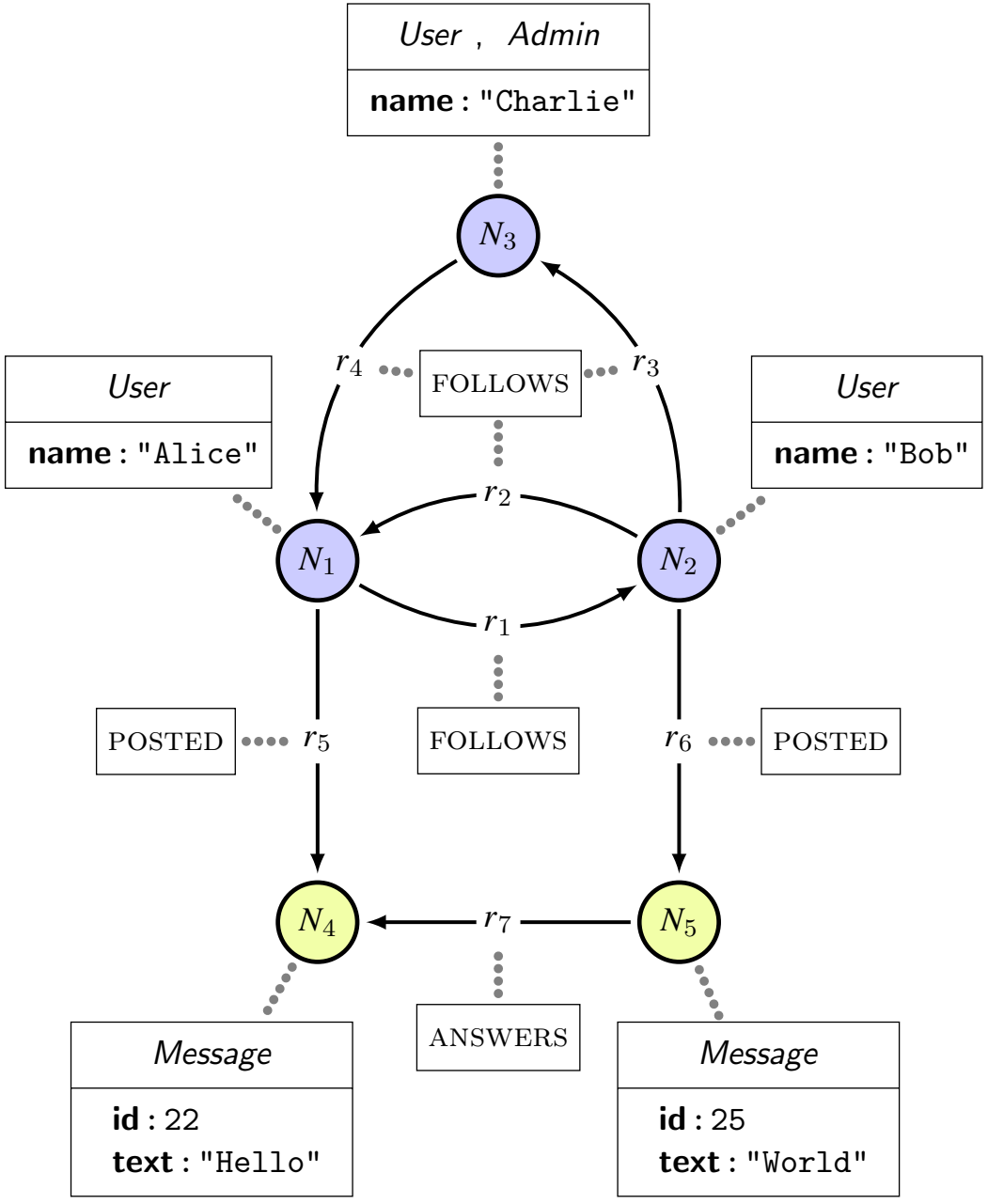

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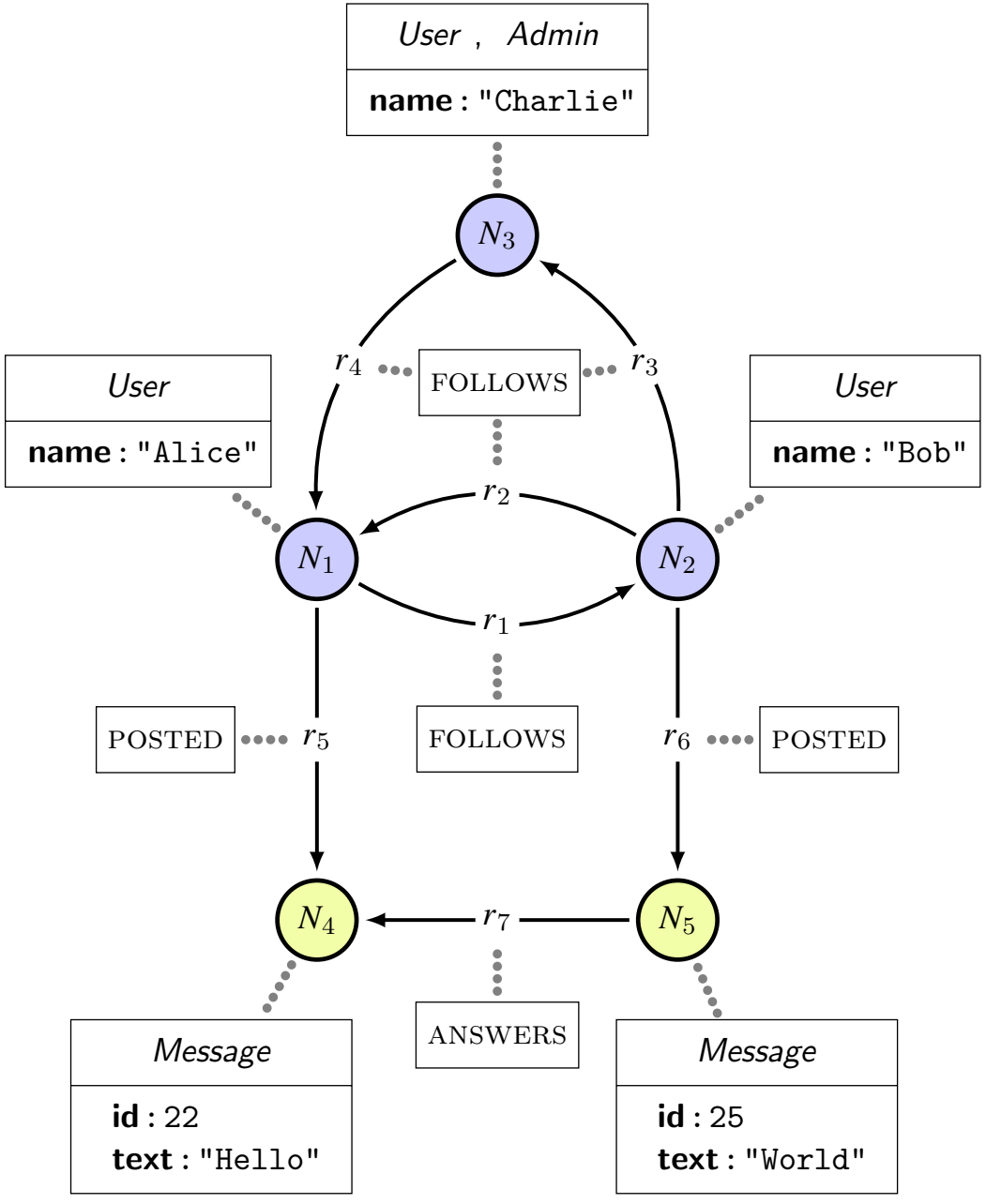
Final result :

u1	p1	t1
N_1	r_5	"Hello"
N_2	r_6	"World"

Line filtering (WHERE clause)



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Après le WITH :

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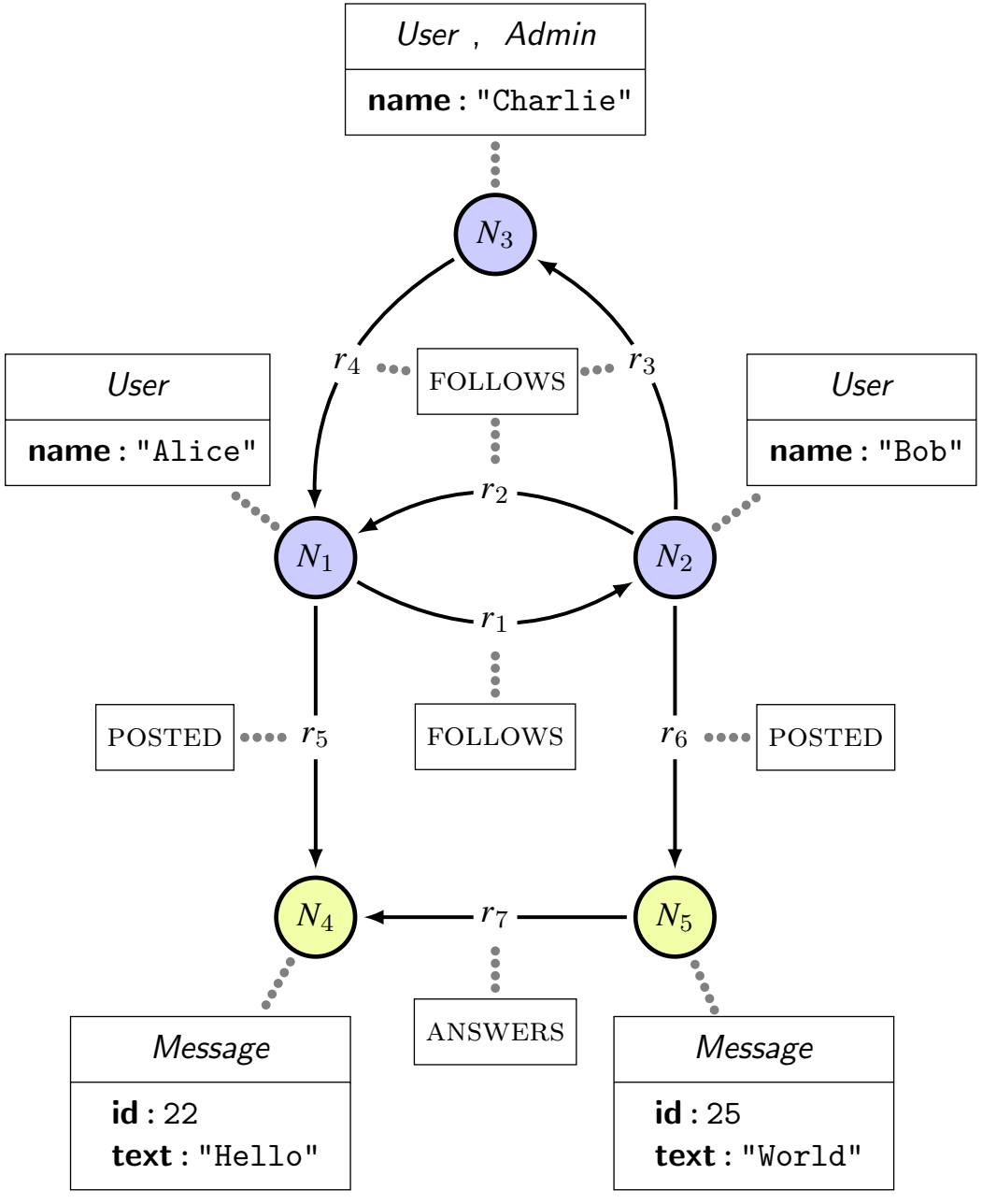

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~~N_2~~	~~r_6~~	~~"World"~~

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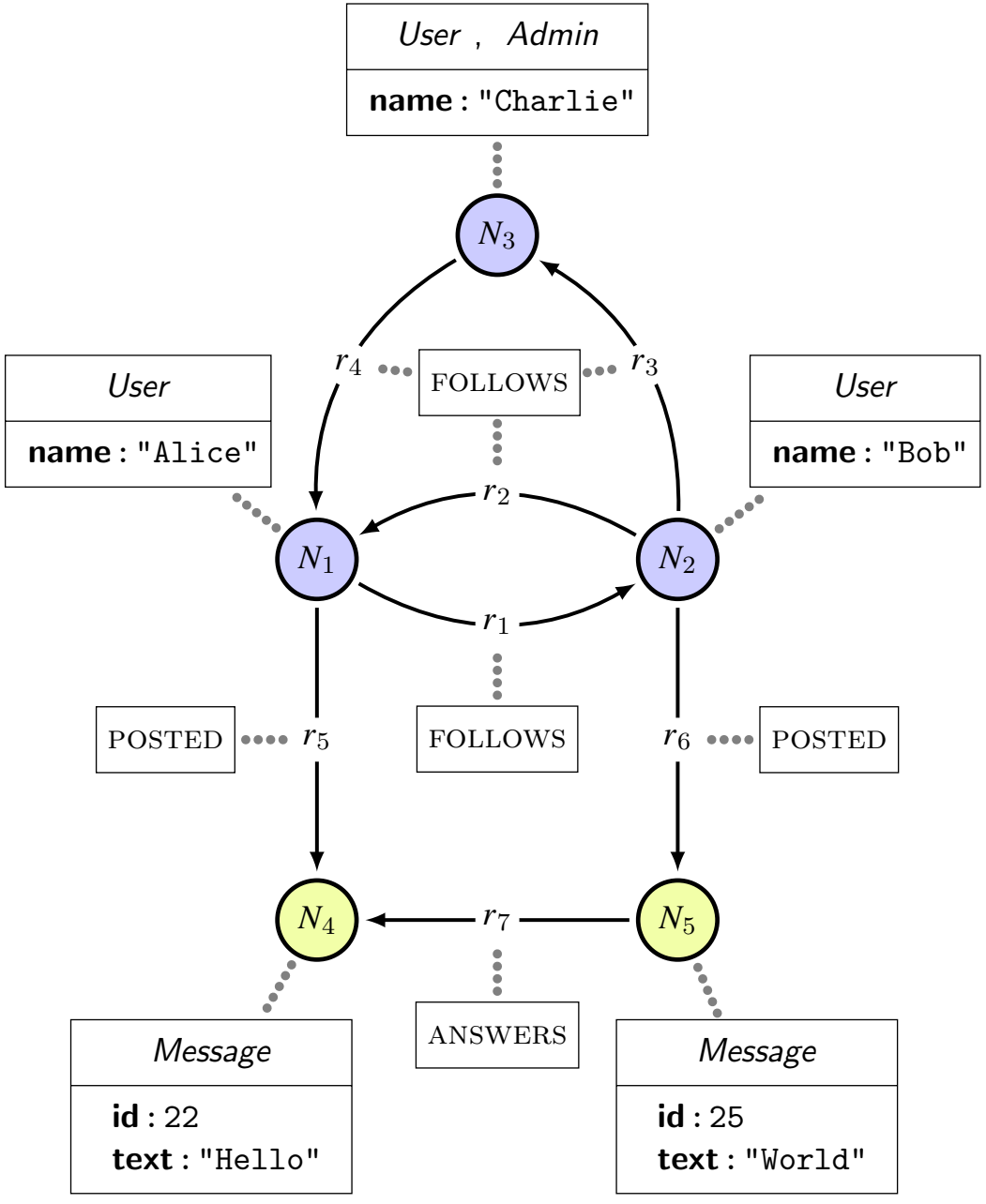

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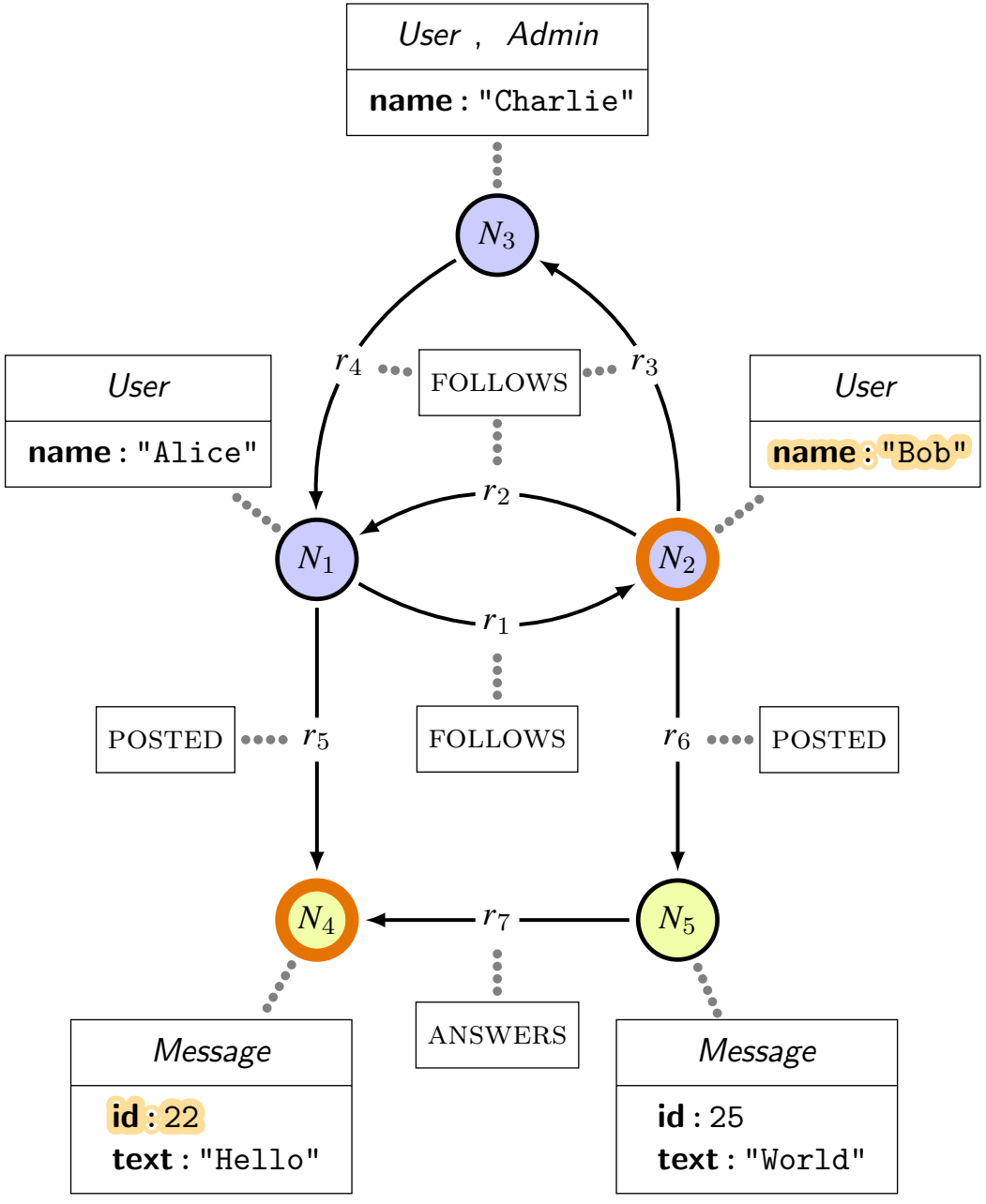
A last example



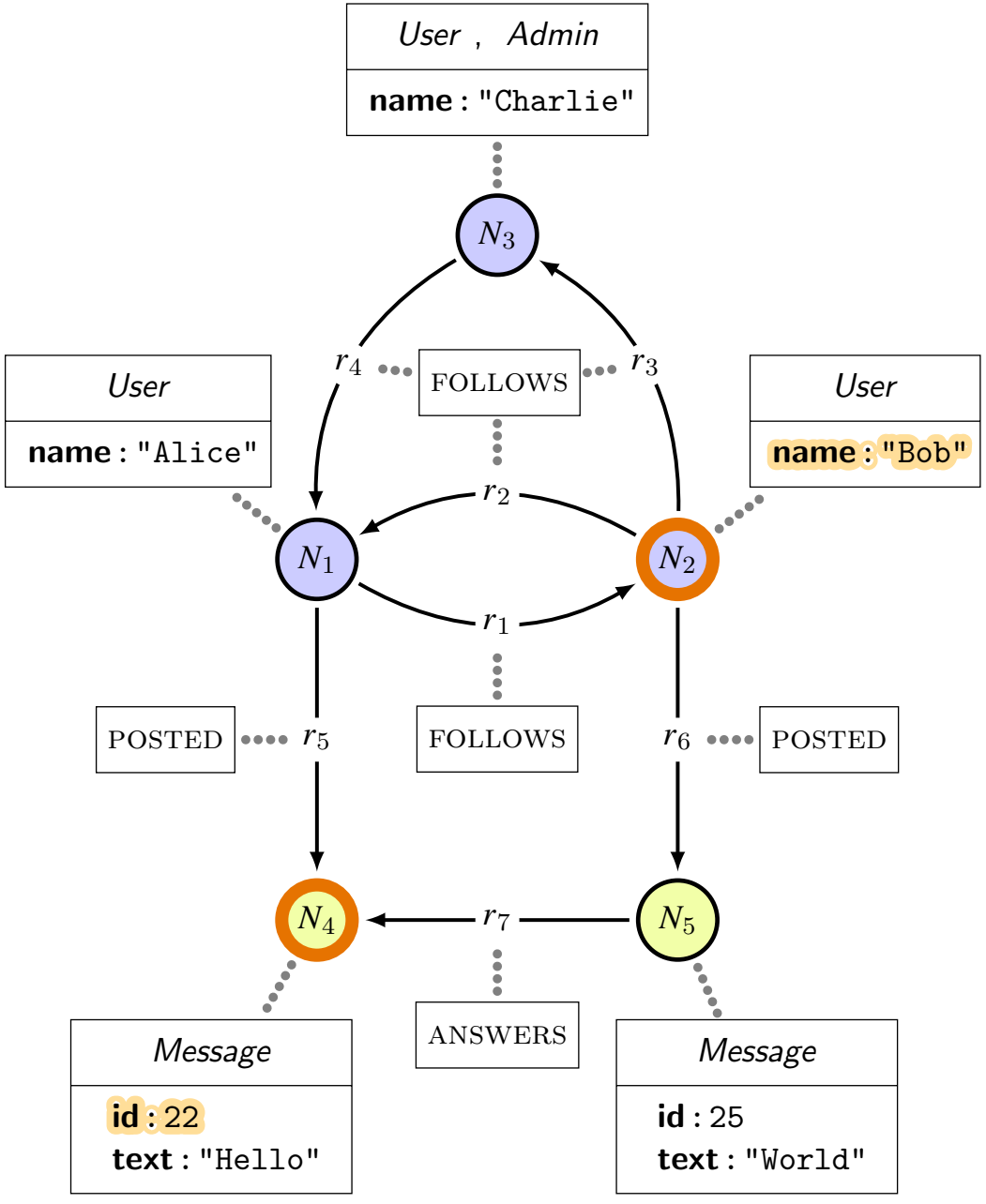
Query :

```
MATCH (a{name:"Bob"})
        -[*]->({id:22})
              <-[*]-(a)
```

Question :

What does this computes ?

A last example



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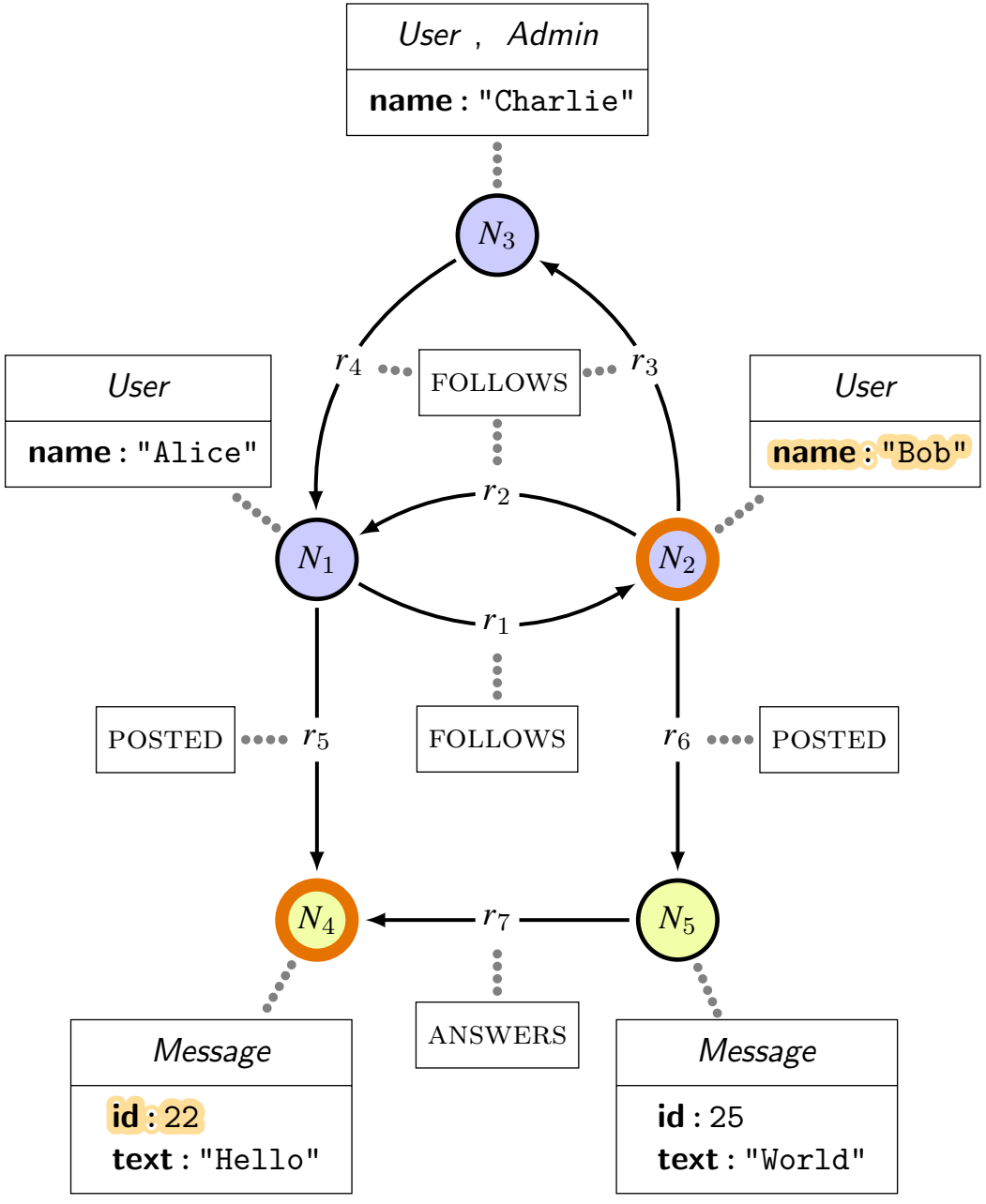
Answer :

The # of pairs of disjoint paths between N_6 and N_2.

A last example



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$\Rightarrow$ Evaluation of Cypher queries is NP-HARD

Outline

Record (table row)

A *record* is a partial function from variables to values.

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x	y
"Bob"	1
"Alice"	999
"Bob"	1

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"Bob"	1

=

x	y
999	"Alice"
1	"Bob"
1	"Bob"

Semantics brackets

G: a graph

Semantics of expressions

$[\![\cdot]\!]_{u,G}$: expression $\longmapsto$ value (where u is a record)

Semantics of clauses

$[\![\cdot]\!]_{G}$: clause $\longmapsto$ (function: Tables $\rightarrow$ Tables)

Semantics of queries

$[\![\cdot]\!]_{G}$: query $\longmapsto$ (function: Tables $\rightarrow$ Tables)

output : (Graph $\times$ Queries) $\longmapsto$ Tables

Principle of the semantics

G: a graph
Q: a query

To compute the output of Q

- Q is a sequence of clauses $Q = C_1\, C_2\, \cdots\, C_n$

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- Compute $[\![C_1]\!]_G\,,\quad [\![C_2]\!]_G\,,\quad \cdots\,,\quad [\![C_n]\!]_G$
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- Let $[\![Q]\!]_G = [\![C_n]\!]_G \circ \cdots \circ [\![C_2]\!]_G \circ [\![C_1]\!]_G$
- output$(G, Q) = [\![Q]\!]_G\left(T_{\text{unit}}\right)$

where T_{unit} is the 1-line 0-column table.

Outline

Node patterns

⟨node_pattern⟩ ::= (⟨name⟩? ⟨labels⟩? ⟨properties⟩?)

Examples

`()`	Any node
`(a)`	Any node; verify/map id to column a
`(:User)`	Nodes bearing label *User*
`({name:"Alice"})`	Nodes with the property **name** set to `"Elin"`
`(a:User{name:"Alice"})`	Nodes of type User and with a property **name** set to `"Elin"`; verify/map id to column a.

Relationship patterns

⟨pattern⟩ ::= **[** ⟨name⟩? ⟨types⟩? ⟨iter⟩? ⟨properties⟩? **]**

Example

`[]`	Any relationship
`[r]`	Any relationship; verify/map relationship id to column r
`[r*]`	Any path; verify/map the list of relationship id to column r
`[*..3]`	Any path of length 1–3
`[:FOLLOWS*3..]`	Any FOLLOWS path of length at least 3
`[*{isNice:true}]`	Path such that every relationship has the property **isNice** set to `true`

Path patterns

⟨pattern⟩ ::=
⟨node_pattern⟩ [<-? ⟨rel_pattern⟩ ->? ⟨node_pattern⟩]*

Examples

```
()
()-[*2..3]-(b)
(a)<-[]->()-[{id:220}]->()
(a)<-[*]-(b)-[*]->(a)
({name:a.name})-[*]->(a)
({name:"Alice"})-[:POSTED]->()<-[:ANSWERS*]-()
```

Rigid path-patterns

Definition

A path pattern is *Rigid* if its length is fixed
(*i.e.* all ⟨iter⟩ are absent or derive to $*i..i$ for some $i \in \mathbb{N}$)

Examples

`(a)<-[]->()-[{id:220}]->()`		Rigid
`()-[]->()-[*42..42]->(:User)`		Rigid
`()-[*2..3]->(b)`	and `(a)<-[]-(b)-[*]-(a)`	Flexible

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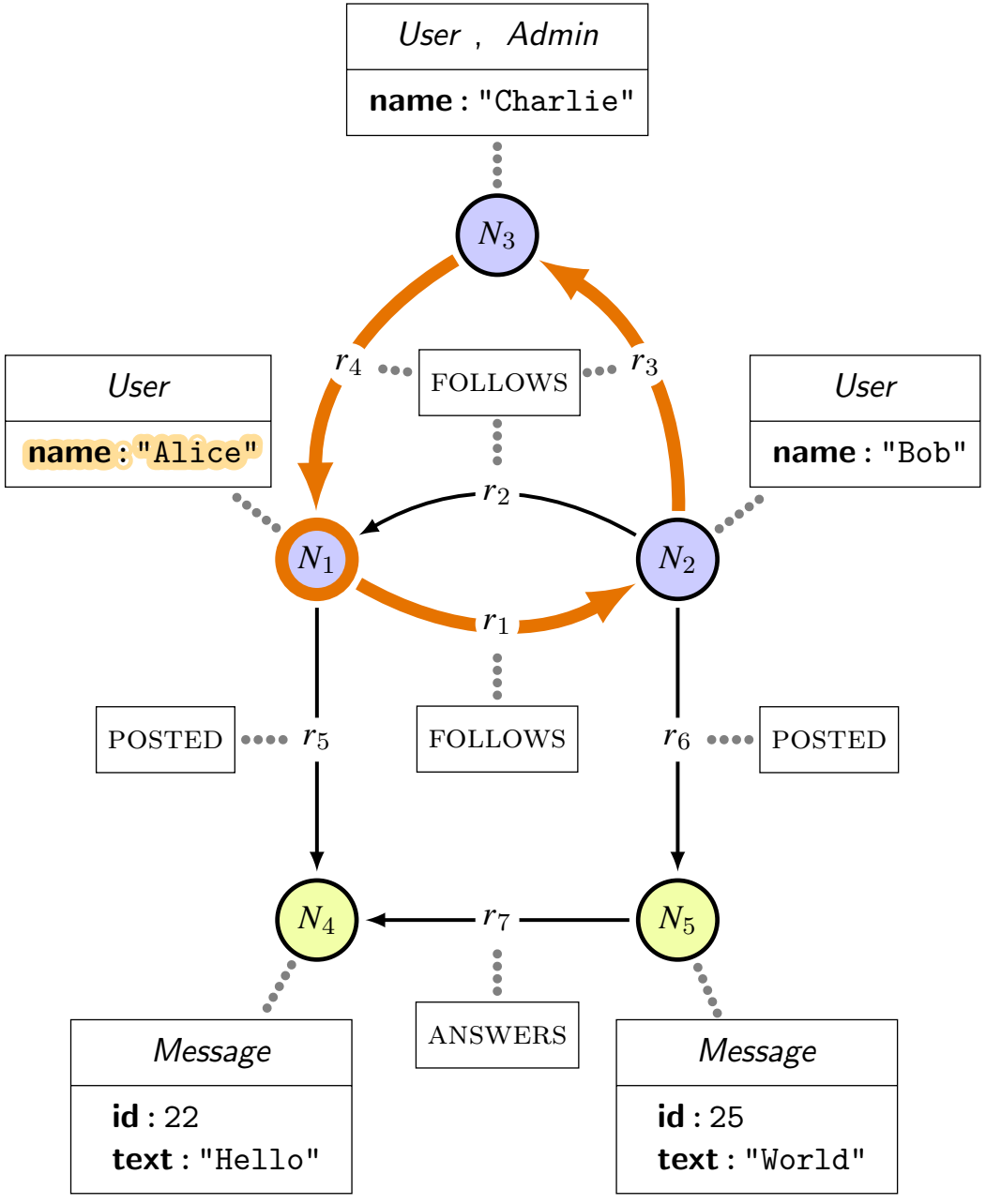
Property

A path has only one way to satisfy a rigid path pattern.

Rigid vs flexible path (1)



Query :

```
MATCH (x {name:Alice})
      -[:FOLLOWS*1..1]->(mid)
      -[:FOLLOWS*2..2]->(x)
```

The path

$$N_1 \xrightarrow{r_1} N_2 \xrightarrow{r_3} N_3 \xrightarrow{r_4} N_1$$

satifies the pattern.

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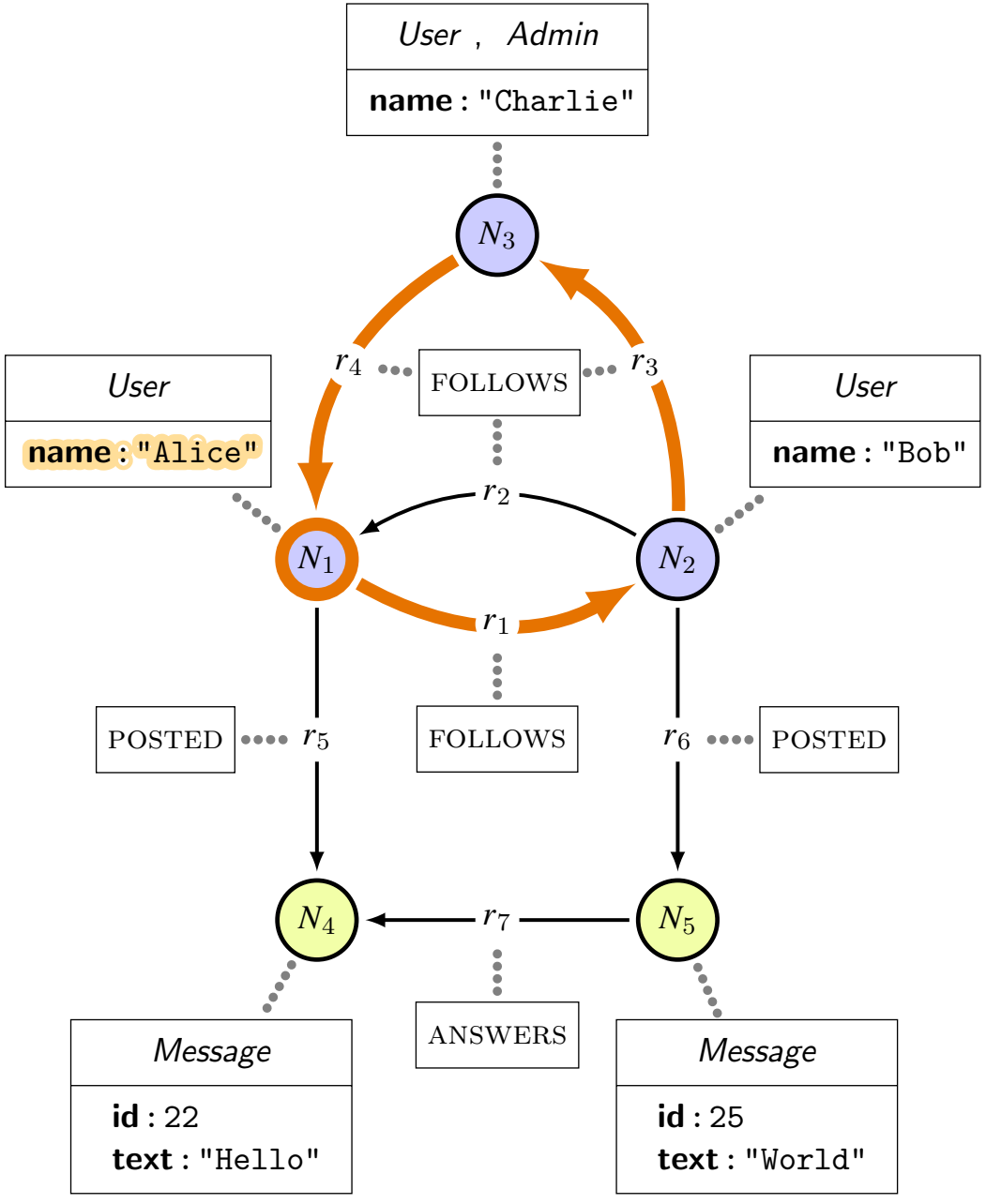

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The path

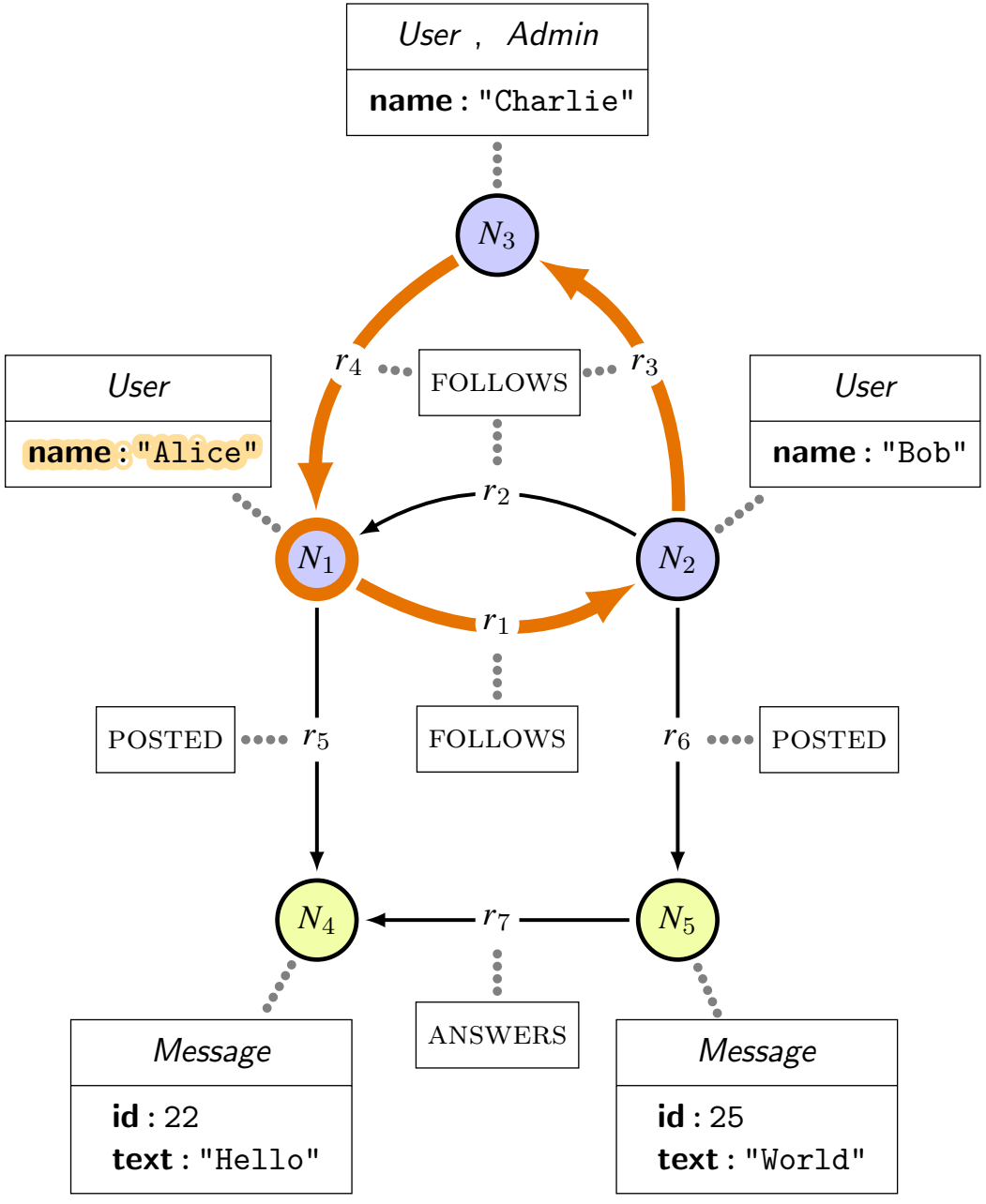
$$N_1 \xrightarrow{r_1} N_2 \xrightarrow{r_3} N_3 \xrightarrow{r_4} N_1$$

satifies the pattern.

Variables are uniquely set

x	mid
N_1	N_2

Rigid vs flexible path (2)



Query :

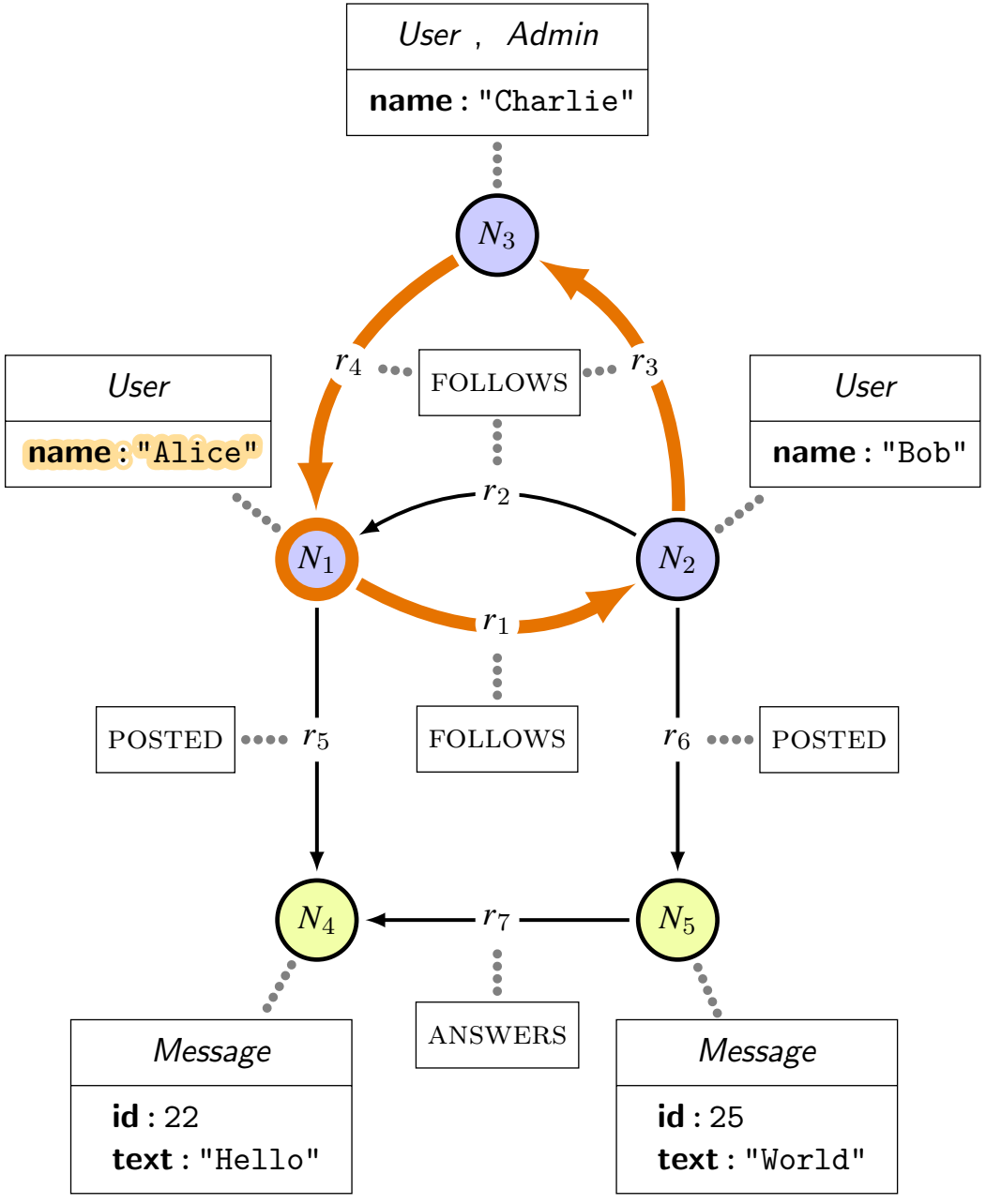
```
MATCH (x {name:"Alice"})
      -[:FOLLOWS*1..2]->(mid)
      -[:FOLLOWS*1..2]->(x)
```

The path

$$N_1 \xrightarrow{r_1} N_2 \xrightarrow{r_3} N_3 \xrightarrow{r_4} N_1$$

satifies this second pattern.

Rigid vs flexible path (2)



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```

The path

$$N_1 \xrightarrow{r_1} N_2 \xrightarrow{r_3} N_3 \xrightarrow{r_4} N_1$$

satifies this second pattern.

Two possible assignments

x	mid
N_1	N_2
N_1	N_3

Path satisfying a rigid path-pattern

u: a record
p: a path
π: a rigid path-pattern
n: length of π

Definition

$G, p, u \models \pi$ if

- p is of length n
- $\forall i \leq p$, the i-th node of p satisfies the i-th node pattern of π (variable, labels, properties)
- $\forall i < p$, the i-th relationship of p satisfies the i-th relationship pattern of π (variable, types, properties)

Rigid path-pattern subsumed by a flexible path-pattern

$\text{rigid}(\pi) = \text{set of all rigid patterns subsumed by } \pi$

Examples

`()-[*]->()` $\mapsto$ {`()-[*1..1]->()` , `()-[*2..2]->()` , $\cdots$}

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`()-[*2..3]-(b)` $\mapsto$ {`()-[*2..2]-(b)` , `()-[*3..3]-(b)`}

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rigid(π) = set of all rigid patterns subsumed by π

Examples

```
()-[*]->()  ↦ {()-[*1..1]->() , ()-[*2..2]->() , ···}
```

```
()-[*2..3]-(b)  ↦ {()-[*2..2]-(b) , ()-[*3..3]-(b)}
```

```
({name:"Alice"})-[*]->()-[:CITES*]->()  ↦
{({name:"Alice"})-[*1..1]->()-[:CITES*1..1]->() ,
({name:"Alice"})-[*1..1]->()-[:CITES*2..2]->() ,
({name:"Alice"})-[*2..2]->()-[:CITES*1..1]->() , ···}
```

Rigid path-pattern subsumed by a flexible path-pattern

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()-[*]->()  ↦  {()-[*1..1]->() , ()-[*2..2]->() , ···}
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({name:"Alice"})-[*2..2]->()-[:CITES*1..1]->() , ···}
```

WLOG: rigid(π) contains no pattern longer than the graph size

The MATCH clause

Semantics of MATCH

$$\llbracket \texttt{MATCH}\ \pi \rrbracket_G : \quad T \quad \longmapsto \quad \biguplus_{u \in T} \mathsf{expand}_{G,\pi}(u)$$

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Table-row expansion

π: a path-pattern
G: a graph

$$\mathsf{expand}_{G,\pi} : \quad \mathsf{Records} \longrightarrow \mathsf{Tables}$$

$$u \quad \longmapsto \quad \biguplus_{\substack{\text{paths } p \text{ in } G \\ \text{patterns } \pi' \text{ in } \mathrm{rigid}(\pi)}} \left\{ \text{records } v \;\middle|\; \begin{array}{l} \bullet\ \mathrm{dom}(v) = \mathrm{dom}(u) \cup \mathrm{var}(\pi') \\ \bullet\ v \text{ extends } u \\ \bullet\ G, p, v \models \pi' \end{array} \right\}$$

Outline

The core fragment

Queries are essentially sequences of clauses

⟨query⟩ :== ⟨query⟩ `UNION` [`ALL`]$^{?}$ ⟨query⟩

| [⟨clause⟩]* ⟨return⟩

A clause is a main statement followed by subclauses

⟨clause⟩ :== ⟨match⟩ [⟨where⟩]$^{?}$

| ⟨with⟩ [⟨where⟩]$^{?}$

| ⟨unwind⟩

Caveat on the MATCH clause (1)

Syntax

⟨match⟩ ::= [OPTIONAL]$^{?}$ MATCH ⟨pattern_tuple⟩

⟨pattern_tuple⟩ ::= ⟨pattern⟩ [, ⟨pattern⟩]*

- MATCH may search for a tuple of path-pattern
  - Cypher-morphism applies to the whole tuple
- MATCH may be "optional" :
  - if MATCH succeeds → no changes
  - if MATCH fails → free variables are set to null instead of deleting the line.

Caveat on the MATCH clause (2)

Semantics of MATCH

$\bar{\pi} = (\pi_1, \pi_2, \ldots, \pi_n)$: a path-pattern tuple
G: a graph

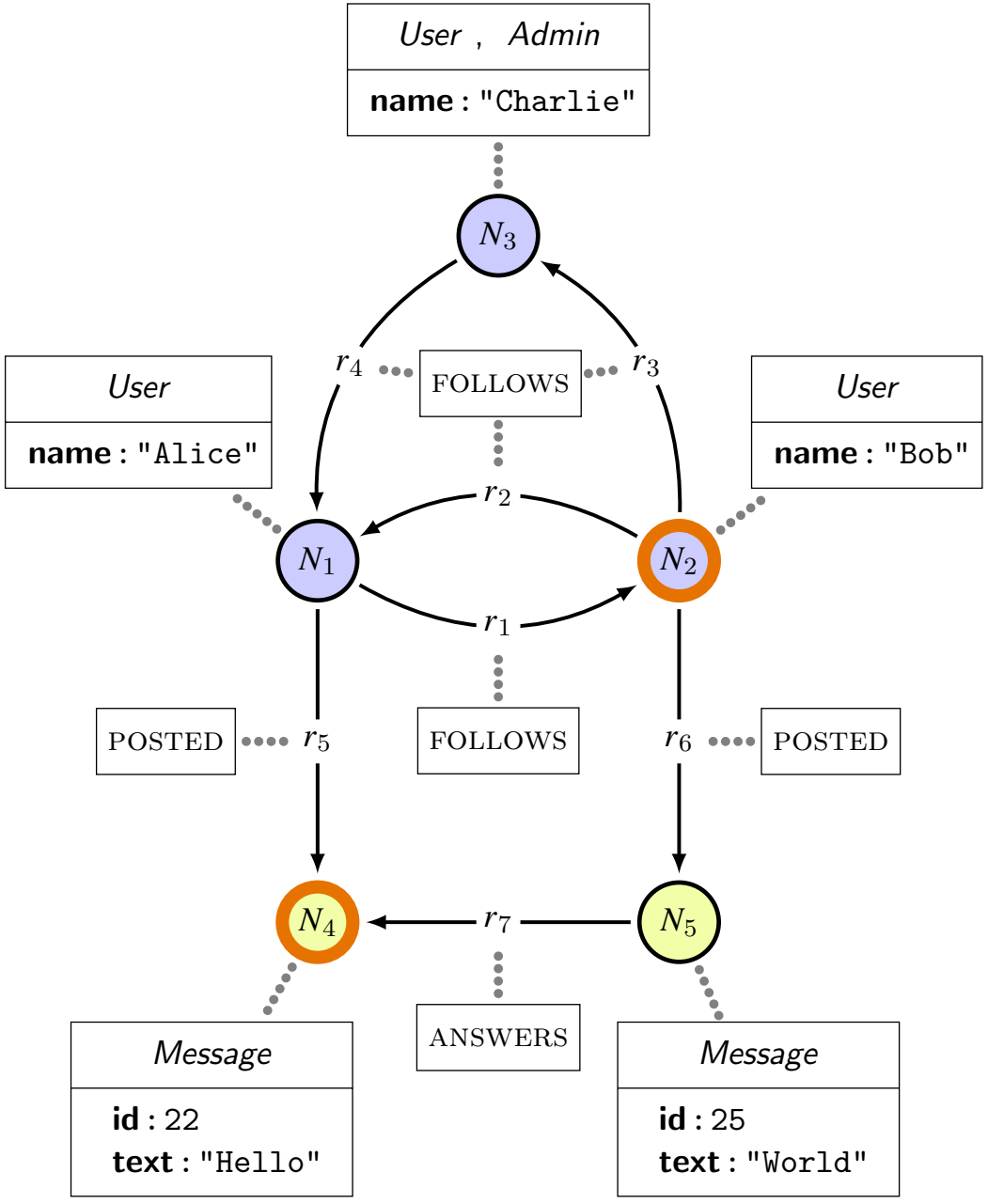
$$\text{rigid}(\bar{\pi}) = \text{rigid}(\pi_1) \times \text{rigid}(\pi_2) \times \cdots \times \text{rigid}(\pi_n)$$

$$\text{expand}_{G,\bar{\pi}} : \quad \text{Records} \longrightarrow \text{Tables}$$

$$u \quad \longmapsto \quad \biguplus_{\substack{\text{paths } \bar{p} \text{ in } G \\ \text{patterns } \bar{\pi}' \text{ in rigid}(\pi)}} \left\{ \text{records } v \;\middle|\; \begin{array}{l} \bullet\ \text{dom}(v) = \\ \qquad \text{dom}(u) \cup \text{var}(\bar{\pi}') \\ \bullet\ v \text{ extends } u \\ \bullet\ G, \bar{p}, v \models \bar{\pi}' \end{array} \right\}$$

$$\llbracket \texttt{MATCH } \bar{\pi} \rrbracket_G : \quad T \quad \longmapsto \quad \biguplus_{u \in T} expand_{G,\pi}(u)$$

Example of OPTIONAL MATCH



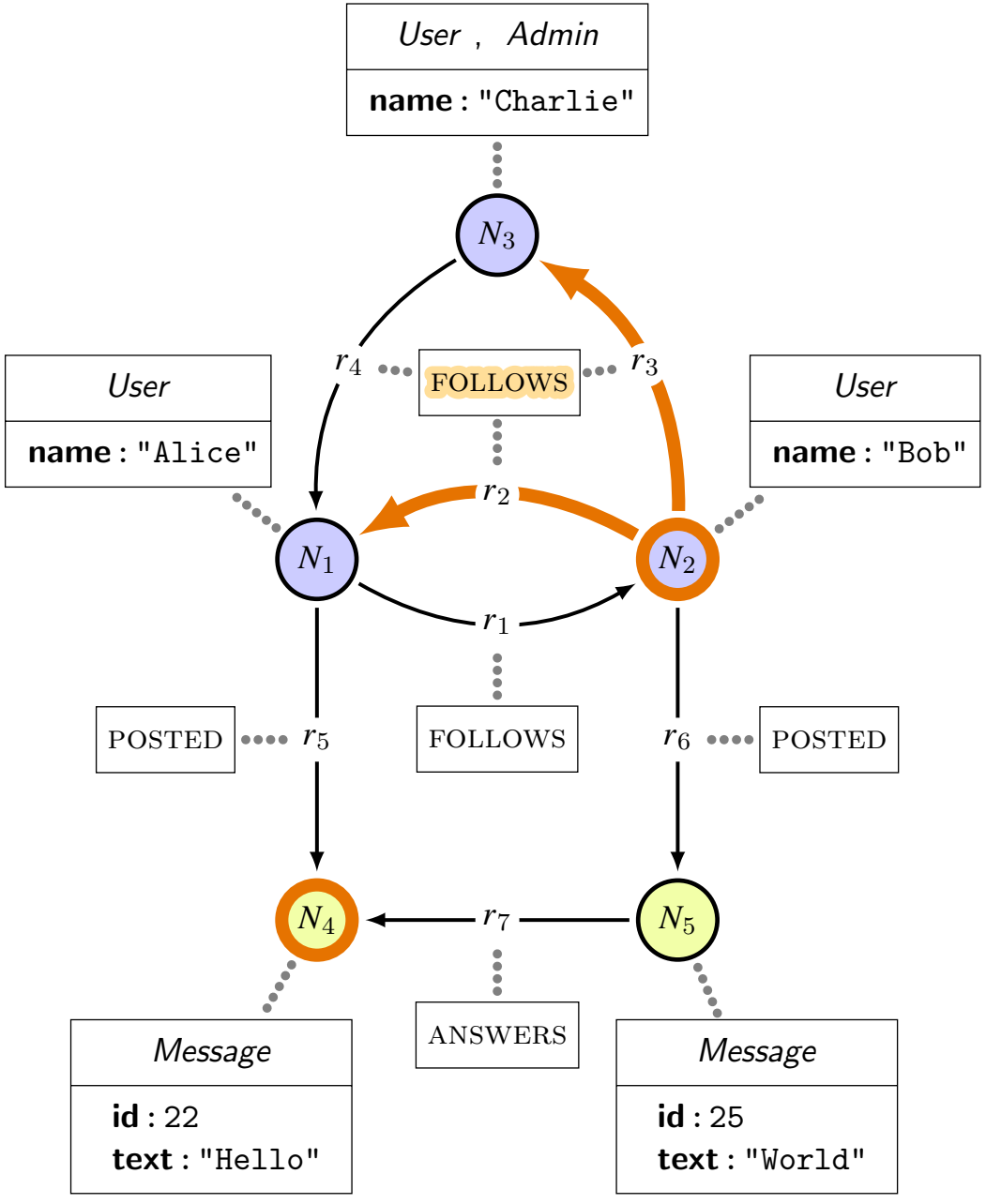
x
N_2
N_4

↓

Query :

```
OPTIONAL MATCH
        (x)-[y:FOLLOWS]->()
```

Example of OPTIONAL MATCH



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N_4

↓

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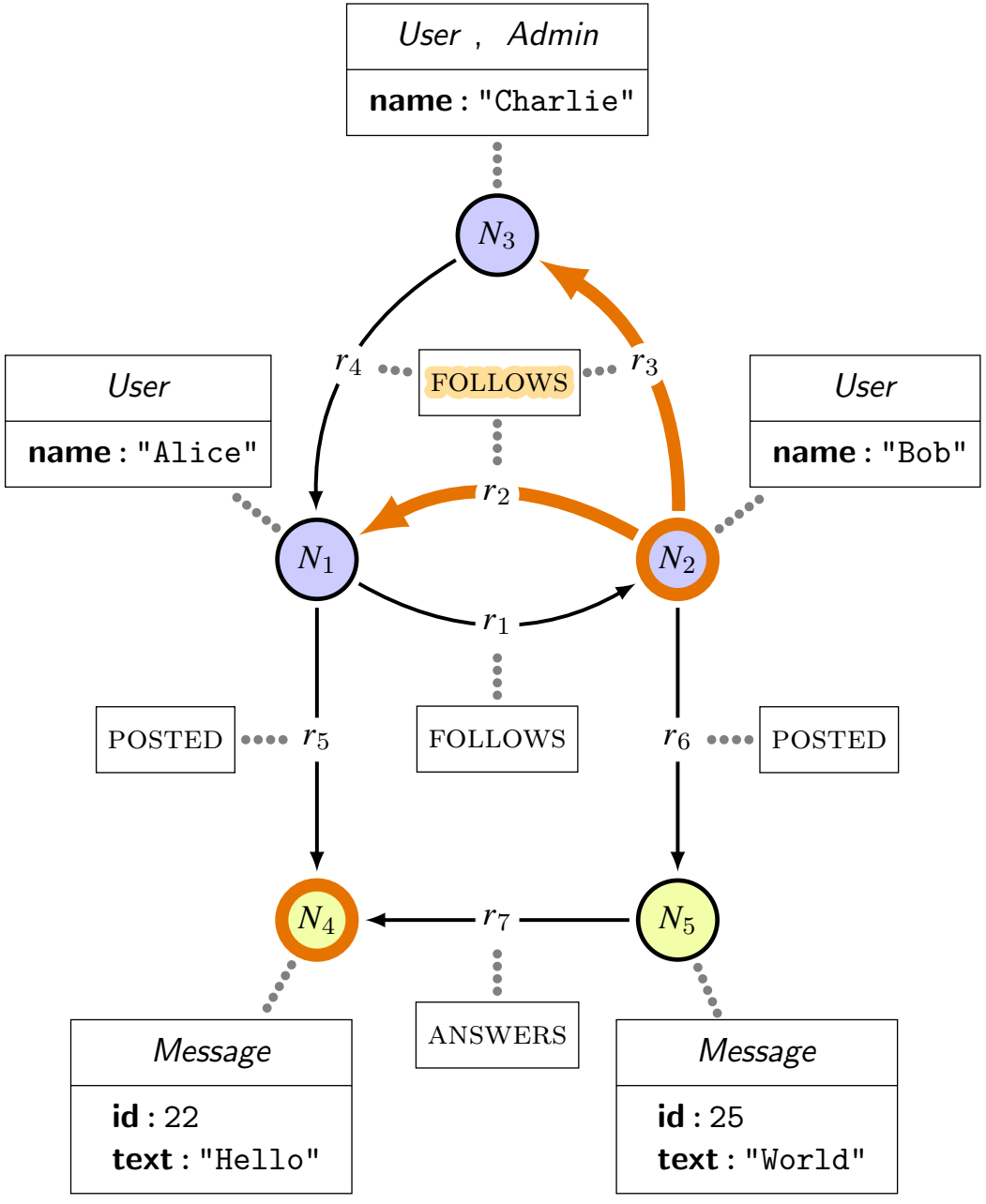

x
N_2
N_4

↓

Query :

```
OPTIONAL MATCH
        (x)-[y:FOLLOWS]->()
```

↓

x	y
N_2	r_2
N_2	r_3
N_4	null

Caveat on the MATCH clause (3)

Semantics of OPTIONAL MATCH

u: a record

$\bar{\pi}$: a path-pattern tuple

compl$(u, \bar{\pi})$ is the record:

- $\text{dom}\Big(\text{cmpl}(u, \bar{\pi})\Big) = \text{dom}(u) \cup \text{var}(\bar{\pi})$
- $a \longmapsto \begin{cases} u(a) & \text{if } a \in \text{dom}(u) \\ \texttt{null} & \text{otherwise} \end{cases}$

$$\Big[\!\Big[\texttt{OPTIONAL MATCH } \bar{\pi} \Big]\!\Big]_G(T) = \biguplus_{u \in T} \begin{cases} T_u & \text{if } T_u \neq \emptyset \\ \{\text{cmpl}(u)\} & \text{otherwise} \end{cases}$$

where $T_u = \Big[\!\Big[\texttt{MATCH } \bar{\pi} \Big]\!\Big]_G(\{u\})$

The WHERE subclause (1)

Syntax

$$\langle \text{where} \rangle ::= \quad \texttt{WHERE} \quad \langle \text{expr} \rangle$$

Semantics

$$[\![\texttt{WHERE}\ e]\!] : \quad T \quad \longmapsto \quad \biguplus_{u \in T} \begin{cases} \{u\} & \text{if } [\![e]\!]_{G,u} = \texttt{true} \\ \emptyset & \text{otherwise} \end{cases}$$

The WHERE subclause (2)

Combined semantics

- $\left[\!\left[\texttt{MATCH}\ \bar{\pi}\ \texttt{WHERE}\ e\right]\!\right]_G(T) = \left[\!\left[\texttt{WHERE}\ e\right]\!\right]\left(\left[\!\left[\texttt{MATCH}\ \bar{\pi}\right]\!\right]_G(T)\right)$
- $\left[\!\left[\texttt{WITH}\ \bar{\pi}\ \texttt{WHERE}\ e\right]\!\right]_G(T) = \left[\!\left[\texttt{WHERE}\ e\right]\!\right]\left(\left[\!\left[\texttt{WITH}\ \bar{\pi}\right]\!\right]_G(T)\right)$
- $\left[\!\left[\texttt{OPTIONAL MATCH}\ \bar{\pi}\ \texttt{WHERE}\ e\right]\!\right]_G(T)$

$$= \biguplus_{u \in T} \begin{cases} T_u & \text{if } T_u \neq \emptyset \\ \{\mathsf{cmpl}(u)\} & \text{otherwise} \end{cases}$$

where $T_u = \left[\!\left[\texttt{MATCH}\ \bar{\pi}\ \texttt{WHERE}\ e\right]\!\right]_G(\{u\})$

(and $\mathsf{cmpl}(u, \bar{\pi})$ is the completion of u w.r.t. variables of $\bar{\pi}$)

The WITH clause (1)

Syntax

⟨with⟩ ::=

| WITH ⟨expr⟩ [AS ⟨name⟩]? , ⋯, ⟨expr⟩ [AS ⟨name⟩]?

| WITH * , ⟨expr⟩ [AS ⟨name⟩]? , ⋯, ⟨expr⟩ [AS ⟨name⟩]?

- Each expression/name pair will be one column in the output table.
- If the name is missing, a default name is provided (implementation dependent).
- * means "every column previously in the table".

The WITH clause (2)

Example

a	b	c
0	1	2
1	1	1
2	1	0

↓

```
WITH   *,   a+b,   a<c AS order
```

↓

a	b	c	‘a+b‘	order
0	1	2	1	true
1	1	1	2	false
2	1	0	3	false

The WITH clause (3)

Semantics

- $\left[\!\left[\texttt{WITH } * \right]\!\right]_G(T) = T$
- $$\left[\!\left[\texttt{WITH } e_1 \texttt{ AS } a_1, \ldots, e_m \texttt{ AS } a_m \right]\!\right]_G(T) = \biguplus_{u \in T} \left\{ (a_1 : \left[\!\left[e_1 \right]\!\right]_{G,u}, \ldots, a_m : \left[\!\left[e_m \right]\!\right]_{G,u}) \right\}$$
- $$\left[\!\left[\texttt{WITH } e_1 \texttt{ [AS } a_1\texttt{]?}, \ldots, e_m \texttt{ [AS } a_m\texttt{]?} \right]\!\right]_G(T) = \left[\!\left[\texttt{WITH } e_1 \texttt{ AS } a'_1, \ldots, e_m \texttt{ AS } a'_m \right]\!\right]_G(T)$$

  where a'_i equals a_i if it is given, or arbitrary otherwise.
- $\left[\!\left[\texttt{WITH } *, \ldots \right]\!\right]_G(T) = \left[\!\left[\texttt{WITH } b_1 \texttt{ AS } b_1, \ldots, b_q \texttt{ AS } b_q, \ldots \right]\!\right]_G(T)$

  where $b_1, \ldots, b_q$ are the column names of T

The UNWIND clause (1)

Syntax

⟨unwind⟩ ::= UNWIND ⟨expr⟩ AS ⟨name⟩

- Given expression is supposed to be evaluated to lists
- Each line of the input table is expanded as multiple lines in the output, one for each element of the list.

The UNWIND clause (2)

Example

n	*list*
0	["Hello","World"]
1	["singleton"]
2	"not_a_list"

↓

```
UNWIND list AS x
```

↓

n	*list*	x
0	["Hello","World"]	"Hello"
0	["Hello","World"]	"World"
1	["singleton"]	"singleton"
2	"not_a_list"	"not_a_list"

The UNWIND clause (3)

Semantics

$$\left[\!\left[\texttt{UNWIND}\ e\ \texttt{AS}\ a\right]\!\right]_G(T) \quad = \quad \biguplus_{u \in T} \biguplus_{v \in E_u} \{(u, a : v)\}\ ,$$

$$\text{where} \quad E_u = \begin{cases} \biguplus_{0 \le i < m} \{v_i\} & \text{if } \left[\!\left[e\right]\!\right]_{G,u} = \text{list}(v_1, \ldots, v_m) \\ \{\} & \text{if } \left[\!\left[e\right]\!\right]_{G,u} = \text{list}() \\ \left\{\left[\!\left[e\right]\!\right]_{G,u}\right\} & \text{otherwise} \end{cases}$$

The RETURN clause

RETURN and WITH clauses have the exact same semantics.

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RETURN clause is the "end-marker" of a clause sequence:

- it is mandatory;
- it cannot appear earlier.

UNION and UNION ALL

Syntax

⟨query⟩ ::= ⟨query⟩ UNION [ALL]? ⟨query⟩

- The left and right queries are assumed to have the same column-tables.
- UNION is the set-union of tables.
- UNION ALL is the bag-union of tables.

UNION and UNION ALL

Syntax

$$\langle \text{query} \rangle ::= \quad \langle \text{query} \rangle \ \texttt{UNION} \ [\texttt{ALL}]? \ \langle \text{query} \rangle$$

- The left and right queries are assumed to have the same column-tables.
- `UNION` is the set-union of tables.
- `UNION ALL` is the bag-union of tables.

Semantics

- $\left[\!\left[Q_1 \ \texttt{UNION ALL} \ Q_2 \right]\!\right]_G(T) = \left[\!\left[Q_1 \right]\!\right]_G(T) \cup \left[\!\left[Q_2 \right]\!\right]_G(T)$
- $\left[\!\left[Q_1 \ \texttt{UNION} \ Q_2 \right]\!\right]_G(T) = \text{distinct}\left(\left[\!\left[Q_1 \right]\!\right]_G(T) \cup \left[\!\left[Q_2 \right]\!\right]_G(T) \right)$

Outline

Aggregation (1)

Principle

- Group lines according to some criteria
- Some column of the output table contains aggregated values.

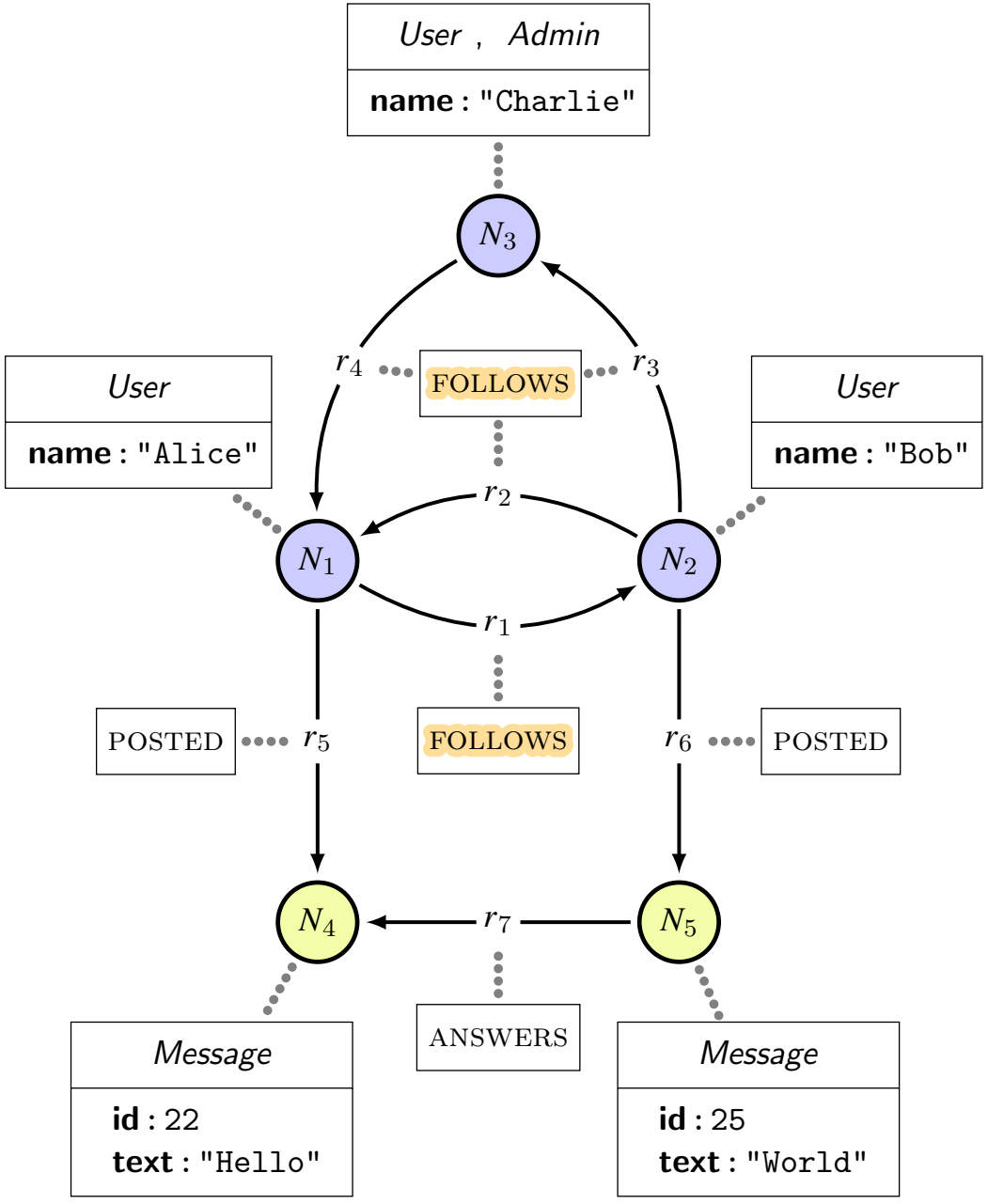
Aggregation functions

$\mathcal{G}$: a set of functions that map value bags to single values

E.g., `count`, `max` $\in \mathcal{G}$

- In `WITH` clauses, we allow *aggregate expression*:
  (*WITH* ⟨aggexpr⟩ [`AS` ⟨name⟩]?)
- Aggregate expressions are of the form: $g(\langle \text{expr} \rangle)$ where $g \in \mathcal{G}$

Aggregation (2) — Example



Query :

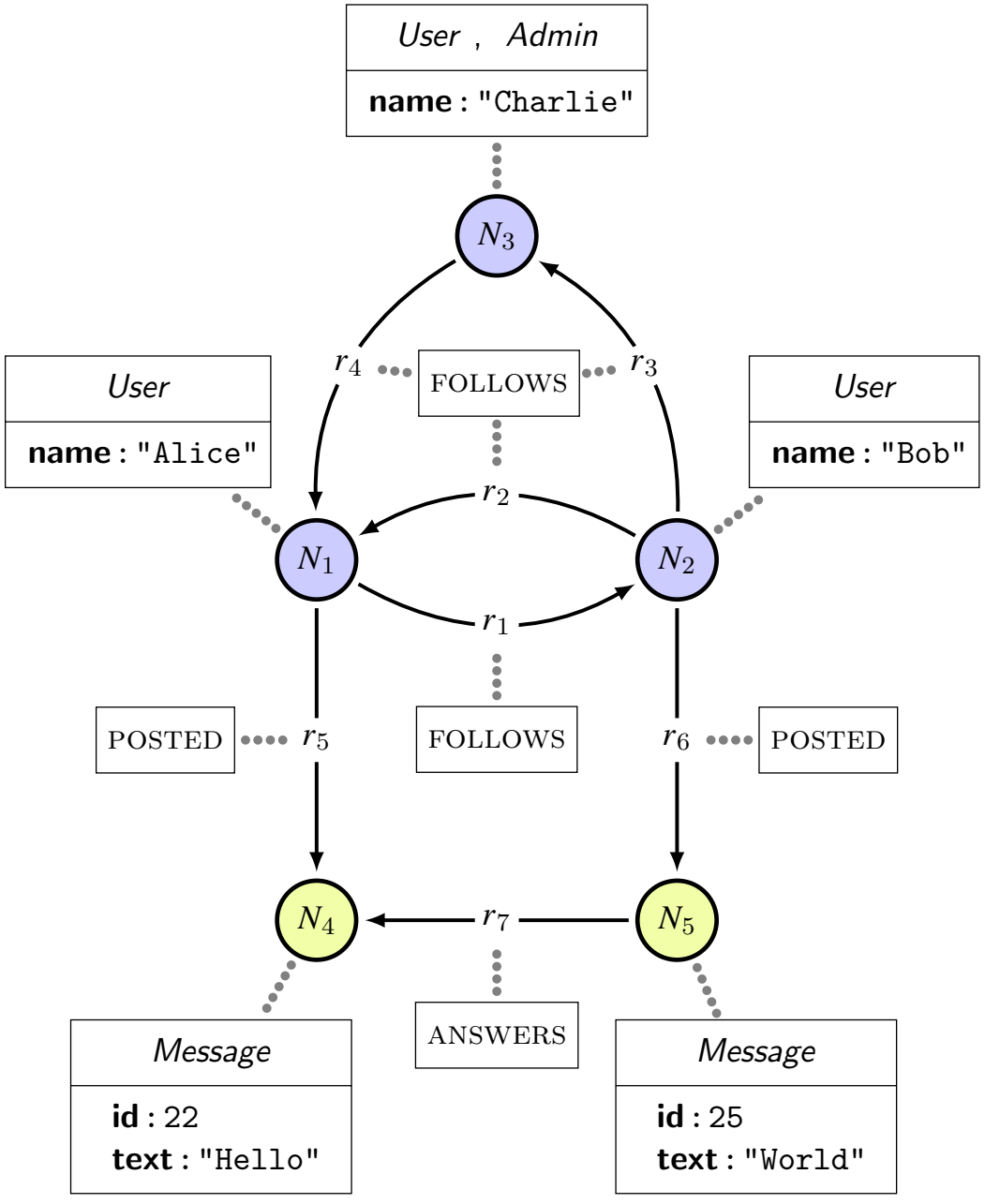
```
MATCH (a)-[r:FOLLOWS]->()
WITH a.name AS b, count(r) AS c
```

Aggregate

- over expression `a.name`
- and count them

b	c
"Alice"	1
"Bob"	1
"Charlie"	2

Aggregation (3) — Example



Query :

```
MATCH (a)-[r:FOLLOWS]->()
WITH a.name AS b, count(r) AS c
WITH max(c) AS d
```

d
2

Aggregation (4)

Semantics on an example

Example: WITH e_1 AS a_1, e_2 AS a_2, $max(e_3)$ AS a_3

e_1, e_2 and e_3 are ⟨expr⟩
$max(e_3)$ is an ⟨aggexpr⟩

We group the lines according to the expressions e_1, e_2:

- We partition the input table as subtables $T_1, \ldots, T_k$
- For each $u, v \in T_i$, $[\![e_j]\!]_{G,u} = [\![e_j]\!]_{G,v}$ (for $j \in \{1, 2\}$)

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- For $i < k$, let $V_i = \left\{ \llbracket e_3 \rrbracket_{G,v} \middle| v \in T_i \right\}$ and let $u \in T_i$.

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The output table has 3 columns and k lines; the i-th line is:

- For $i < k$, let $V_i = \left\{ \left[\!\left[e_3 \right]\!\right]_{G,v} \middle| v \in T_i \right\}$ and let $u \in T_i$.
- The i-th line is $\left(a_1 : \left[\!\left[e_1 \right]\!\right]_{G,u},\ a_2 : \left[\!\left[e_2 \right]\!\right]_{G,u},\ a_3 : max(V_i) \right)$

Line order (1)

Principle

- `ORDER BY` clause: provides an order
- `LIMIT` subclause: keeps only the first lines of the table
- `SKIP` subclause: removes the first lines of the table

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- In tables, columns and lines are not ordered

- Semantics of queries
- Semantics of clauses
- Semantics of subclauses

} are functions from tables to tables

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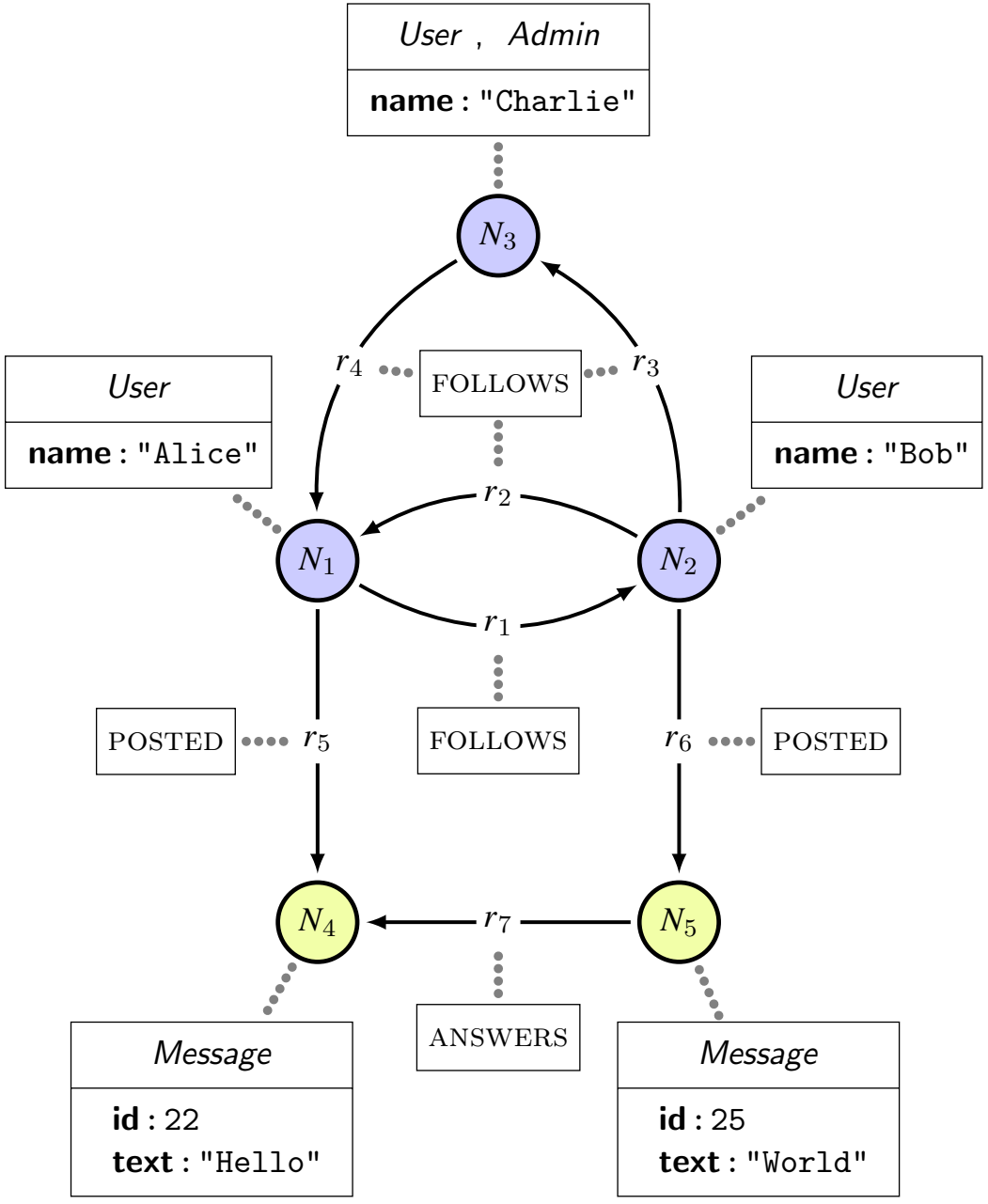
- In tables, columns and lines are not ordered
- Semantics of queries
- Semantics of clauses
- Semantics of subclauses

} are functions from tables to tables

From now on

- `ORDER BY` and its subclause propagate an order

Line order (2) — Semantics through example



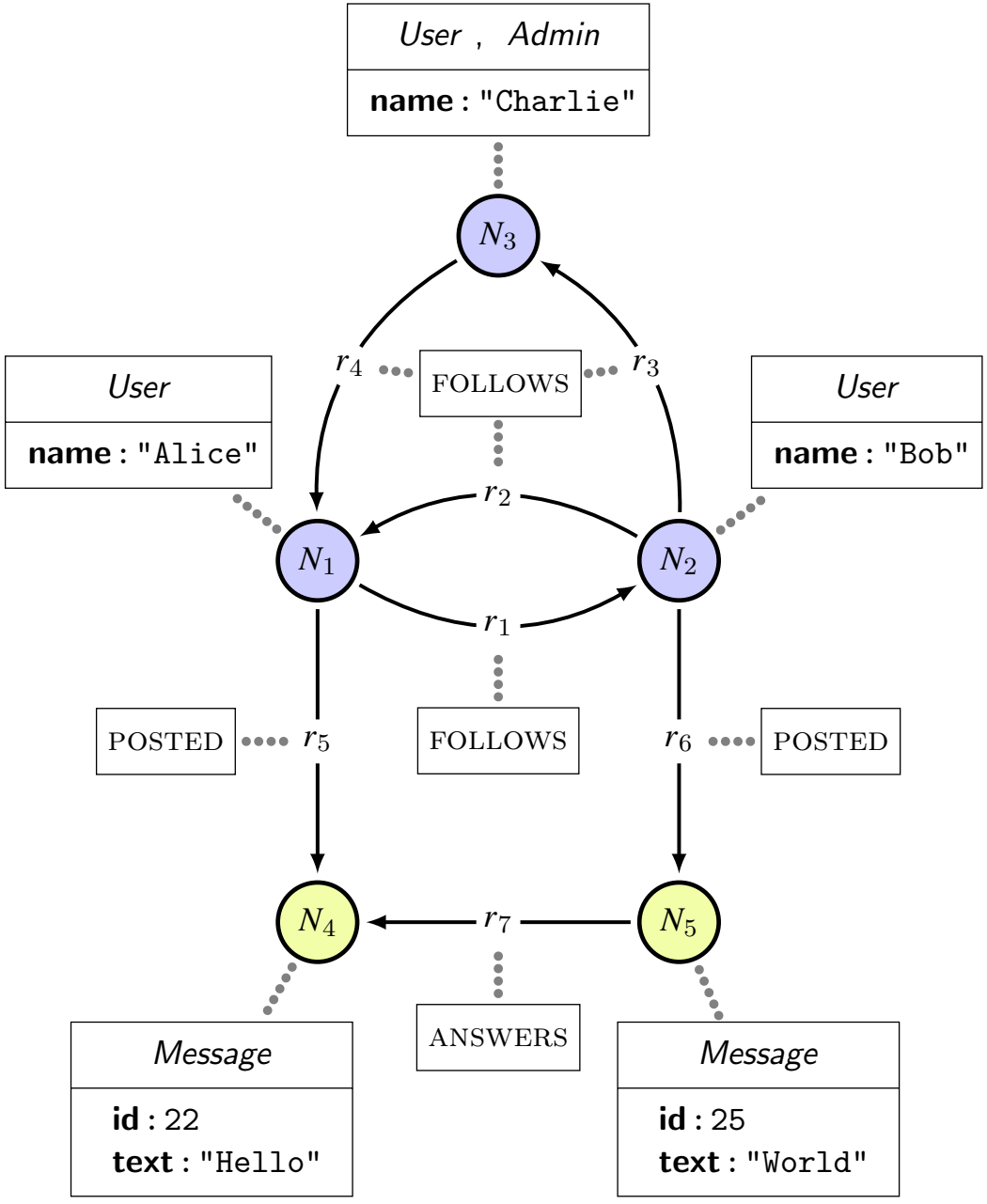
("Alice" < "Bob" < "Charlie")

Query :

```
MATCH (a:User)
ORDER BY a.name
  SKIP 1
  LIMIT 1
RETURN a.name AS b
```

b
"Bob"

Line order (3) — Semantics through example



(`false` < `true`)

Query :

```
MATCH (a:User)
ORDER BY a:Admin
  SKIP 1
  LIMIT 1
RETURN a.name AS b
```

In this case:

- Non-determinism
- 1 column and 1 line
- Cell contains either "Alice" or "Bob"

Outline

Conclusion

Cypher

- Relatively recent query-language for graph databases
- More and more used in industry
- Community-led development since 2015 (openCypher project)

Our goal

Full denotational semantic for Cypher.

Status

Core of the language	Done (SIGMOD'18)
Remainder of read-only part	In finalisation
Updates	Going well

Future work: semantics for updates

Features

- `CREATE` / `DELETE` clause: add/remove nodes or relationships.
- `SET` clause: change labels and properties.
- `MERGE` clause: "match else create".

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  E.g., `MATCH (a:Student) CREATE (a)-[]->(:Project)`
- Query evaluation is ACID ...supposedly
  (Atomicity, Consistency, Isolation, Durability)

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- `CREATE` / `DELETE` clause: add/remove nodes or relationships.
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  (Atomicity, Consistency, Isolation, Durability)

A few hiccups

- Inconsistent syntax accross the update clauses
- Inconsistent graph-state in the middle of transations
- Row-order dependence of `MERGE` and `SET`

⋮